Freidlin-Gartner’s formula for general reaction terms

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Freidlin-Gartner’s formula expresses the asymptotic speed of spreading for spatial-periodic Fisher-KPP equations in terms of the principal eigenvalues of a family of linear operators. One cannot expect the same formula to hold true for other classes of reaction terms (monostable, combustion, bistable). However, these eigenvalues have been later related with the minimal speeds of pulsating travelling fronts, yielding a formula for the spreading speed which is not unreasonable to expect to hold beyond the KPP case. We present a new proof of the formula which applies to general reaction terms, and also provides partial results when the coefficients depend arbitrarily on time and space. This highlights a general connection between the asymptotic speed of spreading and almost planar transition fronts.

On an approximate scheme for a distance function of evolving interfaces

Nao Hamamuki
Hokkaido University

We show that viscosity solutions of a certain Hamilton-Jacobi equation approximate a distance function of evolving interfaces. A difficulty is that the distance function can be discontinuous with respect to time when the interface disappears. We establish convergence results by introducing a notion of a half-relaxed limit of solutions from below in time. This talk is based on a joint work with Eleftherios Ntovoris (Ecole des Ponts ParisTech).
A perturbation problem involving singular perturbations of domains for Hamilton-Jacobi equations

Taiga Kumagai
Waseda University

We consider the problem

\[
\begin{aligned}
\begin{cases}
    u^\varepsilon - \frac{b \cdot Du^\varepsilon}{\varepsilon} + |Du^\varepsilon| &= f & \text{in } \Omega, \\
    u^\varepsilon &= 0 & \text{on } \partial \Omega,
\end{cases}
\end{aligned}
\]

where \( \varepsilon \) is a positive parameter, \( \Omega \) is an open subset of \( \mathbb{R}^2 \), \( u^\varepsilon : \bar{\Omega} \to \mathbb{R} \) denotes the unknown function, \( f : \bar{\Omega} \to \mathbb{R} \) is a given, continuous and nonnegative function and \( b : \mathbb{R}^2 \to \mathbb{R}^2 \) is a Hamiltonian vector field.

We study the asymptotic behavior of solutions \( u^\varepsilon \) as \( \varepsilon \) goes to zero. The limit function is described as a solution of ODE on a graph. Freidlin-Wentzel, Freidlin-Weber, Sowers, by probabilistic techniques, and Ishii-Souganidis, by PDE techniques, studied stochastic perturbation problems for Hamiltonian flows. Our problem can be seen as a deterministic control version of such perturbation problems.

Monotone approximations to stationary mean-field games

Diogo A. Gomes
King Abdullah University of Science and Technology (KAUST)

We consider stationary monotone mean-field games (MFGs) and study the existence of weak solutions. We introduce a regularized problem that preserves the monotonicity and prove the existence of solutions to the regularized problem. Next, using Minty’s method, we establish the existence of solutions for the original MFGs. Finally, we examine the properties of these weak solutions in several examples.

A nonlinear parabolic equation with discontinuity in the highest order and applications

Qing Liu
Fukuoka University

We discuss a class of nonlinear parabolic equations with discontinuities of the sign function type in the second derivatives of the unknown function. We modify the definition of classical viscosity solutions and show uniqueness and existence of the solutions. These results are related to the limit behavior for the motion of a curve by a very small power of its curvature, which has applications in image processing. We also discuss the relation between our equation and the total variation flow in one space dimension.
A viscosity approach for fully nonlinear PDEs with fractional time derivatives

Atsushi Nakayasu
University of Tokyo

In this talk an initial value problem of abstract nonlinear equations with the Caputo fractional time derivative is considered from the viscosity point of view. We will construct a solution with suitable continuity in view of a priori estimate and study asymptotic behavior of the solution as the fractional order tends to one. This talk is based on a recent joint work in progress with Tokinaga Namba (Tokyo).

An estimate for boundary layers in Sobolev-Kato spaces

Christophe Prange
Université de Bordeaux

In this talk I will introduce a method to prove the existence to boundary layer systems in spaces of non localized energy of Sobolev regularity (so-called Sobolev-Kato spaces). The method will be applied to the system of linearized Ekman layers arising in the study of highly rotating fluids in rough domains. The talk is based on works with Anne-Laure Dalibard (University Paris 6) and Carlos Kenig (University of Chicago).

The generalized principal eigenvalue for viscous Hamilton-Jacobi equations with superquadratic Hamiltonians

Naoyuki Ichihara
Aoyama Gakuin University

This talk is concerned with the ergodic problem for superquadratic viscous Hamilton-Jacobi equations with exponent $m$. We prove that, as $m$ tends to infinity, the generalized principal eigenvalue of the equation converges to that of an ergodic problem with gradient constraint. We also investigate some qualitative properties of the generalized principal eigenvalue with respect to a perturbation of the potential function.
Asymptotic speed of solutions to level-set mean curvature flow equations with driving and source terms

Hiroyoshi Mitake
Hiroshima University

In the talk, I will propose a model equation to study the crystal growth, which is described by a level-set mean curvature flow equation with driving and source terms. We establish the well-posedness of solutions, and study the asymptotic speed. Interestingly, a new type of nonlinear phenomena in terms of asymptotic speed of solutions appears because of the double nonlinear effects coming from the surface evolution and the source term, which is sensitive to the shapes of source terms. This is a joint work with Y. Giga (U. Tokyo), and H. V. Tran (U. Wisconsin Madison).

Stochastic conservation laws

Panagiotis E. Souganidis
University of Chicago

I will present a recently developed theory for scalar conservation laws with nonlinear multiplicative rough signal dependence. I will describe the difficulties, introduce the notion of pathwise entropy/kinetic solution and its well-posedness. I will also talk about the long time behavior of the solutions as well as some regularization by noise type results.

On a Bernstein type theorem for some types of parabolic Hessian equations

Kazuhiro Takimoto
Hiroshima University

About a hundred years ago, Bernstein proved that if \( f = f(x, y) \in C^2(\mathbb{R}^2) \) and the graph of \( z = f(x, y) \) is a minimal surface in \( \mathbb{R}^3 \), then \( f \) is necessarily an affine function of \( x \) and \( y \). This theorem gives the characterization of entire solutions to the minimal surface equation in \( \mathbb{R}^2 \). For Monge-Ampère equation, the following result is known; if \( u \in C^4(\mathbb{R}^n) \) is a convex solution to \( \det D^2 u = 1 \) in \( \mathbb{R}^n \), then \( u \) is a quadratic polynomial. This result is called “Bernstein type theorem” for Monge-Ampère equation. In this talk, we shall obtain a Bernstein type theorem for the parabolic \( k \)-Hessian equation of the form

\[
  u_t = \mu \left( F_k(D^2 u)^{1/k} \right) \quad \text{in} \quad \mathbb{R}^n \times (-\infty, 0],
\]

where \( \mu : (0, \infty) \to \mathbb{R} \) is a function, under some assumptions. This talk is based on a joint work with Saori Nakamori (Hiroshima University).
Remarks on the comparison principle for fully nonlinear equations with no zeroth terms

Takahiro Kosugi
Tohoku University

We discuss a comparison principle for viscosity solutions of second-order quasilinear and fully nonlinear elliptic PDEs with no zeroth order terms

\[ F(Du, D^2u) = 0 \quad \text{in } \Omega \]

where \( \Omega \subset \mathbb{R}^n \) is a bounded domain. In 2001, G. Barles and J. Busca showed that the comparison principle holds for (1). We adapt a different transformation from that of Barles-Busca to enable us to deal with more general equations. This is a joint work with Prof. S. Koike in Tohoku University.

On a Gagliardo-Nirenberg type inequality for log-concave functions

Yasuhiro Fujita
University of Toyama

For \( 0 \leq \alpha \leq 1 \), let

\[ 1 \leq q, r, s \leq \infty, \quad 1/r = \alpha(1/s - 1/n) + (1 - \alpha)/q. \]

Then, Gagliardo-Nirenberg inequality states that, for any \( C^1(\mathbb{R}^n) \)-function \( f \) with compact support,

\[ \| f \|_r \leq C(n, p, q, r) \| f \|_q^{1-\alpha} \| Df \|_s^\alpha. \]

In this talk, we provide a Gagliardo-Nirenberg type inequality for log-concave functions on \( \mathbb{R}^n \). In this Gagliardo-Nirenberg type inequality, the parameter \( \alpha \) above can take a value in \([0, n]\). We explain how this fact is important.
Hölder continuity for subsolutions of integro-differential equations

Shigeaki Koike
Tohoku University

In 2010, I. Cazamppo Dolcetta, F. Leoni and A. Porretta showed a Hölder continuity for viscosity subsolutions of fully nonlinear elliptic PDE with super-quadratic first derivative terms. Typically, they study the following PDE in a domain of $\mathbb{R}^n$:

$$\lambda u - \Delta u + |Du|^m = f \quad \text{for } m > 2.$$ 

Our question is what will happen if we change the Laplacian by the fractional Laplacian.

$$\lambda u - \int \{u(\cdot + z) - u(\cdot) - 1_B(Du(\cdot), z)\} d\nu(z) + |Du|^m = f \quad m > \sigma,$$

where $1 < \sigma < 2$, and

$$\nu(z) \approx \frac{dz}{|z|^{n+\sigma}}.$$

This is a joint work with G. Barles, O. Ley and E. Topp.