Spillovers of Equal Treatment in Wage Offers

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Abstract

We analyse a labour matching model with wage posting, where – reflecting institutional constraints – firms cannot differentiate their wage offers within certain subsets of workers. *Inter alia*, we find that the presence of impersonal wage offers leads to wage compression, which propagates to the wages for high productivity workers who receive personalised offers.

1   Introduction

This paper studies a labour market where both workers and firms are vertically differentiated. In such a setting, if firms could offer a personalised wage to each worker the outcome would be efficient matching with (firm-optimal) competitive wages. In practice, however, "equal treatment" is often imposed on offers to certain subsets of workers, either by law or by convenience. For example, employers in the public sector are often required to offer the same or similar salaries to workers whose observable characteristics (such as education, job experience, etc.) are comparable, via

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either salary scales or a more explicit equal treatment rule.[1] At the same time, the workers' productivity is often observable by the employers (through detailed CVs, recommendation letters, interviews etc.) who are restricted to compete in uniform wages for workers with different productivity levels. Our analysis suggests that these practices may have an implication not only on the wages of those who receive "equal treatment" but also on the wages of workers with high productivity who typically receive personalised offers. In particular, we show that the inability to differentiate offers leads to inefficient matching and lower equilibrium wages than when the firms can make an individualised offer to each worker.

While inefficiency is limited to the matching of the equally treated, whom we call "bundled" workers, the equilibrium wages are lower than those in the competitive equilibrium for both the bundled workers and for the workers who are more productive than them. Meanwhile, the workers receiving personalised offers who are of lower productivity than the ones who receive a bundled offer, continue to receive the competitive wage offer from their efficient match. In other words, the bundling creates no downward externality.

Specifically, the equilibrium offers for the bundled workers result from mixed strategies by the firms, and that necessarily leads to locally inefficient matching. It also leads to local wage compression: the wage of the least productive bundled worker is higher while the wage of the most productive bundled worker is lower than in a competitive equilibrium. In addition, the wages of the workers whose productivity is higher than those of the bundled workers are uniformly shifted down by the amount of gap between the most productive bundled worker's expected wage and his

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[1]In this paper, these constraints are assumed exogenous. One possible explanation is political pressures, especially from trade unions, towards "equality" among workers whose job grade/title is the same. Or, particularly for low-level positions for manual work, the cost of making individualised offers may outweigh the benefit of hiring (marginally) better matched workers. An alternative view is that, since wage compression through equal treatment is desirable for firms, they might actively coordinate to sustain it, despite the matching inefficiency it generates.

competitive wage, compared to their competitive wages. That is, wage reduction relative to competitive wages spreads across offers to workers who are more productive than the bundled workers and receive personalised offers. Moreover, we show that it is more efficient and also leads to higher wages if the bundled workers are distributed into many pairs rather than into fewer but bigger groups. We also consider the effect of a quality threshold, where the job in high productivity firms requires sufficiently high skills (productivity) on the worker's side. We find that the presence of such a quality threshold leads to less wage reduction.

The analysis of wages and matching under uniform wages was pioneered by Bulow and Levin (2006). They showed that when firms are unable to differentiate offers at all, wages are compressed and in the aggregate they fall relative to competitive equilibrium. At the same time, firms' profits increase, despite the presence of some matching inefficiency. Their seminal work has been extended in a number of directions. Niederle (2007) allows firms to offer multiple (ordered) contracts, and shows that the firm-optimal competitive outcome is achievable in equilibrium. In a different extension, Kojima (2007) shows that if firms have different capacities then the average wage may exceed the competitive benchmark (and hence firms' profits may decrease). Azevedo (2011) takes a further step and endogenises the firms' capacity choice. He shows that impersonal wage offers may yield an overall more efficient outcome than personalised wages do. Using a different framework that features continua of buyers and sellers, Mailath, Postlewaite and Samuelson (2013) study efficiency in investment and matching with respect to personal and impersonal wages.

In a directed search model, Michelacci and Suarez (2006) investigate a related scenario, where firms *choose* whether to offer uniform wages (for example, because they cannot observe – or measure/demonstrate in court – the workers' productivities) or to bargain with the workers, once they have observed their productivity. When an equilibrium involving both choices exists, it is the low productivity workers who go for the posted wage and the high productivity ones for the negotiated one. This

produces an adverse selection effect, so the presence of the negotiated deals depresses the posted wages. However, the opposite effect, which we find in our model, is ruled out by their assumption that in the *ex post* bargaining the outside options are exogenously normalised to zero. That is, once matched, the negotiated wage between a firm and a worker is assumed not to depend on the equilibrium, only on the worker's productivity and bargaining power.

Matching and wages are affected not only by the characteristics of wage offers but also by the matching procedure itself. Alcalde, Pérez-Castrillo and Romero-Medina (1998) propose simple hiring procedures to implement a stable matching in a sub-game perfect equilibrium where firms can make personalised offers to multiple agents. Konishi and Sapozhnikov (2008) and Sákovics (2011) study alternative settings where each firm can make a personalised offer to a *single* worker at every stage. Since our focus is on the effect of wage bundling, not on the matching procedures, we assume a matching procedure that guarantees a stable matching in equilibrium.

In practice, heterogeneous workers in the labour market do face different wage determination processes. Hall and Krueger (2012) document wage determination of workers with various qualifications through a survey of workers. They find that more educated workers are more likely to negotiate wages individually before they take up their job, while less educated workers tend to work at posted wages that are not personalised. Brenčič (2012) provides consistent evidence studying job advertisements, and moreover she finds that advertisements with posted wages are more common for jobs that require less specific skills or experience. Since there is complete information in our model, impersonal wages in this paper can be interpreted as wages posted for a group of (e.g. low-skilled) workers, and personalised wage offers can be thought of as those that emerge from individual examination of each worker from a (e.g. high-skilled) group. Our analysis points to the interaction between those two different sets of workers.

2   The model

Workers and firms are indexed from 1 to N, with $N \geq 3$. The productivity of a worker is his index, and productivities of firms are denoted by $0 < D_1 < ... < D_N$. The output/revenue of a matched worker-firm pair is the product of their productivities. For simplicity, outside options of the workers and firms are normalised to zero. The firms simultaneously post a wage offer to each worker, but the wages for k workers are bundled: the firms cannot wage discriminate among those workers. We have $2 \leq k \leq N$. We denote the group of bundled workers by $\{h+1, ..., h+k\}$, that is, we assume that the bundled workers are of similar productivity. We denote the $(N-k+1)$-vector of wage offers made by Firm i by $W^i = (w_1^i, ..., w_h^i, w^i, w_{h+k+1}^i, ..., w_N^i)$, where w_j^i is Firm i's offer to worker j and w^i is its common offer to the bundled workers. We choose a simple strategic form to describe the matching procedure: once the wage vectors offered by the firms are public, the workers choose sequentially, in decreasing order of their productivity, the wage offer that they want to accept (if any). Workers always choose the remaining firm offering the highest wage, and when offered the same wage by different firms they prefer to work for the most productive of those firms. If Worker i accepts the offer from Firm j, both the firm and the worker exit the market with payoffs $iD_j - w_i^j$ and w_i^j, respectively.

3   Preliminaries

It is straightforward to see that, due to the complementarity in productivities, positive assortative matching (PAM) is the efficient outcome in our model. The benchmark result is that, without restrictions on the wage offers, PAM is indeed guaranteed in equilibrium:

Proposition 1 *If firms can make personalised offers to every worker, then the resulting matching will be PAM, with the actual wages paid given by* $w_1 = 0$ *and*

$w_i \in \left[\sum_{j=1}^{i-1} D_j, \sum_{j=2}^{i} D_j \right]$ *for $i = 2, 3, ..., N$. These are a subset of the competitive wages*

that are given by $w_i \in \left[\sum_{j=1}^{i-1} D_j, \sum_{j=1}^{i} D_j \right]$ for $i = 1, 2, ..., N$.

Proof. First, it is immediate that all workers (and firms) must get matched in equilibrium. Next, suppose the matching were not PAM. Then we would have i, j, k, l such that Firm $i < k$ hires Worker $j > i$ and Firm k hires Worker $l < j$. Firm k could deviate and match i's offer to Worker j. Worker j would prefer this wage, so that the deviation payoff of Firm k is at least[2] $jD_k - w_j^i$, which cannot exceed Firm k's putative equilibrium payoff $lD_k - w_l^k$. Similarly, Firm i could deviate by slightly outbidding Firm k's wage offer to l. For this not to occur we must have $jD_i - w_j^i \geq lD_i - w_l^k$. Combining those two conditions we have

$$D_i(j - l) \geq w_j^i - w_l^k \geq D_k(j - l),$$

that is, $D_i \geq D_k$, contradicting the hypothesis that $i < k$.

Given PAM, we know that Firm 1 will be matched with Worker 1, who is the last to choose, so the firm can hold him to his reservation wage, which is zero. Hence, $w_1^1 = 0$ and Firm 1's equilibrium payoff is D_1. If Firm 1 hired Worker 2 for $w_2^2 + \varepsilon$ instead, then its payoff would be $2D_1 - w_2^2 - \varepsilon$, which by the above argument cannot exceed D_1. Thus we have $w_2^2 \geq D_1$. Note that the highest wage Firm 2 is willing to pay Worker 2 given that it could hire Worker 1 for zero is D_2. As it is costless for Firm 1 to make an offer to Worker 2 as long as it is not accepted, it could push up Worker 2's wage to D_2. Thus $w_2^2 \in [D_1, D_2]$. We proceed by induction. Fix $w_i^i \in \left[\sum_{j=1}^{i-1} D_j, \sum_{j=2}^{i} D_j \right]$. Then by the above argument we must have that $iD_i - w_i^i \geq (i + 1)D_i - w_{i+1}^{i+1}$, and $w_{i+1}^{i+1} \in [D_i + w_i^i, D_{i+1} + w_i^i] = \left[\sum_{j=1}^{i} D_j, \sum_{j=2}^{i+1} D_j \right].$

The argument for the set of competitive equilibria only differs for Worker 1. There, a firm other than Firm 1 may drive Worker 1's wage up to the maximum Firm 1 is

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[2]The firm can guarantee this payoff by setting all other offers to zero.

willing to pay: D_1. This then has a knock-on effect on the rest of the wages by the above logic, leading to the set of competitive wages in the statement of the proposition. ∎

The multiplicity of equilibrium wage profiles arises from the fact that the firms can costlessly drive up the wages for workers above their equilibrium match. As can be gleaned from the proof of Proposition 1, in order to have uniqueness, we need to prohibit firms to make offers that they would not like to be accepted.

Definition 1 *Given an equilibrium, a firm's wage vector is* equilibrium undominated *if and only if the firm's equilibrium profit is no more than the profit it would obtain if another of its wage offers were accepted.*

Focusing on equilibrium undominated wage vectors, we can rule out any equilibria supported by unaccepted wage offers that would otherwise (i.e. if accepted) reduce firms' profits, and select a unique equilibrium:

Corollary 1 *There is a unique equilibrium in equilibrium undominated wage vectors, which is the firm-optimal competitive equilibrium, with the accepted wages* $w_i^c = \sum_{j=1}^{i-1} D_j$ *for* $i = 1, 2, ..., N$.

Our second benchmark is at the other end of the spectrum, where we have full equal treatment: each firm has to offer a single wage open to all comers.

Proposition 2 *(Bulow-Levin, 2006): When firms are restricted to offering a single wage, the equilibrium is in mixed strategies, with combined support* $\left[0, \overline{w}_{BL}^{1,N}\right]$. *There is no explicit formula for* $\overline{w}_{BL}^{1,N} \leq w_N^c$, *but it is well-defined via a finite algorithm.*

The mixed strategy equilibrium necessarily leads to mismatch/inefficiency, but firms are still no worse off than in their most favourable competitive equilibrium. As a result, workers (in aggregate) are strictly worse off due to equal treatment.

As it will become clear, it is useful to relax the definition of positive assortative matching to fit our pooled-offer scenario:

Definition 2 *We call a matching generalised positive assortative (GPAM) if the workers whose wages are pooled are matched with the firms from the same index set, while the rest of the matching is PAM.*

4   Equilibrium

We start by stating our main characterisation result. The rest of this section consists of a sequence of partial characterisations that ultimately lead to the proof of the theorem. Unlike in Bulow and Levin (2006) – and just as in the case of personalised offers discussed for Corollary 1 – we focus on equilibrium undominated strategies to have a unique prediction.[3]

Theorem 1 *The unique equilibrium outcome resulting from undominated strategies features GPAM with the accepted wages given by w_i^c for $i \in \{1, 2, ..., h\}$, a mixed strategy pooled wage à la Bulow-Levin over the interval[4] $\left[w_{h+1}^c, \overline{w}_{BL}^{h+1,h+k} + w_{h+1}^c \right]$,*

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[3]This restriction is not without loss of generality. If, say, there is ex post competition in the product market, offering wages that are equilibrium dominated could potentially harm a competitor (by bidding up its wage bill) and thus indirectly benefit the firm.

[4]$\overline{w}_{BL}^{h+1,h+k}$ corresponds to the upper bound of the mixing interval in Bulow and Levin (2006) when there are k firms and workers, indexed between $h+1$ and $h+k$. It can only be determined via an algorithm, presented in Bulow and Levin (2006), and has no closed form solution.

and

$$w_i = \left(\overline{w}_{BL}^{h+1,h+k} + w_{h+1}^c\right) + \sum_{j=h+k}^{i-1} D_j = \left(\overline{w}_{BL}^{h+1,h+k} + w_{h+1}^c\right) + w_i^c - w_{h+k}^c$$

$$= w_i^c - \left[w_{h+k}^c - \left(\overline{w}_{BL}^{h+1,h+k} + w_{h+1}^c\right)\right]$$

for $i \in \{h+k+1, h+k+2, ..., N\}$.

The main insights from the Theorem are that i) "wage compression" occurs not only for workers who receive equal treatment but also for those who are above the pooled range; and that ii) the equilibrium is built "from down up", in the sense that the existence of higher productivity firms/workers does not affect the equilibrium outcome of the "bottom section" below the bundled range. The externalities are small even for those above the pooled range, as it is only the lower bound on wage offers that is affected by the behaviour of lower productivity firms. In other words, wage differentials among the firms that hire a worker above the bundled range are the same as those in the competitive equilibrium, while their wages in absolute terms are lower than the competitive ones. The size of the wage reduction for every worker above the bundled range is the difference between the unbundled competitive wage of the most productive worker in the bundle w_{h+k}^c and the upper bound of the support of wages offers to the bundled workers $\bar{w} \equiv \overline{w}_{BL}^{h+1,h+k} + w_{h+1}^c$, which, if offered, must be accepted by Worker $h+k$. We will also see later that $w_{h+k}^c \geq \bar{w}$, and in particular $w_{h+k}^c > \bar{w}$ if there are three or more bundled workers (i.e. $k \geq 3$).

As stated earlier, we assume that the wage vectors offered by the firms are public and the workers choose sequentially, in decreasing order of their productivity. Since workers have strict preferences, the sequential procedure leads to a unique matching conditional on any wage vector, which makes the analysis of equilibrium wage offers tractable. However, by imposing a further restriction on unaccepted offers in the equilibrium wage vectors, the equilibrium outcome in the Theorem can also be

achieved via any centralised matching procedure that leads to a stable matching.[5]

To prove the Theorem, we first characterise the unique vector of accepted wages in any equilibrium that results in GPAM. Next, we show that an equilibrium with GPAM indeed exists. Finally, we show that there are no equilibria which do not result in GPAM.

Our first lemma partially characterises the wages of non-pooled workers under GPAM:

Lemma 1 *Consider any pair of adjacent workers in GPAM, i and $i+1$ such that $i+1$ (but not necessarily i) is unbundled. In any equilibrium with equilibrium undominated wage vectors we must have $w_{i+1} = \overline{w}_i^i + D_i$, where $\overline{w}_i^i$ is the upper end of the support of Firm i's (possibly mixed) offer strategy to Worker i.*

Proof. First note that the offer to an unbundled worker must be in pure strategy, since in GPAM it is common knowledge whose offer will be accepted. Next, note that in equilibrium Firm i must not strictly prefer hiring Worker $i + 1$ at w_{i+1} to hiring Worker i at $\overline{w}_i^i$. Thus we have $iD_i - \overline{w}_i^i \geq (i+1)D_i - w_{i+1} \Rightarrow w_{i+1} \geq \overline{w}_i^i + D_i$. The only reason to bid more than that would be if another firm (with a smaller index) would be offering more as well. However, that would be loss making if accepted, and thus it is ruled out by the equilibrium undominated wages restriction. Hence we have $w_{i+1} = \overline{w}_i^i + D_i$. ∎

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[5]To see this, fix the accepted equilibrium wages in the Theorem and suppose further that each firm makes offers to unmatched workers in such a way that, if accepted, the offers to the workers above the matched worker generate the same profits as their equilibrium profits, and the offers to the workers below him are the same as their accepted wages in the equilibrium. This implies that all workers have the same preference ordering for the firms (i.e. a more productive firm is better) and hence the stable matching conditional on the wage profiles is unique (e.g. Clark, 2006). Just as in the case of the sequential procedure, the firms do not have incentive to deviate.

It is easy to see that any offers to bundled workers that have a positive probability of being accepted must be in mixed strategies. If there was an equilibrium in pure strategies, then a more productive firm would prefer to match a less productive firm's bundled offer, while the lesser firm would want either to strictly undercut – thereby hiring the same worker as if she matched the higher firm's offer but at a lower wage – or offer more, leapfrogging the better firm.

Lemma 2 *In any equilibrium with equilibrium undominated wage vectors that results in GPAM, the firms hiring the bundled workers $\{h+1, ..., h+k\}$ play a Bulow-Levin mixed strategy equilibrium over the support $[w_h + D_h, \overline{w}_{BL}^{h+1,h+k} + w_h + D_h]$.*

Proof. In GPAM Firms $h+1, ..., h+k$ play mixed strategies for the workers whose wages are bundled. Standard arguments imply that i) the support must be continuous; ii) two or more firms make offers anywhere on the support; and iii) there is no atom except for the bottom of the support.

Suppose that the lower bound of the support were less than $w_h + D_h$. Then by offering $w_{h+1}^h = w_h + D_h - \varepsilon$, Firm h would have a positive probability of hiring a worker of productivity no less than $h+1$, leading to a higher profit than in equilibrium. At the same time the lower bound cannot be higher than $w_h + D_h$, since then the firms would strictly prefer to bid less than the lower bound to offering the lower bound (as they would hire Worker $h+1$ anyway, but for less). Because of the equilibrium undominated wages assumption, no firm below $h+1$ offers a higher wage.

Due to GPAM, the bundled wage offers made by Firms $i > h+k$ do not affect the outcome, so that the rest follows from Proposition 2 above. It follows from the Bulow-Levin algorithm for the computation of the mixed strategies that, since the lowest wage bid here is $w_h + D_h$ instead of zero, all offer densities by the mixing firms simply shift upwards by $w_h + D_h$. ∎

We are now ready to complete the proof of the Theorem:

Proof of the Theorem. We first show that there is an equilibrium that features GPAM. Consider the wages according to Theorem 1, with $w_{i+1}^i = w_{i+1}$ for $i \in \{1, 2, ..., h, h+k+1, h+k+2, ..., N-1\}$ and the rest of the wage offers to unbundled workers equal to zero.

Note that by construction the wage differentials among the unbundled workers are such that no firm matched with an unbundled worker has incentive to deviate and attract another unbundled worker by matching his wage. Each unbundled worker (except Worker 1) receives the same two identical offers, and they choose the one from the more productive firm. Lemma 2 ensures that no firm matched with a bundled worker has incentive to change its mixed offers to hire a bundled worker.

It remains to be shown that, with respect to the strategy profile, no firm matched with an unbundled worker has incentive to hire a bundled worker instead; and that no firm matched with a bundled worker has incentive to hire an unbundled worker instead.

Firm $j \in \{h+1, h+2, ..., h+k\}$ matched with a bundled worker cannot gain from hiring any unbundled worker h or below instead: the deviation profit is bounded by hiring Worker h at w_h^c, but by construction the Bulow-Levin mixed strategy guarantees an expected profit equal to hiring Worker $h+1$ at w_{h+1}^c, which is higher for any Firm $j > h$. Likewise, Firm $j \in \{h+1, h+2, ..., h+k\}$ cannot gain from hiring any unbundled Worker $l \in \{h+k+1, h+k+2, ..., N\}$: the deviation profit is bounded by hiring Worker $h+k+1$ at a wage slightly higher than $w_{h+k+1}^c - \left[w_{h+k}^c - \left(\overline{w}_{BL}^{h+1,h+k} + w_{h+1}^c \right) \right]$, but by construction the Bulow-Levin mixed strategy guarantees an expected profit equal to hiring Worker $h+k$ at $\overline{w}_{BL}^{h+1,h+k} + w_{h+1}^c$, which is strictly higher for any Firm $j \leq h+k$.

Also, no firm matched with an unbundled worker would profit from deviating and hiring a bundled worker. Firms $i \in \{1, 2, ..., h\}$ would have to bid more than w_{h+1}^c to attract a bundled worker. However, Firm $h+1$ (who is mixing) is indifferent

to making such a raise which implies that firms below it are strictly worse off than offering exactly w_{h+1}^c and hiring Worker $h+1$. By the construction of the competitive wages they are at best indifferent between the resulting payoff and their putative equilibrium payoff. Similarly, Firms $l \in \{h+k+1, h+k+2, ..., N\}$ would be at best indifferent between hiring Worker $h+k$ at $\overline{w}_{BL}^{h+1,h+k} + w_{h+1}^c$ and their putative equilibrium payoff. Moreover, Firm $h+k$ (who is mixing) is indifferent to offering a lower wage to the bundled workers, which implies that such a lowering would strictly decrease the expected payoff of any Firm $l \in \{h+k+1, h+k+2, ..., N\}$.

So far we have established that GPAM is an equilibrium outcome, in which the accepted wages must be those specified in Lemma 1 and Lemma 2. Finally, we show that any equilibrium allocation must feature GPAM. If the bundled workers are matched with the firms of the same index set then the rest of the matching must be PAM, by the argument establishing PAM in the proof of Proposition 1. Thus, all we need to show is that a bundled worker cannot be matched with a firm outside of the index set of the bundled workers in equilibrium. Suppose that in equilibrium Firm $i \in \{1, ..., h\}$ hires Worker $g \in \{h+1, ..., h+k\}$. Then it must be that some Firm $j \in \{h+1, ..., h+k\}$ hires either i) Worker $l \in \{1, ..., h\}$; or ii) Worker $m \in \{h+k+1, ..., N\}$.

Consider Case i). The equilibrium wage offers to the bundled workers cannot be in pure strategy, so Firm i must adopt a mixed strategy, being indifferent among all of its offers made to the bundled workers. Let the highest wage in the support of Firm i's mixing be $\bar{w}^i$. Let the expected worker index when firm i offers $\bar{w}^i$ be $\pi(\bar{w}^i)$. Then Firm j could deviate and offer $\bar{w}^i$, which gives the expected worker index of at least $\pi(\bar{w}^i)$. Thus the deviation payoff is at least $\pi(\bar{w}^i)D_j - \bar{w}^i$, which cannot exceed firm j's putative equilibrium payoff $lD_j - w_l^j$, that is, $\pi(\bar{w}^i)D_j - \bar{w}^i \leq lD_j - w_l^j$. Similarly, Firm i could deviate by offering a single wage, which just exceeds w_l^j. Note that Firm i's expected payoff is the same regardless of the (mixed) offer it makes in equilibrium. Hence it must be that $\pi(\bar{w}^i)D_i - \bar{w}^i \geq lD_i - w_l^j$. Combining those two conditions we

have

$$D_i(\pi(\bar{w}^i) - l) \geq D_j(\pi(\bar{w}^i) - l),$$

contradicting the hypothesis that $j > i$.

Consider Case ii). Now in order for Firm j not to deviate and make an (only) offer to the bundled workers equal to $\bar{w}^i$, we must have $mD_j - w_m^j \geq \pi(\bar{w}^i)D_j - \bar{w}^i$, and Firm i's no deviation condition is $\pi(\bar{w}^i)D_i - \bar{w}^i \geq mD_i - w_m^j$. Combining both we obtain

$$D_i(m - \pi(\bar{w}^i)) \geq D_j(m - \pi(\bar{w}^i)),$$

contradicting the hypothesis that $j > i$. ∎

5   The effects of bundling restrictions

So far we have assumed that there is only one class of bundled workers from $h+1$ to $h+k$, but clearly Theorem 1 can be extended to the cases where there are multiple groups of bundled workers, as long as each bundle consists of workers with consecutive indices. The number and size of the bundles have important implications for payoffs and efficiency. As it will become clear, the case of pairs of workers receiving a common wage is qualitatively different from the cases where three or more workers are bundled. We start with the former case.

5.1   Bundles of two

Our first observation is that a two-worker bundle has no externalities on the remaining matches and wages.

Corollary 2 *When there are two bundled workers, namely Workers $h+1$ and $h+2$, the upper bound of the common wage distribution is $\bar{w} = w_h^c + D_h + D_{h+1} = w_{h+2}^c$, which is the firm-optimal competitive wage for Worker $h+2$.*

Proof. Let us have a closer look at the mixed strategy equilibrium, with mixing densities denoted by $f_i(w)$ for Firm i. Note that the benefit from offering a slightly higher wage $w + dw$ than w is given by $D_j \sum_{i \neq j} f_i(w) \cdot dw$, while the additional cost is dw. Thus for any w in the support, there must be two or more consecutive firms within $\{h+1, ..., h+k\}$ that actually offer w, and the density functions of the mixed strategies for those mixing firms j solve

$$D_j \sum_{i \neq j} f_i(w) = 1. \tag{1}$$

If there are two bundled workers, the solution is given by $f_{h+1} = 1/D_{h+2}$ and $f_{h+2} = 1/D_{h+1}$. As both firms must mix over the same range and the only mass point is for the weaker firm at the lower bound, Firm $h + 2$ mixes uniformly over $[w^c_{h+1}, \bar{w}]$, implying that $\bar{w} = w^c_{h+1} + D_{h+1} = w^c_{h+2}$. ∎

The corollary implies that when only two workers are bundled, the support of the mixed strategies is between the two competitive wages that correspond to the respective workers ($h + 1$ and $h + 2$). By bidding w^c_i Firm i ($i \in \{h + 1, h + 2\}$) is matched with the same worker as in the competitive equilibrium. Therefore the expected profits of both firms are the same as in the firm-optimal competitive equilibrium. Meanwhile, clearly the expected wage for Worker $h+1$ is higher, and for Worker $h + 2$ is lower relative to their competitive wages. It is immediate that the workers as a group are worse off – since they bear the cost of the potential inefficiency of the match – and, therefore, the expected gain of Worker $h + 1$ must be less than the expected loss of Worker $h + 2$.

The remaining question we wish to answer is: Conditional on there being a two-bundle, where should it be to maximise welfare?

Corollary 3 *The least inefficient two-bundle is at* $\arg\min_h \frac{D_{h+1}}{D_{h+2}} \left(D_{h+2} - D_{h+1} \right).$

Proof. The probability of an inefficient match is the probability that the more productive firm makes the lower bundled offer: $\int_{w^c_h}^{w^c_{h+1}} f_{h+1}(x) F_{h+2}(x) dx = \int_{w^c_h}^{w^c_{h+1}} \frac{1}{D_{h+2}} \cdot$

$\frac{x - w_h^c}{D_{h+1}} dx = \frac{\left(w_{h+1}^c - w_h^c\right)^2}{D_{h+1} D_{h+2}} = \frac{(D_{h+1})^2}{D_{h+1} D_{h+2}} = \frac{D_{h+1}}{D_{h+2}}$. The deadweight loss of the mismatch is $(h+2)D_{h+2} + (h+1)D_{h+1} - (h+1)D_{h+2} - (h+2)D_{h+1} = D_{h+2} - D_{h+1}$. Thus the expected loss due to mismatch (the only inefficiency) is $\frac{D_{h+1}}{D_{h+2}}\left(D_{h+2} - D_{h+1}\right)$. ∎

Note that $\frac{D_{h+1}}{D_{h+2}}\left(D_{h+2} - D_{h+1}\right) = \frac{D_{h+1}}{D_{h+1} + (D_{h+2} - D_{h+1})}\left(D_{h+2} - D_{h+1}\right)$ is increasing in $D_{h+2} - D_{h+1}$. Thus a sufficient condition for the optimality of bundling the bottom two workers is that the difference between productivities is non-decreasing in h.

Finally, it is important to observe that Corollary 2 generalises to any number of size two bundles, by the very nature of the result that there are no externalities. In line with Corollary 3, all the size two bundles should happen at the bottom of the distribution if the difference between productivities is non-decreasing in h.

5.2   Large bundles

Before we derive the offer distribution when $k \geq 3$, let us first present the following result, which says that for any wage in the mixed range, the offers by firms with higher productivity first-order stochastically dominate those by firms with lower productivity (c.f. Lemma 1 in Bulow and Levin, 2006).

Lemma 3 *If $D_j > D_l$, then in equilibrium for all w, $F_j(w) \leq F_l(w)$.*

Recall that we denote the upper bound of the mixed wage offers by $\bar{w}$, which, if offered must be taken by the most productive bundled worker, $h+k$. Using the above lemma we can show that larger bundles lead to further wage compression:

Corollary 4 *If $k \geq 3$, then $\bar{w} < w_{h+k}^c$.*

Proof.   Let V_j be Firm j's profit with Worker j at w_j^c, and notice that the difference in profits that Firm $h+1$ and Firm j such that $j \in \{h+2, ..., h+k\}$ make

in the firm-optimal competitive equilibrium is given by $V_j - V_{h+1} = (D_j - D_{h+1})j$. Instead, in equilibrium the difference is given by

$$\Pi_j(\hat{w}_{h+1}) - \Pi_{h+1}(\hat{w}_{h+1}) = D_j \cdot j - D_{h+1} \cdot [j + F_j(\hat{w}_{h+1}) - F_{h+1}(\hat{w}_{h+1})]$$
$$= (D_j - D_{h+1})j + D_{h+1}[F_{h+1}(\hat{w}_{h+1}) - F_j(\hat{w}_{h+1})],$$

where $\hat{w}_{h+1}$ denotes the upper end of the support of Firm $h+1$'s strategy. From Lemma 3 the offers by a higher firm stochastically dominate those by a lower firm, so that the second term is non-negative. Thus we have $\Pi_j - \Pi_{h+1} \geq V_j - V_{h+1}$. We can guarantee a strict inequality if the upper bound of Firm $h+1'$s offer is below $\bar{w}$. By construction the (expected) profit of Firm $h+1$ is the same whether or not a personalised offer to Worker $h+1$ is possible. Therefore, the expected profits of all other firms matched with bundled workers are weakly higher than those under the firm-optimal competitive equilibrium. Since a mixing firm's expected profit is the same for any wage offer it makes with positive probability, any firm that expects to hire a worse worker than in the efficient match (with positive probability) must offer strictly lower wages than the firm-optimal competitive one.

Thus, all we have left to show is that if there are at least three bundled workers, the highest offer Firm $h+1$ makes is strictly below $\bar{w}$. The solution of (1) if all the firms bid over the same support is $f_i(w) = \frac{1}{k}\sum_{j=h+1}^{h+k} D_j^{-1} - D_i^{-1}$. It is straightforward to see that $f_{h+1}(w) < 0$, contradicting the hypothesis that all firms bid over the same support. By Lemma 3 it must be that Firm $h+1$ does not bid near $\bar{w}$. ∎

Theorem 1 and Corollary 4 imply that, if $k \geq 3$, the wages of the more productive workers (strictly above $h+k$) who receive personalised offers are reduced relative to the competitive wages, but the wage differentials among them are the same.

Again, given the non-externality result of Theorem 1, for firms and workers in the bundle, Propositions 2 and 3 of Bulow and Levin (2006) apply: all firms except Firm $h+1$ enjoy a strict expected gain, while Firm $h+1$ is indifferent; given this and the resulting inefficiency, workers are strictly worse off in the aggregate but lower

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productivity workers ($h+1$ for sure) benefit. The novelty here is that the lower wages propagate to the high productivity firms hiring with personalised wages. That is, all the firms above $h+k$ are strictly better off by exactly as much as Firm $h+k$ is (since they hire the same worker as with personalised wages, but for $w_{h+k}^c - \overline{w}_{BL}^{h+1,h+k} > 0$ less). Naturally the opposite is the case for the high productivity workers above $h+k$.

6   Quality thresholds

Our analysis so far has assumed that every firm's output is the product of D_i and its worker's index. In practice, it may be that tasks involved in a high productivity firm/job requires particular (high) skills in order to produce anything at all. This section considers how such a quality threshold may affect our results. Suppose for simplicity that the output is zero if Firm i is matched with a worker of index $i-q$ or below, where $q \in \{1, 2, ..., i-1\}$.

If there is no bundling, this restriction does not affect the matching (PAM) or firm-optimal wages because, focusing on equilibrium undominated strategies, the equilibrium wage of Worker i is determined in such a way that Firm i is indifferent between hiring Worker i and Worker $i+1$. In other words, Firm $i+1$'s offer to Worker i does not play a role in equilibrium whether or not it can produce positive output with the Worker i, since it is better off hiring Worker $i+1$ by matching the (rational) offer from Firm i, than to hire Worker i (or lower) by matching the offers he has received from firms below $i+1$. This feature comes from the supermodularity of output, which implies the output differential between Worker i and Worker $i+1$ is larger for Firm $i+1$ than for Firm i.

Consider the effect of quality threshold on the match and wages of bundled workers. If $q \geq k$, that is, if the threshold is relatively low, then the quality threshold does not change the equilibrium matching and wages with bundling. This is

because i) from the above argument the equilibrium wages for Workers $1, ..., h$ are not affected; and because ii) the equilibrium wage of Worker $h + k + 1$ is determined by Firm $h + k$'s incentive to bid for Worker $h + k + 1$, which is to make it indifferent between hiring a bundled worker and hiring Worker $h + k + 1$ for certain. This means the quality threshold is irrelevant for the equilibrium construction, and we have $w_{h+k+1} = \overline{w}_{BL}^{h+1,h+k} + w_{h+1}^c + D_{h+k}$, and hence as we have seen in Theorem 1, $w_i = \overline{w}_{BL}^{h+1,h+k} + w_{h+1}^c + \sum_{j=h+k}^{h+k+i-1} D_j = w_i^c - \left(w_{h+k}^c - \overline{w}_{BL}^{h+1,h+k} \right)$ for $i \in \{h + k + 1, h + k + 2, ..., N\}$.

If $q < k$, that is, if the threshold is tight, then the situation becomes different. Due to equal treatment, a firm offering a bundled wage runs the risk of hiring a worker who is completely unproductive. This will lead to relatively aggressive bidding by more productive firms offering bundled wages, and as a result the matching inefficiency is reduced. To see this, consider the following example where Workers h and $h + 1$ are bundled and the workers are not productive in a firm above their index ($q = 1$). Now the equilibrium offer densities of the respective firms are given by $f_{h+1} = \frac{1}{(h+2)D_{h+2}}$ and $f_{h+2} = \frac{1}{D_{h+1}}$. Firm $h + 2$'s offer density is the same as in the case where there is no quality threshold, which implies the support of the offers also remains unchanged. However, the density of Firm $h + 1$ is lower than in the case without the quality threshold (and hence has a larger mass at w_{h+1}^c). Consequently Firm $h + 2$ is more likely to be matched with Worker $h + 2$.

The probability of mismatch is given by $\int_{w_h^c}^{w_{h+1}^c} f_{h+1}(x) F_{h+2}(x) dx = \int_{w_h^c}^{w_{h+1}^c} \frac{1}{(h+2)D_{h+2}} \cdot \frac{x - w_h^c}{D_{h+1}} dx = \frac{(w_{h+1}^c - w_h^c)^2}{D_{h+1}D_{h+2}} = \frac{(D_{h+1})^2}{D_{h+1}D_{h+2}} = \frac{D_{h+1}}{(h+2)D_{h+2}}$. The deadweight loss of the mismatch is $(h+2)D_{h+2} + (h+1)D_{h+1} - (h+2)D_{h+1} = (h+2)D_{h+2} - D_{h+1}$. Thus the expected loss due to mismatch (the only inefficiency) is $\frac{D_{h+1}}{(h+2)D_{h+2}} ((h+2)D_{h+2} - D_{h+1}) = \frac{D_{h+1}}{D_{h+2}} (D_{h+2} - \frac{D_{h+1}}{h+2})$. Since the expected loss from mismatch in the case with no quality threshold is $\frac{D_{h+1}}{D_{h+2}} (D_{h+2} - D_{h+1})$,[6] we can see that the cost of the loss in pro-

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[6]See the proof of Corollary 3.

duction from mismatch outweighs the efficiency gain from the reduced probability of mismatch.

When a bundle contains three or more workers/firms, the upper end of the wage distribution (of the bundled workers) becomes higher in the presence of a quality threshold. The offer densities for $q = 1$ with three types are as follows:

$$f_{h+1} = \frac{1}{(h+2)D_{h+2}} \text{ for } w_{h+2} \in [w_{h+1}^c, w_{h+1}^c + D_{h+1} - \frac{D_{h+1}D_{h+2}}{(h+3)D_{h+3}}] \text{ with mass at } 0$$

$$f_{h+2} = \begin{cases} \frac{1}{D_{h+1}} \text{ for } w_{h+2} \in [w_{h+1}^c, w_{h+1}^c + D_{h+1} - \frac{D_{h+1}D_{h+2}}{(h+3)D_{h+3}}] \\ \frac{1}{(h+3)D_{h+3}} \text{ for } w_{h+3} \in [w_{h+1}^c + D_{h+1} - \frac{D_{h+1}D_{h+2}}{(h+3)D_{h+3}}, w_{h+1}^c + D_{h+1} - \frac{D_{h+1}D_{h+2}}{(h+3)D_{h+3}} + D_{h+2}] \end{cases}$$

$$f_{h+3} = \frac{1}{D_{h+2}} \text{ for } w_{h+3} \in [w_{h+1}^c + D_{h+1} - \frac{D_{h+1}D_{h+2}}{(h+3)D_{h+3}}, w_{h+1}^c + D_{h+1} - \frac{D_{h+1}D_{h+2}}{(h+3)D_{h+3}} + D_{h+2}].$$

Without the quality threshold, the densities are

$$f_{h+1} = \frac{1}{D_{h+2}} \text{ for } w_{h+2} \in [w_{h+1}^c, w_{h+1}^c + D_{h+1} - \frac{D_{h+1}D_{h+2}}{D_{h+3}}] \text{ with mass at } 0$$

$$f_{h+2} = \begin{cases} \frac{1}{D_{h+1}} \text{ for } w_{h+2} \in [w_{h+1}^c, w_{h+1}^c + D_{h+1} - \frac{D_{h+1}D_{h+2}}{D_{h+3}}] \\ \frac{1}{D_{h+3}} \text{ for } w_{h+3} \in [w_{h+1}^c + D_{h+1} - \frac{D_{h+1}D_{h+2}}{D_{h+3}}, w_{h+1}^c + D_{h+1} - \frac{D_{h+1}D_{h+2}}{D_{h+3}} + D_{h+2}] \end{cases}$$

$$f_{h+3} = \frac{1}{D_{h+2}} \text{ for } w_{h+3} \in [w_{h+1}^c + D_{h+1} - \frac{D_{h+1}D_{h+2}}{D_{h+3}}, w_{h+1}^c + D_{h+1} - \frac{D_{h+1}D_{h+2}}{D_{h+3}} + D_{h+2}].$$

Therefore it is easy to see that the upper bound of the mixing support is higher with quality threshold, though still lower than in the firm-optimal competitive equilibrium.[7] From Theorem 1 we know that the size of the wage reduction for every worker above the bundled range is the difference between the unbundled competitive wage of the most productive worker in the bundle, w_{h+k}^c, and the upper bound of the support of wages offers to the bundled workers $\bar{w} \equiv \overline{w}_{BL}^{h+1,h+k} + w_{h+1}^c$. This implies that the wage reduction effect for the workers above the bundled workers is weaker, when there is a quality threshold.

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[7]Following the notation for Theorem 1, in the absence of the threshold the upper bound of the support is given by $w_{h+1}^c + \overline{w}_{BL}^{h+1,h+3} = w_{h+1}^c + D_{h+1} - \frac{D_{h+1}D_{h+2}}{D_{h+3}} + D_{h+2}$.

7   Conclusion

Wage determination processes vary according to worker types: while high-skilled workers receive personalised (often negotiated) wages, many low-skilled workers work at posted wages that do not differentiate between workers with slightly different qualifications or productivities. We have rigorously shown that the models of personalised and bundled wages can be integrated seamlessly. We have also demonstrated how those two different wage determination processes can interact with each other. In particular, impersonal wage offers lead to wage compression, which propagates to the wages for high productivity workers (and firms).

We have also derived comparative statics results about the size and number of bundles. Under reasonable assumptions on productivities, bundling is more efficient – or rather less inefficient – at lower levels of productivity. This is in line with the wide-spread salary policy of paying uniform wages at entry level positions but personalised ones higher up the echelon (Hall and Krueger, 2012; Brenčič, 2012).

Throughout this paper we have focused on undominated wage offers (hence firm-optimal competitive wages) for unbundled workers. If competing firms had an incentive to reduce the profits of others – say, because they competed in the same product market –, then the wages could be higher than the firm-optimal competitive level even if such incentive were very small.

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