# Earth Pressure Applied to Tunnel Due to the Settlement of Soft Clay

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SYNOPSIS In the case of design of underground structures at the coastal area of Tokyo Bay, it is very important to estimate the magnitude of earth pressure applied to the tunnel-like structures such as subways and other public facilities due to the settlement of newly reclaimed soft ground. In this paper, the deformation of soft clay ground and the earth pressure applied to the tunnel were examined using small model tests and finite element analyses based on Stokes' law. The additional pressure applied to the crown of tunnel due to the ground displacement was much greater than the value obtained by the conventional method. Consequently the finite element method based on Stokes' law was shown to be very useful to estimate the additional earth pressure applied to the underground structures due to the settlement of very soft clay whose vane shear strength is less than about 1.0(kN/m$^2$).

## INTRODUCTION

In the recent years many underground structures such as subways and other public facilities have been constructed in soft clay ground at the coastal area of Tokyo Bay, according to the increased demands of new office and residential space. Because most of the coastal area of Tokyo Bay was recently reclaimed, the consolidation settlement of newly reclaimed ground has not been completed yet. Moreover the consolidation settlement could occur due to the disturbance of soft clay associated with the construction work of underground structures, i.e. shield tunneling, excavation and piling.(Mori et al. 1985) Therefore it is very important in the case of design of underground structures at the coastal area of Tokyo Bay to estimate the magnitude of additional earth pressure applied to the tunnel-like underground structures due to the settlement of surrounding ground.
Practically the magnitude of earth pressure applied to the tunnel constructed in the consolidating soft ground was obtained by the method similar to the theory proposed by Spangler.(Spangler 1948; Tottori 1978) This conventional method assumed the two vertical slip planes above the circular tunnel. The additional earth pressure applied to the crown of tunnel was calculated by using the equilibrium equations with regard to the weight of soil prism separated by the two vertical slip planes and the sum of the shear strength of soil along the slip planes.

In this study, the deformation of soft clay and the magnitude of earth pressure applied to the tunnel were measured under the uniform relative displacement between tunnel and sur-

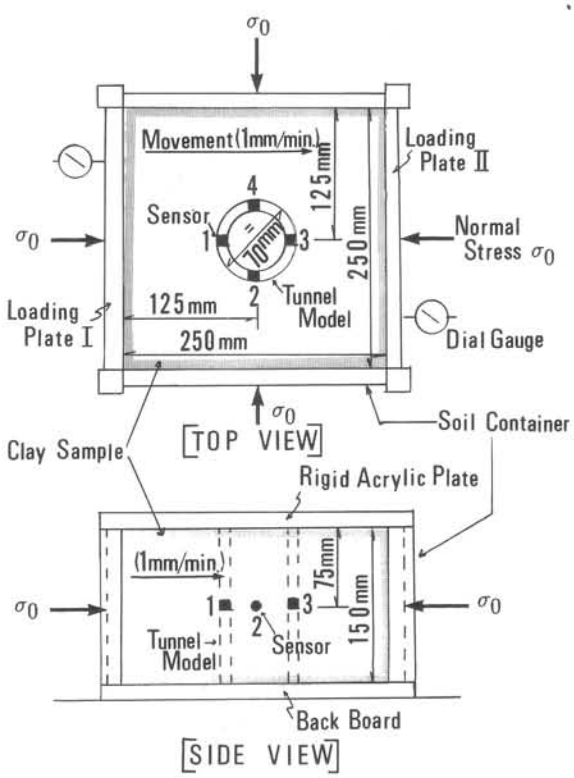


Fig.1 Laboratory Model Test Apparatus

rounding clay, using small model test apparatus. Noting the similarity of the situation examined in this study to the laminar flow of Newtonian liquid around the circular obstacle, the model test results were compared with the numerically calculated value based on Stokes' law using finite element method.

## EXPERIMENTAL INVESTIGATION

Schematic diagram of the small model test apparatus used in this experiment is shown in Fig.1. The model test apparatus is made of soil container(250mmx250mmx150mm) and tunnel model($\phi$=70mmx150mm). Clay sample was filled in the soil container and the initial pressure was applied to clay sample by the four loading plates using pneumatic pressure, in order to attach the loading plates to clay sample and make the stress state of clay sample around the tunnel model determinate. All values of the initial earth pressure measured by the four pieces of small pressure sensor embedded in tunnel model were 55(kN/m$^2$). The clay sample used in the experiments was prepared by mixing two types of commercial clay at various water contents. The physical properties of clay samples were as follows: $w_L$=123%, $w_P$=20%. The value of vane shear strength of clay sample $Sv$ was varied from 0.3(kN/m$^2$) to 4.3(kN/m$^2$) according to its water content between 151% and 72%.
Uniform relative displacement between tunnel model and adjacent soil took place by moving the loading plates I and II manually to the same direction at the rate of about 1(mm/min.), because the tunnel model was fixed to the back board of soil container. The magnitude of displacement of loading plate, i.e. the relative displacement between tunnel model and adjacent soil was measured by dial gauge. The change of the earth pressure applied to tunnel model during relative displacement was measured by the small pressure sensor.
The movement of loading plates was continued until the reading of each pressure sensor became almost constant and the final displacement of loading plate was about 3(mm). The deformation of clay sample took place under plane strain condition, because the rigid acrylic plate was attached to the cross section of tunnel model and clay sample. In order to reduce the adhesional force between clay and loading plate, teflon seat and rubber membrane with silicone grease were used between clay and loading plate.

## DEFORMATION OF SOFT CLAY AND EARTH PRESSURE APPLIED TO TUNNEL

Typical photographic view of the shape of the slip planes observed by the experiment is shown in Fig.2. The shape of the observed slip planes was different from the vertical ones assumed by the conventional method and resembled the streamline of viscous fluid around the circular obstacle. In Fig.3, the values of the earth pressure measured by the sensor 1, 2 and 3 are shown against the relative displacement between tunnel model and adjacent soil. Each of the pressure measured by the sensor 1, 2 and 3 corresponds to the earth pressure applied to the crown, the springline and the invert of tunnel respectively. The magnitude of earth pressure was increased up to the almost constant value in the

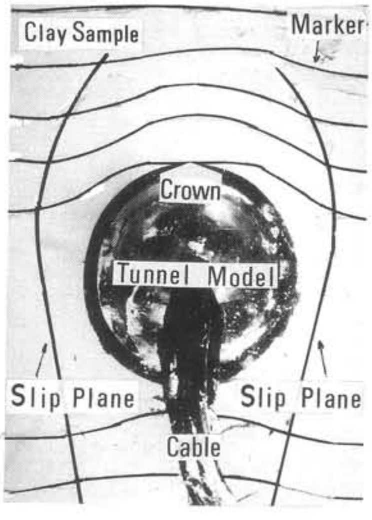


Fig.2 Slip Planes Observed by Model Test (Vane Shear Strength=0.9kN/m$^2$)

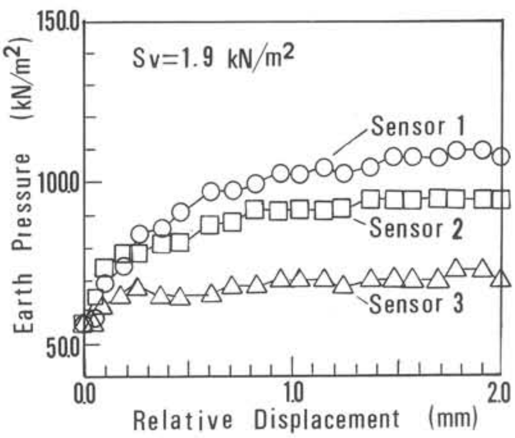


Fig.3 Change of Earth Pressure Applied to Tunnel Model Due to Ground Displacement (Vane Shear Strength=1.9kN/m$^2$)

range of the relative displacement greater than 1.5(mm) and the increase in earth pressure due to the movement of clay was greatest at the crown of tunnel model.

Fig.4 demonstrates the relations between the ratio of earth pressure increase(= (the difference between the final pressure and the initial pressure measured in the experiments)/(the initial pressure)) and the vane shear strength of clay samples. The ratio of earth pressure increase at sensor 1 and sensor 2 was increased in proportion to the logarithmic value of the vane shear strength of clay samples.

However the ratio of earth pressure increase at sensor 3 was decreased as the vane shear strength of clay sample became greater. Especially when the vane shear strength became greater than 2.8(kN/m$^2$), the ratio of earth pressure increase was less than zero, i.e. the earth pressure applied at the point of sensor 3 became smaller than the initial value due to the ground displacement.

According to the conventional method, the ratio of earth pressure increase $R$ at the crown of tunnel is obtained by the following equation.

$$R = \frac{2S_v \cdot d}{b \cdot \sigma_0} \qquad (1)$$

where $S_v$ is the vane shear strength of clay, $b$ is the distance between vertical slip planes, $d$ is the length of vertical slip plane and $\sigma_0$ is the initial pressure applied to the tunnel. Fig.5 shows the comparison between the ratio of earth pressure increase obtained by Eq.(1) and the experimental value measured by sensor 1. The calculated value of the ratio of earth pressure increase using conventional method is much smaller than the experimental.

## NUMERICAL ANALYSES BASED ON STOKES' VISCOUS FLUID THEORY

The shape of the slip planes shown in Fig.2 is similar to the streamline observed in the viscous fluid around the circular obstacle. Thus by using finite element method, a series of numerical analyses were carried out in order to examine the validity of the application of Stokes' viscous fluid theory to the estimation of change of earth pressure applied to tunnel. If the laminar flow of viscous fluid took place under the two-dimensional condition and the body force was ignored, Stokes' equations of motion are obtained as follows.

$$\frac{\partial p}{\partial x} = \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \qquad (2)$$

$$\frac{\partial p}{\partial y} = \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \qquad (3)$$

where $p$ is the pressure of liquid, $u$ is the velocity of $x$ direction, $v$ is the velocity of $y$ direction and $\mu$ is the coefficient of viscosity. And the equation of continuity is represented by the following equation.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (4)$$

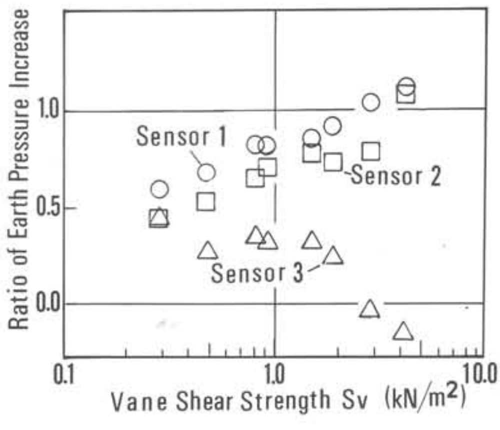


Fig.4 Ratio of Pressure Increase versus Vane Shear Strength of Clay Sample

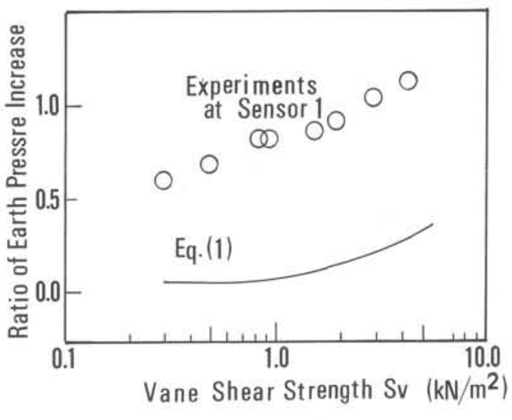


Fig.5 Comparison of the Ratio of Earth Pressure Increase

Because Stokes' viscous fluid theory makes use of Newton's law of viscosity for constitutive law, it is very important to confirm whether the relation between the shear stress and the rate of shear strain of the material satisfies the Newton's law of viscosity. There are some experimental methods for obtaining the relation between the shear stress and the rate of shear strain of the viscous material. In this study, this relation was obtained by measuring the resistant force applied to the steel sphere buried in soft clay, when the steel sphere moved upward at constant velocity in the gravitational field.
As shown in Fig.6, the steel sphere was hung through the rigid wire in clay sample contained in the mold. In order to measure the resistant force applied to the steel sphere, the other end of rigid wire was connected to the force transducer. Clay sample used in this experiment was the same as used in the small model test and the vane shear strength of clay sample was varied from 0.05($kN/m^2$) to 2.0 ($kN/m^2$). The mold moved downward at constant velocity $v$ and the resistant force applied to the steel sphere $F$ was measured, until the indicated value of force transducer became constant. In the series of the experiments, two types of the steel spheres were used, which had different radius($r$=15.08(mm) and 12.70(mm)). Two kinds of the velocity were adopted($v$=1.67x$10^{-5}$(m/s) and 8.33x$10^{-6}$(m/s)). The shear stress and the rate of shear strain $\dot{\gamma}$ were obtained by the following equations.(Mizuguchi et al.,1973)

$$\tau = \frac{F}{4\pi r^2} \qquad (5)$$

$$\dot{\gamma} = \frac{3v}{2r} \qquad (6)$$

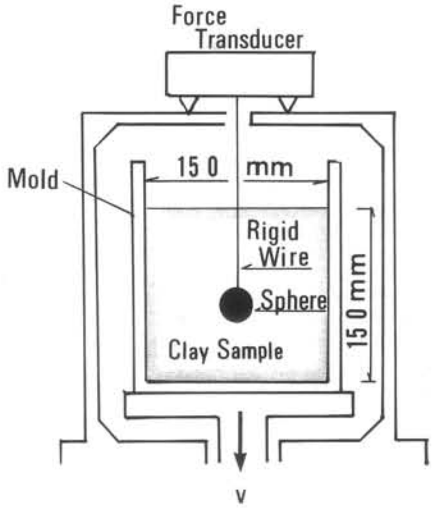


Fig.6 Determination of Viscous Properties

Fig.7 shows the relations between shear stress and the rate of shear strain. With regard to the clay samples whose vane shear strength are less than 1.1($kN/m^2$), the shear stress is in proportion to the rate of shear strain. The straight lines can be drawn which pass the origin and the gradients of these lines are different dependent on those vane shear strength. When the vane shear strength is 2.0($kN/m^2$), the straight line can be also drawn. However this straight line cannot pass the origin.
As a result, the clay samples whose vane shear strength are less than about 1.0($kN/m^2$) can be considered to satisfy the Newton's law of viscosity. Therefore the Stokes' viscous fluid theory can be applied for calculating the earth pressure applied to tunnel due to the settlement of soft clay, whose vane shear strength is less than about 1.0($kN/m^2$).
The coefficient of viscosity of clay used in the analyses can be obtained by the gradients of the straight lines representing the relations between the shear stress and the rate of

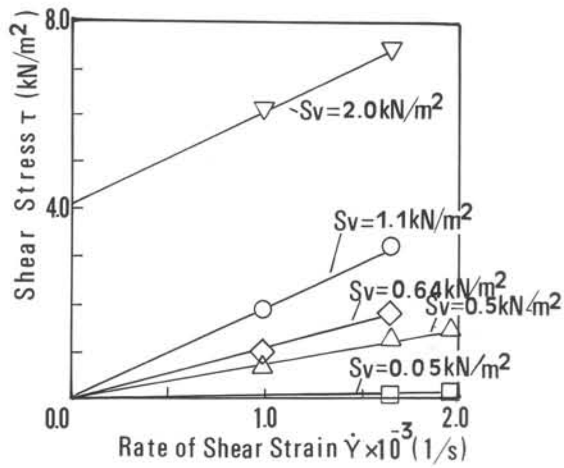


Fig.7 Change of Viscous Properties of Clay Samples According to those Vane Strength

shear strain shown in Fig.7. Fig.8 shows the relation between the coefficient of viscocity obtained from Fig.7 and the vane shear strength of clay. The coefficient of viscosity of clay sample $\mu$ is nearly in proportion to the value of vane shear srength and this relation is represented by the following equation.

$$\mu(kN/m^2 \cdot s) = 1680 \cdot Sv(kN/m^2) \quad (7)$$

where $Sv$ is the vane shear strength of clay.

In these analyses, the coefficients of viscosity of clays were obtained by Eq.(7). Finite element model used in these analyses is shown in Fig.9. This finite element model represents the cross-sectional view of the model test apparatus used in the experiment.
Fig.10 shows the comparison between the numerical results based on Stokes' viscous fluid theory and the model test results. The numerical analyses were carried out in the three cases of clay samples whose vane shear strength were less than about 1.0(kN/m$^2$). At the points of sensor 1 and sensor 2, the numerical results coincide fairly with the test results. It is noted again that the method of numerical analysis used in this study is only valid in the case of very soft clay whose vane shear strength is less than about 1.0(kN/m$^2$).

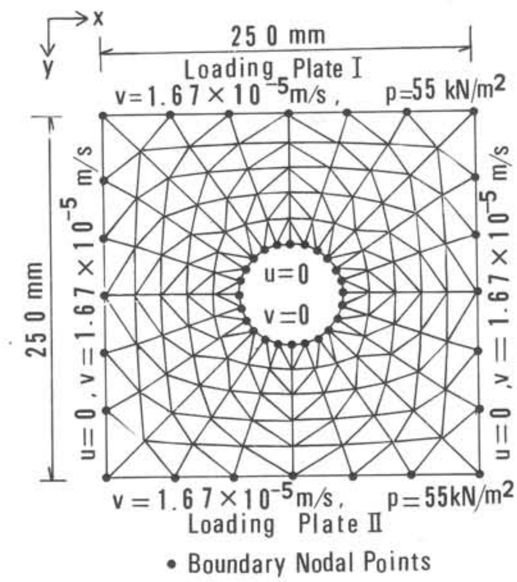


Fig.9 Finite Element Model

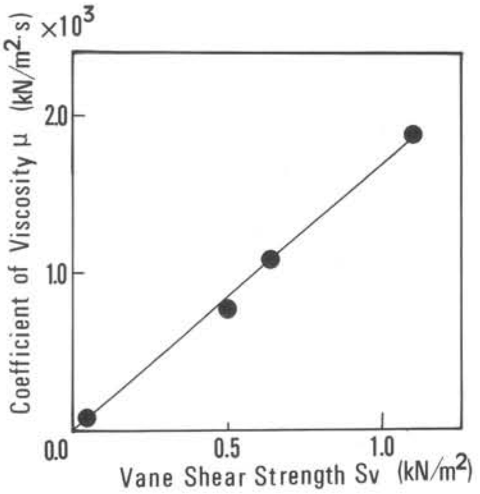


Fig.8 Relation between Coefficient of Viscosity and Vane Shear Strength of Clay Samples

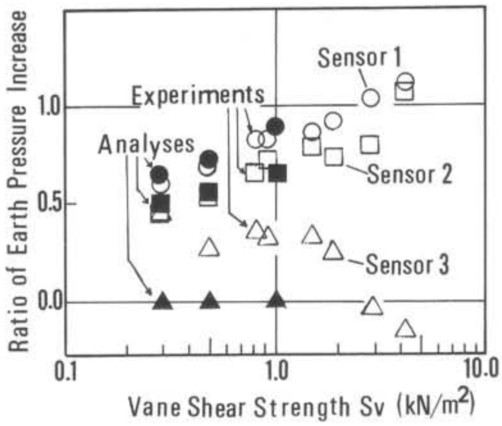


Fig.10 Comparison of Ratio of Pressure Increase Obtained by Numerical Analyses and Experiments

## CONCLUSION

In this study, the magnitude of earth pressure applied to the tunnel-like underground structures due to the uniform relative displacement between tunnel and surrounding ground were examined by small model tests and finite element analyses based on Stokes' viscous fluid theory. According to the results of these small model tests and finite element analyses, it is concluded that:

1. The shape of the observed slip planes due to the ground subsidence is different from the vertical ones assumed by the conventional theory and resembled the streamline of viscous fluid around the circular obstacle.

2. The ratio of earth pressure increase at the crown of tunnel becomes greater in proportion to the logarithmic value of the vane shear strength of clay. However the earth pressure applied to the invert of tunnel becomes smaller than its initial value especially in the case of clay whose vane shear strength is greater than 2.8($kN/m^2$).

3. When the vane shear strength of clay is less than about 1.0($kN/m^2$), the relations between the shear stress and the rate of shear strain of clay are assumed to satisfy the Newton's law of viscoity. In this extent, the earth pressure applied to the tunnel-like underground structures due to the settlement of soft clay can be estimated by the finite element analyses based on the Stokes' viscous liquid theory.

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