Efficient Equilibrium in Common Interest Voting Games: A Note*

Kohei Kawamura† Vasileios Vlaseros‡

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Abstract

This note presents an alternative and unified derivation of an asymmetric pure strategy equilibrium and its optimality in the canonical common interest voting model of Austen-Smith and Banks (1996). We argue that the equilibrium points to a distinctive purpose and outcome of pre-voting deliberation. Some implications of the equilibrium with respect to the symmetric mixed strategy equilibrium commonly studied in the literature are also discussed.

Keywords: committee decision making, asymmetric equilibria, public information, private information, strategic voting
JEL Classification: C92, D72, D82

1 Introduction

Even for simple strategic voting games, it is customary in the literature to focus on the most efficient *symmetric* strategy equilibrium, not least because of the large strategy space and the presence of multiple equilibria. While Feddersen and Pesendorfer (1998) and Wit (1998) derived the efficient *symmetric* equilibrium in the canonical binary common interest voting model of Austen-Smith and Banks (1996), it has been well known that the most efficient equilibrium is in asymmetric pure strategies.

In the literature, Chakraborty and Ghosh (2003) showed that the efficient equilibrium features a certain number of agents always voting according to the private signal, and the

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†Graduate School of Economics, Faculty of Political Science and Economics, Waseda University. kkawamura@waseda.jp

‡Piraeus Bank and School of Economics and Political Sciences, National and Kapodistrian University of Athens. vlaseros@aueb.gr

rest always voting for one of the alternatives. Maug and Rydqvist (2008) explicitly derived an asymmetric pure strategy equilibrium in line with the strategy profiles, but they did not relate the equilibrium to the earlier observation by Chakraborty and Ghosh (2003) and hence did not establish the optimality.[1] As the approaches adopted by Chakraborty and Ghosh (2003) and Maug and Rydqvist (2008) are very different from each other, it has been difficult to relate the results and grasp the structure and intuition behind the asymmetric equilibrium and its optimality among all strategy profiles. Also, since the main focus of Maug and Rydqvist (2008) was on the application of their theoretical result to empirical corporate finance, the implications of the efficient asymmetric equilibrium with regards to the strategic voting literature have not been sufficiently explored.

This note offers a unified approach to the derivation of the asymmetric equilibrium and its optimality, so that the intuition can be much more clearly obtained relative to the earlier contributions. Specifically, we first pin down a class of pure strategy profiles the most efficient strategy profile must belong to, and subsequently derive the efficient asymmetric equilibrium by searching for the (unique) equilibrium strategy profile in the class. Throughout we consistently focus on each voter's incentive to deviate. Our derivation and presentation make it clear that, in the most efficient equilibrium, when a supermajority rule excessively favours an alternative that is ex ante more desirable, then some agents always vote *against* the alternative to offset the superfluous advantage imposed by the supermajority rule. Meanwhile, if the rule in place favours an ex ante more desirable alternative too little or even handicaps it, then some agents always vote for it regardless of their private signals.[2]

The asymmetry of the strategies that maximize the efficiency and a potentially large number of voters would call for some coordination before they vote. The efficient equilibrium points to a novel rationale for pre-voting deliberation, where the agents endogenously "correct" the exogenously given rule by deciding a specific number of agents who vote or against the alternative that is ex-ante desirable. For example, those agents may vote early before they learn their private information, which allows the other agents to simply vote according to their private signals.

We also resort to the clear comparative statics of the efficient asymmetric equilibrium to make comparison with the symmetric mixed equilibrium in terms of equilibrium strategies and efficiency. For example, it is well known that in a symmetric strategy equilibrium under unanimity, the probability of "convicting the innocent" ("acquitting the guilty) may become higher (lower) as the size of the committee/jury increases, and the expected payoff may be decreasing in the committee size. The efficient asymmetric pure strategy equilibrium, on the other hand, involves a fixed number of agents always voting

[1]Maug and Rydqvist (2008) also assume way the non-generic cases that involve multiple equilibria. We complete the equilibrium characterization and show that the equilibria feature the same efficiency.

[2]For example, a voting rule that handicaps an ex ante desirable outcome may be associated with situations where a supermajority is required to implement a promising reform, which is objectively more likely to be beneficial than an unsatisfactory status quo.

against the alternative the unanimity rule favours (i.e. they vote for "convict" regardless of their private information), in such a way that the number of informative votes and the probability of convicting the innocent and the expected payoff are not affected by the committee size.

In what follows, we present the model in Section 2, and derive the most efficient equilibrium in Section 3. We discuss interpretations of the equilibrium strategy profile in Section 4 and see some numerical examples in Section 5 in order to gain intuition behind the efficient equilibrium and compare it with the well-known symmetric mixed strategy equilibrium. Section 6 concludes.

2 Model

Consider a committee that consists of an odd number of strategic agents n. Each agent $i \in N \equiv \{1, 2, .., n\}$ simultaneously casts a costless binary vote, denoted by $x_i \in X = \{A, B\}$, for a collective decision with respect to the binary alternative $y \in Y = \{A, B\}$. The collective decision is determined by a k-majority rule favouring $y = A$, where $k \in \{0, 1, 2, ..., \frac{n-1}{2}\}$ and $\frac{n+1}{2} - k$ or more votes are required for the alternative A to be selected. Naturally, simple majority rule features $k = 0$, and unanimity rule features $k = \frac{n-1}{2}$.

The binary state of the world is denoted by $s \in S = \{A, B\}$. The agents have identical preferences $u_i : Y \times S \to \mathbb{R}$ and, specifically we denote the vNM payoff by $u_i(y, s)$ and assume $u_i(A, A) = u_i(B, B) = 1$ and $u_i(A, B) = u_i(B, A) = 0$ for any $i \in N$.[3] This implies that the payoff depends only on whether the committee decision matches the state.

Before voting, each agent has two pieces of information. One is a private signal about the state $\sigma_i \in K = \{A, B\}$, for which the probability of the signal and the state being matched is given by $\Pr[\sigma_i = A \mid s = A] = \Pr[\sigma_i = B \mid s = B] = p$, where $p \in (1/2, 1]$. We also have $\Pr[\sigma_i = A \mid s = B] = \Pr[\sigma_i = B \mid s = A] = 1 - p$. In addition to the private signal, all agents in the committee have a public signal $\sigma_0 \in L = \{A, B\}$. Specifically, we assume $\Pr[\sigma_0 = A \mid s = A] = \Pr[\sigma_0 = B \mid s = B] = q$ and $\Pr[\sigma_0 = A \mid s = B] = \Pr[\sigma_0 = B \mid s = A] = 1 - q$, where $q \in [1/2, 1]$. Throughout this paper we refer to the public signal as the "prior", since q can be interpreted as a common prior distribution of the state before the agents receive their private signals. The prior in the model can also be thought of as a posterior given both a common prior and an additional public signal. The agents do not communicate before they vote and are not allowed to abstain. We say that a k-majority rule *favours* the prior if $\sigma_0 = A$ and *handicaps* the prior if $\sigma_0 = B$.

Before investigating the efficient equilibrium let us make the following observation:

Fact. *The voting game described above under a k-majority rule is equivalent to the voting game under simple majority rule with $n + 2k$ agents where $2k$ agents deterministically vote for A and the rest vote strategically.*

[3] As Austen-Smith and Banks (1996) noted, the assumption of symmetric payoffs with respect to the state is for expositional convenience only.

Clearly, this transformation into the simple majority voting game preserves the original feature of k-majority rule that there are n strategic agents and $\frac{n+1}{2} - k$ or more votes are required for the alternative A to be selected. We resort to this observation throughout the paper to simplify the exposition.

3 Efficient Equilibrium

In what follows, we investigate the equilibrium of the transformed simple majority voting game with $n + 2k$ agents where $2k$ agents are non-strategic and vote for A regardless of the information they receive. In particular, we focus on a class of pure strategy profiles $M(n, k, c, d)$ for n strategic agents under k-majority rule, where all variables are non-negative integers, such that c agents consistently vote for the prior (i.e. regardless of the private signal), d agents consistently vote against the prior and $n - c - d$ agents vote consistently according to the private signal. In what follows, by voting for the prior (the private signal), we mean voting for the prior (the private signal) with probability 1 regardless of the other signal. Let us note that when the accuracy of the prior q is very high, the efficient equilibrium may well be such that a sufficient number of agents vote for the prior that the committee decision is σ_0.

Let us focus on the case where an efficient equilibrium is not deterministic so that the decision is B (i.e. against the prior) with positive probability. We first establish through the following lemmas that in such an efficient equilibrium, i) no agent's strategy is to vote against the private signal; and ii) the strategy profile must not involve both an agent whose strategy is to consistently vote for the prior and an agent whose strategy is to consistently vote against the prior. The latter implies "no vote must be wasted" since a vote for and a vote against the prior cancel each other and together they have no consequence on the final majority decision. Formally, as we will demonstrate shortly, the efficient equilibrium strategy profile has to feature $M(n, k, c, d)$ and moreover we must have $(c \geq 0) \wedge (d = 0)$ or $(c = 0) \wedge (d > 0)$.

Lemma 1. *The efficient strategy profile does not involve any strategy to vote against the private signal regardless of the prior.*

Proof. Let the probability that an arbitrary signal profile $\sigma_{-i} = \{\sigma_0, \sigma_1, ..., \sigma_{i-1}, \sigma_{i+1}, ..., \sigma_n\}$ is realized given the state is A be $r \equiv P(\sigma_{-i} \mid s = A)$. We have

$$P(s = A|\sigma_{-i}, \sigma_i = A) = \frac{rp}{rp + (1 - r)(1 - p)}, \tag{1}$$

$$P(s = A|\sigma_{-i}, \sigma_i = B) = \frac{r(1 - p)}{rp + (1 - r)(1 - p)}. \tag{2}$$

Since $p \in \big(1/2, 1]$, from (1) and (2) we have

$$P(s = A|\sigma_{-i}, \sigma_i = A) > P(s = A|\sigma_{-i}, \sigma_i = B). \tag{3}$$

for any realization of σ_{-i}.

Consider any combination of a strategy profile (except agent i's strategy) and a realization of all signals such that agent i is pivotal. Agent i either votes for or against the private signal. Suppose the agent votes for A and the majority decision is A. If the agent's private signal was A and thus voted for the private signal, then the ex post accuracy of the signal is denoted by $P(s = A \mid \sigma_{-i}, \sigma_i = A)$. Meanwhile, if the agent's private signal was B and thus voted against the the private signal, the ex post accuracy is $P(s = A \mid \sigma_{-i}, \sigma_i = B)$. Thus the accuracy with the pivotal vote by the agent is higher when he votes for the private signal. □

Lemma 2. *In the efficient equilibrium, the strategy profile must feature* $M(n, k, c, d)$*, and moreover we must have* $(c \geq 0) \wedge (d = 0)$ *or* $(c = 0) \wedge (d > 0)$.

Proof. From Lemma 1, unless the efficient equilibrium is obedient to the prior, there must be at least one agent who votes for the private signal. Consider any such agent. If the agent is pivotal, the probability that the majority decision is correct, conditional on the agent being pivotal, is p due to signal independence.

Let us add two extra agents who vote for the private signal to the committee. Those two agents change the outcome and hence the conditional accuracy of the majority decision a) when the majority decision is correct and both voters receive incorrect signals; and b) when the majority decision is incorrect and the two voters receive correct signals. The probability that a) occurs is $p(1-p)^2$. The probability that b) occurs is $(1-p)p^2$. Since $p > 1/2$, we have $(1-p)p^2 > p(1-p)^2$. Thus the two new voters increase the probability that the majority decision is correct, which implies that a strategy profile that involves a vote for the prior regardless of the private signal and a vote against the prior regardless of the private signal cannot be efficient. This is because those two votes cancel each other and do not affect the majority decision, while if the two votes are for their respective private signals, they strictly increase the accuracy of the majority decision. Thus we conclude that, in an efficient strategy profile, we must have $(c \geq 0) \wedge (d = 0)$ or $(d > 0) \wedge (c = 0)$. □

Definition. Let us define the following log odds ratio:

$$g \equiv \frac{ln(q) - ln(1-q)}{ln(p) - ln(1-p)}.$$

As we will see shortly, this ratio plays a key role in the optimal weight on the prior, in terms of the number of agents who vote for it. It is easy to see that g is positive under our assumptions on p and q, strictly increasing q and strictly decreasing in p. Also, if $q \geq p$ then $g \geq 1$. We will discuss the interpretation and intuition through simple examples in Section 5.

Proposition. *For the generic cases where* g *is not an integer. The efficient equilibrium of the model is uniquely characterized by non-negative integers* c *and* d *as follows:*

1. *If the k-majority rule favours the prior and $g \geq 2k$, the efficient equilibrium features $c \in [g-2k-1, g-2k]$ and $d = 0$;*

1′. *If $k = 0$ (simple majority rule), the efficient equilibrium features $c \in [g-1, g]$ and $d = 0$;*

2. *If the k-majority rule favours the prior and $g < 2k$, the efficient equilibrium features $c = 0$ and $d \in [2k-g-1, 2k-g]$;*

3. *If the k-majority rule handicaps the prior, the efficient equilibrium features $c \in [g+2k-1, g+2k]$ and $d = 0$.*

Under each k-majority rule above, if g above prescribes c or d such that the committee decision is σ_0 (the prior) with probability 1, then any c or d that does not change the outcome leads to an outcome equivalent equilibrium that is efficient.

For the non-generic cases where g is an integer, there are two equilibria that lead to the same expected payoff for each case above, where c or d corresponds to both endpoints of the respective interval.

Proof. Let $m \equiv \frac{n+2k-1}{2}$, which denotes the the number of votes for each alternative that makes an agent pivotal in the committee with $n+2k$ agents under simple majority rule. Throughout the proof, in order to derive the equilibrium under a supermajority rule, we follow the Fact we saw earlier and consider a committee with $n+2k$ agents under simple majority rule, where $2k$ agents deterministically vote for A.

Before voting, an agent $i \in \{1, 2, .., n\}$ computes the difference in expected payoff $W_j(\sigma_i, \sigma_0)$ between voting for A and B, conditional on his private signal σ_i and the prior σ_0 in the event where he is pivotal. The index $j \in \{O, D, I\}$ indicates the agent's pure strategy, such that O indicates that the agent is one of c agents to vote for the prior (*obedient* voting), D indicates that the agent is one of d agents to vote against the prior (disobedient voting) and I indicates that the agent is one of $n-c-d$ agents to vote for the private signal (*individually informative* voting). Recall from Lemma 2 that we must have $(c \geq 0) \wedge (d = 0)$ or $(c = 0) \wedge (d > 0)$, so that we do not have to consider i) voting against the private signal or ii) the coexistence of agents who vote for and against the prior.

Case 1: k-majority rule favours the prior and $(c \geq 0) \wedge (d = 0)$

Suppose the prior $\sigma_0 = A$ so that k-majority rule favours the prior. We will now derive $c \geq 0$ such that neither the c obedient voters nor individually informative agents have incentive to deviate. Consider one of the c agents who follow the prior and suppose that the prior and his private signal disagree. In order for him not to deviate from the obedient strategy, we must have $W_O(B, A) \geq 0$. That is,

$$\begin{aligned} W_O(B,A) = q(1-p)&\binom{n-c}{m-2k-(c-1)}p^{m-2k-(c-1)}(1-p)^m \\ &- p(1-q)\binom{n-c}{m}p^m(1-p)^{m-2k-(c-1)} \geq 0 \\ &\Rightarrow q(1-p)p^{-2k-(c-1)} \geq (1-q)p(1-p)^{-2k-(c-1)} \\ &\Rightarrow \frac{q}{1-q} \geq \left(\frac{p}{1-p}\right)^{c+2k} \\ &\Rightarrow c + 2k \leq g. \end{aligned} \tag{4}$$

If the prior and the agent's private signal agree, (4) readily implies $W_O(A, A) \geq 0$.

Next, let us consider one of the $n - c$ agents who vote for the private signal. If such an agent has received a private signal B, no deviation implies we must have $W_I(B, A) \leq 0$ and hence

$$c + 2k + 1 \geq g \tag{5}$$

If the agent's private signal agrees with the the prior, we must have $W_I(A, A) \geq 0$, which implies

$$c + 2k - 1 \leq g. \tag{6}$$

Note that (4) implies (6). Thus from (4) and (5), we conclude that for Case 1, we must have

$$c \in [g - 2k - 1, g - 2k]$$

for $g \geq 2k$ in equilibrium. For simple majority rule ($k = 0$) we have

$$c \in [g - 1, g]$$

as stated in the proposition.

Case 2: k-majority rule favours the prior and $(c = 0) \wedge (d > 0)$

So far our analysis of k-majority rule that favours the prior is incomplete, as the solution we have obtained with $(c \geq 0) \wedge (d > 0)$ is valid only for $g \geq 2k$. Let us now examine $(c = 0) \wedge (d > 0)$. Consider one of the d agents who vote against the prior (disobedient voting) and suppose the prior and the private signal agree and indicate A. The no deviation condition for the disobedient agent is given by $W_D(A, A) \leq 0$ (i.e. voting for B is weakly is better), which implies

$$2k - d \geq g. \tag{7}$$

Suppose $\sigma_i = B$ and thus the prior and the private signal disagree. Then (7) implies $W_O(B, A) \leq 0$ and hence the agent is better off voting against the prior (and thus voting for B).

Next, let us consider one of $n - d$ agents who vote for the private signal. If the prior and the private signal agree we must have $W_I(A, A) \geq 0$ and thus

$$2k - d - 1 \leq g. \tag{8}$$

If the prior and the private signal disagree (i.e. $\sigma_i = B$), we must have $W_I(B, A) \leq 0$, which implies

$$2k - d + 1 \geq g. \tag{9}$$

It is easy to see that (7) implies (9). Therefore, from (7) and (8) we conclude that

$$c = 0 \text{ and } d \in [2k - g - 1, 2k - g]$$

for $2k > g$ in equilibrium, as stated in the proposition.

Case 3: k-majority rule handicaps the prior and $(c \geq 0) \wedge (d = 0)$

Suppose $\sigma_0 = B$ so that k-majority rule handicaps the prior. We will now derive $c \geq 0$ such that neither the c obedient voters nor $n - c$ individually informative agents have incentive to deviate. Consider one of c agents who vote for the prior and suppose the prior and the private signal disagree (i.e. $\sigma_i = A$). The no deviation condition $W_O(A, B) \leq 0$ leads to

$$c - 2k \leq g. \tag{10}$$

For the case where the prior and the private signal agree, (10) implies $W_O(B, B) \leq 0$.

Next, let us consider one of the $n - c$ agents who vote for the private signal. If such an agent has received a private signal A (and thus the prior and the private signal disagree), no deviation implies we must have $W_I(A, B) \geq 0$ and hence

$$c - 2k + 1 \geq g. \tag{11}$$

If the agent's private signal agrees with the the prior, we must have $W_I(B, B) \leq 0$ (i.e. voting for A is weakly worse) and thus

$$c - 2k - 1 \leq g. \tag{12}$$

Note that (10) implies (12). Therefore, combining (10) and (11) we conclude that for

Case 3, we must have in equilibrium

$$c \in [g + 2k - 1, g + 2k] \text{ and } d = 0.$$

as stated in the proposition.

Case 4: k-majority rule handicaps the prior and $(c = 0) \wedge (d > 0)$

Suppose $\sigma_0 = B$ so that k-majority rule handicaps the prior. We show by contradiction that $(c = 0) \wedge (d > 0)$ does not hold in equilibrium. Consider one of the d disobedient agents who vote against the prior, and suppose the prior and the private signal agree. For him not to deviate, we must have $W_D(B, B) \geq 0$ (i.e. voting for A is weakly better), which implies

$$-d - 2k \geq g \Leftrightarrow d \leq -g - 2k. \tag{13}$$

That is, if the prior and the private signal agree, the agent deviates and votes according to the signals regardless of d. This contradicts $d > 0$ and we conclude that when the k-majority rule handicaps the prior there is no equilibrium that features $(c = 0) \wedge (d > 0)$.

Optimally deterministic decision for the prior

Finally, by construction, for Cases 1 and 3 if $c \in \{0, 1, ..., \frac{n-1}{2} - k\}$ in equilibrium, a higher expected payoff is achieved than in any equilibrium where $c \geq \frac{n+1}{2} - k$ (i.e. there are $\frac{n+2k+1}{2}$ or more deterministic votes) and hence the committee decision is for the prior with probability 1. This is because from (4) and (10) an agent who votes for the prior deviates if $c = \frac{n+1}{2} - k$ (note that the expected payoff is the same for $c > \frac{n+1}{2} - k$ although no vote is then pivotal). Likewise for Case 2, if $d \in \{0, 1, ..., \frac{n+2k-1}{2}\}$ in equilibrium, a higher payoff is achieved than any equilibrium where the decision is against the prior with probability 1, since the expected payoff therein is smaller than 1/2.

Equal payoff for the two equilibria when g is an integer[4]

Let us focus on Case 1 above. The statement is proved similarly for the other Cases. From (4) the no deviation condition for the obedient agent whose private signal disagrees with the prior is $c \geq g - 2k$, while from (5) the no deviation condition for the individually informative agent whose private signal disagrees with the prior is $c \leq g - 2k - 1$. Thus for an integer g, there are two equilibria, namely one with $c = g - 2k$ and the other with $c = g - 2k - 1$, while $d = 0$ in both.

Suppose $c = g - 2k$. Then (4) implies that the obedient agent whose private signal disagrees with the prior is indifferent between following the private signal and the public signal, so that he is indifferent between the two equilibrium strategy profiles with $c = g - 2k$ and $c = g - 2k - 1$.[5] Suppose $c = g - 2k - 1$. Then (5) implies that the individually

[4] Maug and Rydqvist (2008) assume away integer g.

[5] Note that (4) is for the pivotal event but the conditional expected payoff for the non-pivotal events remains unchanged as we fix the other agents' strategies.

informative agent whose private signal disagrees with the prior is indifferent between following the private signal and the public signal, so that he is indifferent between the two equilibrium strategy profiles. Thus conditional upon receiving a private signal that disagrees with the prior, the agents' expected payoffs are the same the same in both equilibria.

Consider an obedient agent who has received a private signal that agrees with the prior in one of the equilibria. His conditional expected payoff is given by $(p/(1-p))\, L_O$, where L_O is the expected payoff of the obedient agent when the signals disagree in the equilibrium. Similarly, the conditional expected payoff of an individually informative agent in one of the equilibria is given by $(p/(1-p))\, L_I$, where L_I is the expected payoff of the individually informative agent when the signals disagree in the equilibrium. Clearly, L_O and L_I are unchanged between the two equilibria, and thus $(p/(1-p))\, L_O$ and $(p/(1-p))\, L_I$ are also the same. Since the conditional payoffs are unchanged between the two equilibria whether the private signal agrees or disagrees with the prior, we conclude that the ex ante payoff is also the same in both equilibria. □

4 Deliberation as Coordination

Now that we have established the optimality of the asymmetric pure strategy equilibrium, it would be natural to consider coordination to play the equilibrium, given the multiplicity of equilibria and a potentially large number of agent, which may well make playing the equilibrium challenging.

As we mentioned in the Introduction, the equilibrium derived above points to a distinctive rationale for pre-voting deliberation, where the agents endogenously "correct" the exogenously given voting rule. Specifically, the deliberation in this context is not for the exchange of private information, but for the coordination on the efficient equilibrium. Such deliberation would lead to an agreement as to which (or how many) agents should vote or against the ex-ante desirable alternative and which agents vote according to their private information, given g and the voting rule k in place. According to this interpretation, the "role assignment" should be the focus of this type of deliberation.

Since by construction the most efficient equilibrium is also the most efficient strategy profile, no agent deviates in different information structures, as long as the "role assignment" is consistent with the efficient strategy profile. For example, a certain number ($c \in [g-1, g]$) of agents may vote early based only on the prior knowledge before they receive their private signals, and the other agents vote for the private signals as they become available. If there is such an agreement through pre-voting deliberation, it is self-enforcing.

Note that the prior (public information) in the model can also be interpreted as an "expert" signal/opinion for $q > p$, where the agents listen to an expert who does not vote but has more accurate information than each agent before they vote. Kawamura and Vlaseros (2017) find experimentally that expert information tend to have excessive influence on voting behaviour in a common interest voting game under majority rule without pre-voting deliberation. Consequently, the committee may be better off without expert information, even though it increases efficiency in theory if the agents play the efficient equilibrium.[6] Their findings indicate that coordination is very difficult in practice when all agents have both private and public information. However, the efficient outcome might be achieved when a specific number ($n - c$, where $c \in [g - 1, g]$) of committee members choose, or are instructed *not* to listen to the expert opinion, so that they could be committed to voting for their private signals and the rest of the members listen to and vote for the expert information. That is, deliberation may potentially lead to a consensus on changing the information structure, so as to incorporate expert information into voting behaviour more efficiently.

5 Examples

Let us discuss a few simple examples to gain intuition behind the asymmetric equilibrium presented in the Proposition. We first discuss the efficient equilibrium under simple majority rules where the CJT holds. For unanimity rule, we make an explicit comparison between the symmetric mixed strategy equilibrium studied by Feddersen and Pesendorfer (1998) and the efficient equilibrium we have derived above, in order to highlight some qualitative differences.

5.1 Majority and Supermajority Rules

As briefly mentioned earlier, the log odds ratio $g \equiv \frac{ln(q)-ln(1-q)}{ln(p)-ln(1-p)}$ specifies the "weight" the prior (whose accuracy is q) has with respect to private signals (whose accuracy is p) in the asymmetric equilibrium, independently of the committee size n.[7] Suppose $p = 0.6$ and $q = 0.8$, in which case we have $g \approx 3.419$. This means the prior is "worth" three private signals under simple majority rule ($k = 0$), regardless of the committee size. Indeed, the efficient pure strategy equilibrium prescribed for $k = 0$ in the Proposition features two agents voting for the prior, and the rest voting for their private signals.

Figure 1 compares the efficiency of the efficient equilibrium and the symmetric equilibrium. We can see that for $n = 3$ and $n = 5$, both equilibria lead to the same expected payoff since the best is for the committee decision to follow he prior with probability 1, which achieves the accuracy of $q = 0.8$. While the number of agents who actually vote for the prior may differ, the two equilibria are outcome equivalent. Since $g \approx 3.419$, three votes must be cast for the prior independently of private signals, but they already

[6] The efficiency benchmark of Kawamura and Vlaseros (2017) is that of the symmetric mixed strategy equilibrium. They also show that much of the excessive influence was caused by the subjects' failure to fully understand information aggregation.

[7] Recall that g is increasing in q and decreasing in p.

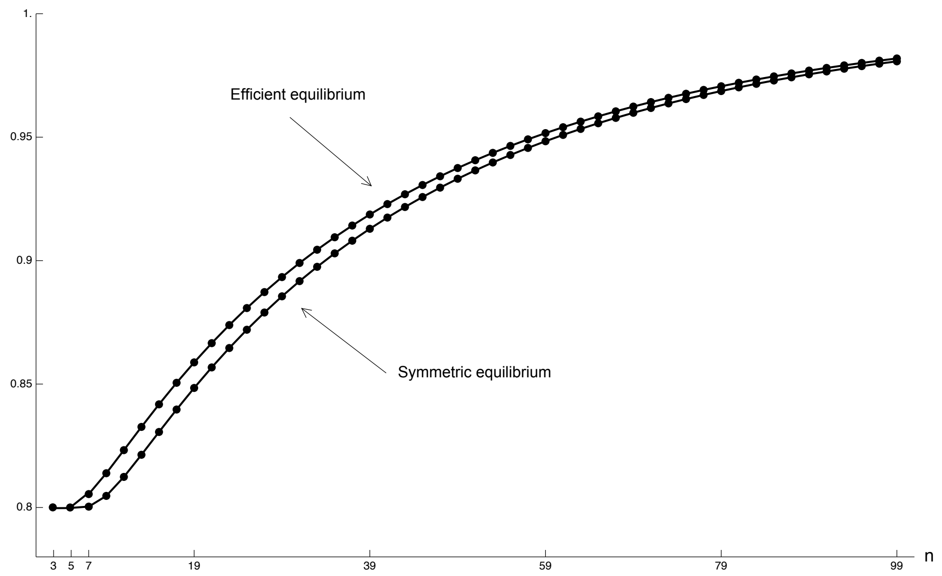

Figure 1: Efficiency comparison under simple majority

constitute a majority for $n = 3$ and $n = 5$. At $n = 7$ the accuracy levels diverge, but they both converge to 1 as n becomes larger.

A supermajority rule may either favour or handicap the prior. Suppose as earlier that $p = 0.6$, $q = 0.8$, and in addition let $k \geq 1$ and $\sigma_0 = A$ so that a k-majority rule favours the prior. According to the Proposition, for $g > 2k$, if $k = 1$ then one agent votes for the prior and the rest vote according to their private signals in the efficient pure strategy equilibrium. Recall our observation that the voting game under a k-majority rule is equivalent to the voting game under simple majority rule with $n + 2k$ agents where $2k$ agents deterministically vote for A. This implies the rule $k = 1$ itself gives the prior a weight of two votes, and thus there must be one additional vote for the prior in the efficient pure strategy equilibrium, since the prior is worth three private votes.

Meanwhile, if $k = 3$ the rule gives a weight of six private votes to the prior, which is more than the prior is worth. In this case ($g < 2k$), the Proposition states that there must be two votes *against* the prior and the rest vote according to the private signals. The votes against the prior are to offset the excessive weight the rule puts on the prior.

Now suppose $k = 1$ and $\sigma_0 = B$ so that the supermajority rule handicaps the prior. The Proposition indicates that the efficient pure strategy equilibrium in this case features five agents voting for the prior and the rest vote for the private signals. Intuitively, two ($= 2k$) of the five votes for the prior are to "correct" for the bias against the prior imposed by the rule, and the remaining three votes are to give the appropriate weight to the prior, after the correction to simple majority rule.

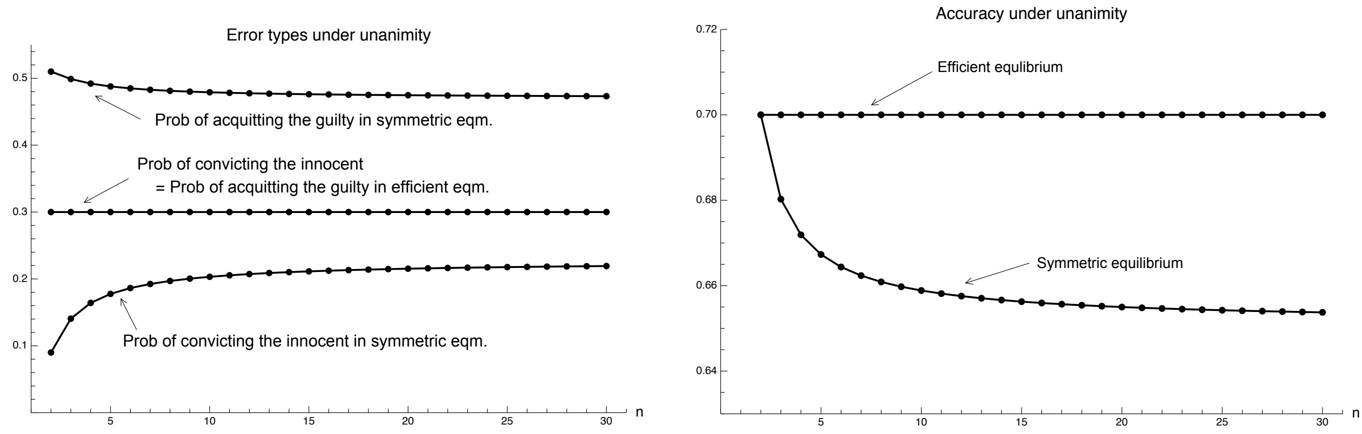

Figure 2: Comparison between efficient equilibrium and symmetric equilibrium under unanimity

5.2 Unanimity Rule and "Convicting the Innocent"

Let us consider some interesting implications of the efficient pure strategy equilibrium on voting behaviour and efficiency under unanimity rule. It has been well known since Feddersen and Pesendorfer (1998) that information aggregation fails and in addition there are some peculiar properties in the symmetric mixed strategy equilibrium. For example, under unanimity rule the probability of "convicting the innocent" becomes higher, and the probability of "acquitting the guilty" becomes lower as the committee size increases. Here we demonstrate that, while the CJT fails in both equilibria, the comparative statics in the efficient pure strategy equilibrium with respect to committee size is different, and the equilibrium strategy profile has a simple and intuitive structure.

For comparison, let $p = 0.7$ and $q = 0.5$ following the leading example in Feddersen and Pesendorfer (1998), so that the strategic environment is identical. In our terms the prior is uninformative, and in their terms the "standard of reasonable doubt" is 0.5 for conviction. Also, $y = A$, which is the alternative unanimity rule favours in our model, corresponds to "acquittal" and $y = B$ corresponds to "conviction" in Feddersen and Pesendorfer (1998),[8] and states $s = A$ and $s = B$ correspond to the defendant being innocent and guilty, respectively.[9]

Under unanimity our calculation does not suffer from indivisibility so that we let n be any positive integer larger than or equal to 2. Since $g = 0$ (i.e. in our terms, the prior is worthless) and unanimity implies $k = \frac{n-1}{2}$, according to the Proposition, the efficient pure strategy equilibrium features $d = n - 1$. This means that in the equilibrium all but one agent vote for conviction, so that the committee decision is determined solely by one agent's vote for his private signal. As a result the probability of convicting the innocent is

[8] That is, all agents have to vote for B (conviction) for B to be selected, while A (acquittal) is chosen as long as one or more agents vote for A.

[9] The payoff scale differs but it has no bearing on the characterization of equilibrium strategies. The comparative statics with respect to efficiency is also qualitatively the same between the model of Feddersen and Pesendorfer (1998) and ours.

the same as the probability of acquitting the guilty, which is $1 - p = 0.3$ regardless of the committee size. This in contrast to the error probabilities in Feddersen and Pesendorfer (1998) where, as shown in Figure 2, each error probability converges monotonically to a different value as the committee size increases. Note that, even though the probability of convicting the innocent is indeed lower in the symmetric mixed strategy equilibrium, the probability of acquitting the guilty is so much higher that the overall efficiency is still higher in the asymmetric pure strategy equilibrium.

The right-hand side of Figure 2 compares the efficiency. We can see that in the symmetric equilibrium, the expected payoff is decreasing in n and weakly lower than in the efficient equilibrium, while the expected payoff in the efficient equilibrium is constant.

Generally, the efficient equilibrium under unanimity for given p and q is characterized by the fixed effective number of agents who vote according to the private signals, which is independent of n. This independence leads to the failure of CJT in the efficient equilibrium as the information from the private signals is not aggregated. We have seen from the Proposition that when $p > q$ and thus $g < 1$, there is one voter who solely determines the committee decision by voting for the private signal. If $p = 0.6$, $q = 0.8$, and thus $g \approx 3.419$ as seen in Section 5.1, the Proposition implies that $2 \times \frac{n-1}{2} - 3 = n - 4$ agents must vote against A (hence for B) regardless of their private signals, which leaves four agents who vote according to their private signals. In other words, under unanimity, $\lfloor g \rfloor + 1$ agents vote for their private signals and the rest vote against A in the efficient equilibrium, where $\lfloor g \rfloor$ denotes the largest integer that does not exceed g. Thus the feature that the probability of "convicting the innocent", the probability of "acquitting the guilty", and the expected payoff are all independent of sufficiently large n holds true beyond the example we have looked at in Figure 2.

6 Conclusion

We have presented a unified approach of the asymmetric pure strategy equilibrium and its optimality in the voting model of Austen-Smith and Banks (1996). The asymmetry would require sophisticated coordination especially when the committee size is large, which points to a specific role pre-voting deliberation may play to achieve efficiency. While the difference in the efficiency between the equilibrium and the symmetric mixed strategy equilibrium often studied in the strategic voting literature may not be large, we have observed that the efficient equilibrium has a remarkably simple and intuitive structure. For example, the equilibrium demonstrates the peculiarity of unanimity rule in a striking way, such that the effective number of informative voters and thus the efficiency remain unchanged for any number of voters.

Appendix

The expected payoff for the efficient equilibrium in Figure 1 is derived from the following expression

$$q \sum_{i=\frac{n+1}{2}-k-c}^{n-c-d} \binom{n-c-d}{i} p^i (1-p)^{n-c-d-i} + (1-q) \sum_{i=\frac{n+1}{2}+k-d}^{n-c-d} \binom{n-c-d}{i} p^i (1-p)^{n-c-d-i},$$

where $k = 0$ (simple majority rule) and $d = 0$ (no agent votes against the prior). The expected payoff in the symmetric mixed strategy equilibrium is given by

$$q \sum_{i=\frac{n+1}{2}-k}^{n} \binom{n}{i} r_A^i (1-r_A)^{n-i} + (1-q) \sum_{i=\frac{n+1}{2}+k}^{n} \binom{n}{i} r_B^i (1-r_B^i)^{n-i},$$

where $k = 0$, $r_A = p + (1-p)(1-\beta)$ and $r_A = p\beta$, such that β is the equilibrium probability that the agents vote according to the private signal when their private signal and the prior/public signal disagree (and they vote according to the signals when they agree). The equilibrium strategy is explicitly derived in Wit (1998).

The expected payoff in the symmetric mixed equilibrium under unanimity rule in Figure 2 can be found in Feddersen and Pesendorfer (1998).

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