A Model of Public Consultation: Why is Binary Communication So Common?

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October 2010

Abstract

This paper studies information transmission between multiple agents with heterogeneous preferences and a welfare maximizing decision maker who chooses the quality or quantity of a public good (e.g. size of a public project; pace of lectures in a classroom; government regulation) that is consumed by all of them. As the number of agents becomes larger, the quality of information transmission diminishes. The use of binary messages (e.g. 'yes or no') is shown to be a robust mode of communication even when the preferences and policy space are non-binary.

Short Title: PUBLIC CONSULTATION

In many social environments, policy decisions are made following consultation with interested parties. For example, a local authority may try to find the optimal public services policy for the community by discussing with the residents, or a teacher may ask her students how fast they would like her lectures to be. Non-binding referendums can also be thought of as a communication device between the government and citizens. Suppose that a benevolent decision maker chooses the quality or quantity of a public good that is consumed by members of a group with heterogeneous preferences, but no monetary transfers are allowed. In order to inform her decision, the decision maker may communicate with the members. Are they willing to reveal their private information truthfully? How does the number of members affect the nature of communication? Why is it very often the

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†I thank Vincent Crawford, Wouter Dessein, Mathias Dewatripont, Navin Kartik, David Myatt, Marco Ottaviani, Andrea Patacconi, John Quah, Jose V. Rodríguez Mora, József Sákovics, Joel Sobel, Koichi Tadenuma, an anonymous referee, and seminar participants at Aberdeen, Edinburgh, Oxford, and Southampton for helpful discussions and comments. I am particularly grateful to James Malcomson and Margaret Meyer for their guidance. All errors are my own.

case that, in communication with many individuals, the decision maker asks questions in a binary form (e.g. 'yes or no', 'agree or disagree') even when policy choice, preferences and state (distribution of preferences) may not be binary?[1]

This paper addresses these questions by modelling communication as 'cheap talk', where each agent (interested party) receives a private signal about his preference and sends a message to an uninformed decision maker who, on the basis of the information received from all agents, makes a decision that affects their welfare. We demonstrate that communication becomes distorted by agents' incentive to 'exaggerate' their preferences, as the number of agents becomes larger and hence each agent has less influence on the decision.[2]

Specifically, in the presence of multiple agents, extreme messages become less informative about the agents' preferences. For example, suppose that local government communicates with its residents to determine the size of a public project. Then a resident who is only slightly in favour of (against) the project may nonetheless exaggerate his enthusiasm (doubt) for it, since he knows that the local government will take into account the opinions of the other residents, who may have the opposite preference, as well as his own. It may thus be that the residents' responses should not be taken literally, to adjust for the incentive to exaggerate their preferences. Similarly, when asked about the pace of lectures in a large classroom, a good (weak) student who finds it 'somewhat' slow (fast) may however say the lecture is 'very' slow (fast), in an attempt to influence the teacher more than if he answered completely truthfully. In such circumstances, extreme messages tend to be used by people with non-extreme as well as extreme preferences and therefore become less informative.[3]

Interestingly, however, we find that the incentive to exaggerate does not itself completely eliminate the possibility of mutually beneficial communication. Regardless of the number of agents, there always exists an equilibrium where binary messages (e.g. 'yes or no') are credibly communicated. Moreover, as the number of agents becomes larger the most informative equilibrium communication converges to binary communication. The intuition

[1]Needless to say, certain policy decisions, such as the ratification of a treaty, are binary in nature. However, in many circumstances the policy space is much richer, even though it may be presented as a binary choice. Examples include gun control, abortion rights, immigration, and devolution, where there is a wide range of possibilities regarding the extent to which they are advanced or retreated.

[2]By incentive to 'exaggerate', we mean an agent's incentive to misreport in such a way that, if words are taken literally and believed by the decision maker, the agent whose type is high (low) 'overstates' ('understates') his type by saying his type is even higher (lower). In other words, incentive to 'exaggerate' includes both downward and upward biases simultaneously in a one dimensional policy/message space.

[3]In our formal analysis (and in many other cheap talk models) messages used are completely arbitrary and do not have to be taken literally. What matters for the equilibrium outcome is the correspondence between each agent's preference and the decision maker's induced action, so that what word (or language) is used to induce a particular action is irrelevant. Throughout this paper we interpret our results with the assumption that messages either refer to the agents' types directly, or have a meaning appropriate to the contexts we consider.

is very simple: if an agent is to choose to send only one of two messages, he cannot possibly exaggerate the intensity of his preference for what is meant by each message. Hence to the extent that the potential source of informational distortion is exaggeration, binary communication eliminates any incentive to misreport. When the number of agents is very large, informative communication becomes effectively binary or near binary to allow for the strong incentive to exaggerate. This may explain why the 'choice between the two' ('yes or no', 'agree or disagree', etc.) is a very common way of communication in the presence of multiple interested parties, even when (as in the size of a public project, pace of lectures, or strictness of regulation) their preferences and/or the decision made after communication are non-binary and binary communication does not necessarily provide sufficient information for efficient decision making.

In some situations, interested parties may have individual bias as well as incentive to exaggerate. For instance, people living close to a planned incineration plant would always be negatively biased with respect to its potential effects on their lives. Such individual bias is typically pointed in a particular direction regardless of the agent's private information. This is in contrast to incentive to exaggerate, which is pointed in either extreme (positive or negative) depending on the relative position of an agent's preference. Furthermore, individual bias can distort information transmission even if there is only a single agent affected by the decision, as it represents an intrinsic conflict of interest between the decision maker and agent. In this paper we also examine the interplay between those two different sources of informational distortion, namely individual bias caused by an intrinsic conflict of interest; and incentive to exaggerate due to the presence of multiple agents.

In this paper we extend the standard cheap talk model of Crawford and Sobel (1982). If there is only one informed agent, our model collapses to that of Crawford and Sobel (1982). This enables us to contrast the effect of the presence of multiple agents, which is our primary focus, and the effect of individual bias in one-to-one communication, which is the focus of their analysis. Melumad and Shibano (1991) and Gordon (2007) study exaggeration incentive in one-to-one communication settings and derive equilibria with a similar structure to ours.[4] Alonso *et al.* (2008) examine communication and delegation for decision making that requires coordination between multiple agents with possibly different preferences. In a similar framework, Carrasco and Fuchs (2008) propose a simple dynamic allocation rule that implements the optimal outcome. Both Alonso *et al.* (2008) and Carrasco and Fuchs (2008) focus on communication with two agents, while we are concerned with how the nature of communication changes according to the number of agents.

Morgan and Stocken (2008) study communication between a single decision maker and a continuum of agents in the context of polls.[5] They develop a cheap talk model related

[4]See also Blume *et al.* (2007).

[5]Feddersen and Pesendorfer (1997) analyze an information aggregation problem with a voting model. They assume that the message (voting) space is binary.

to ours but assume that the message space is binary while preferences and policy space are continuous. We do not restrict the message space a priori, and derive binary and near-binary communication endogenously. Also in Morgan and Stocken (2008), despite strategic communication, the decision maker can obtain sufficient information for the first-best policy as the size of the poll goes to infinity. In contrast, by explicitly incorporating complex uncertainty in the distribution of preferences, we are able to observe how equilibrium communication increases welfare but at the same time leads to inefficient information aggregation even when the number of agents is very large.

Kawamura (2010) considers optimal sample size in a model similar to the one in the present paper, but with a continuum of agents as in Morgan and Stocken (2008). Kawamura (2010) assumes that the decision maker is able to choose and commit to the number of agents she communicates with. In the present paper, the decision maker communicates with all the agents and the primary focus is on the effects of the number of interested parties (agents), rather than sample size.[6]

Grüner and Kiel (2004) and Rosar (2010) also study related models but they focus on the performance of different voting schemes (i.e. mechanisms) rather than cheap talk communication. Also, unlike the model developed in this paper, their models do not feature uncertainty in the probability distribution of preferences.[7]

Other papers that study communication with multiple informed parties include Gilligan and Krehbiel (1987, 1989), Baliga *et al.* (1997), Krishna and Morgan (2001), Ottaviani and Sørensen (2001) and Battaglini (2002, 2004), where agents observe the same or correlated states of nature while each agent has a different bias or ability. Among models of communication with multiple agents, our model is closer to Austen-Smith (1993) and Wolinsky (2002) where agents observe independent signals. Austen-Smith (1993) focuses on the comparison between simultaneous and sequential reporting, and Wolinsky (2002) considers information sharing between agents.

This paper proceeds as follows. The following section describes the model and examines informative equilibria when the distribution of agents' types is known to the decision maker. Section 2 extends the analysis to the case where the type distribution is imperfectly known, which is of particular relevance when the number of agents is large. Section 3 concludes.

[6] In fact, as we will see later, if the decision maker lacks commitment to a predetermined sample size, then she ex post wishes to communicate with all the affected agents. In certain collective decision making environments the decision maker may be required to communicate with all interested parties on the basis of fairness.

[7] As a result, as the population size goes to infinity, a welfare maximizing decision maker in Grüner and Kiel (2004) and Rosar (2010) can implement the first-best policy with probability 1 even if she is completely uninformed about the agents' private information.

1 Model

Let us consider communication between a single decision maker and n agents labelled by $i \in \{1, 2, .., n\}$. Each agent may have a different preference for the decision maker's action, denoted by $y \in \mathbb{R}$, and the utility of agent i is given by

$$U^{Ai}(y, \theta_i) = -(y - \theta_i)^2 \text{ for all } i, \tag{1}$$

where θ_i represents the agent's preference for y and is private information to agent i. In the context of the public project example, the government chooses the size of the project y that affects the residents with heterogeneous preferences. Since (1) is a quadratic loss function, each agent has an ideal policy $y = \theta_i$ that maximizes his utility. The decision maker maximizes the sum of all agents' utilities

$$U^{DM}(y, \boldsymbol{\theta}) = -\sum_{i=1}^{n}(y - \theta_i)^2, \tag{2}$$

where $\boldsymbol{\theta} = [\theta_1, \theta_2, ..., \theta_i, ..., \theta_n] \in [0, 1]^n$. The decision maker can be thought of as a utilitarian social welfare maximizer who nonetheless acts as a 'player'. She is a 'player' in the sense that she determines her action strategically after receiving the messages. In other words, she does not commit to a pre-determined mechanism that automatically implements y according to the reported messages. There is no conflict at the individual level. That is, if $n = 1$, both the decision maker and the agent share the same utility function.

The agent's type θ_i is independently and uniformly distributed on $[0, 1]$.[8] Let $m_i \in M$ be the message agent i reports to the decision maker, where the message space M has enough elements to cover all types, and is shared by all agents. This means that, unlike the voting literature where the message space is typically *assumed* to be binary, we do not impose a strong a priori restriction on the messages to be reported.

Every agent reports a costless message after learning his type but before the decision maker takes her action.[9] Prior to choosing y, the decision maker updates her belief on θ_i according to the message from agent i, since the decision maker cannot observe the agent's type directly. The timing is restated as follows:

1. All agents privately learn their types;
2. The agents send messages to the decision maker;
3. The decision maker chooses her action y.

The first order condition for the decision maker's maximization problem gives her best response function

$$y(m_1, m_2, ..., m_n) \equiv \frac{1}{n}\sum_{i=1}^{n} E[\theta_i \mid m_i]. \tag{3}$$

[8] We relax this independence assumption in Section 2.

[9] In the current setup the decision maker is strictly better off communicating with all the agents because, as we will see shortly, each agent's strategy is independent from the other agents' strategies.

In other words, the decision maker's best response coincides with the average of the expected type of each agent.

Let us consider agent i's strategy. From his viewpoint, after sending his message the decision maker's action is still a random variable, because it depends on the other agents' messages too. In other words, each message induces a corresponding distribution of the decision maker's action y. Since the utility functions are quadratic and the choice of message does not change the variance of the induced distribution, it suffices to consider the expected value of the decision maker's action. Since each agent does not observe the other agents' types or messages, (3) implies that the expected action from the agent's viewpoint conditional on his own message is given by

$$\begin{aligned} y_A(m_i) &= \frac{1}{n}E[\theta_i \mid m_i] + \frac{n-1}{n}E\left[E[\theta_{-i} \mid m_{-i}]\right] \\ &= \frac{1}{n}E[\theta_i \mid m_i] + \frac{n-1}{n}\frac{1}{2}, \end{aligned} \tag{4}$$

where θ_{-i} denotes the type of any agent other than i. The second equality follows from the fact that

$$E\left[E[\theta_{-i} \mid m_{-i}]\right] = E[\theta_{-i}] = \frac{1}{2}.$$

We call $y_A(\cdot)$ the *reaction* function, or the decision maker's expected *reaction* to the message from a particular agent. This is to be distinguished from the *best response* function $y(m_1, m_2, ..., m_n)$ in (3), which is a function of the messages from all agents. (4) implies that the decision maker's expected reaction is independent of the other agents' strategies. Therefore, to derive an equilibrium strategy of any agent, we can focus on his individual best response to (4) without taking into account the other agents.[10]

Let us illustrate how $n \geq 2$ gives rise to incentive not to reveal truthfully. From (4) we can see that an agent's message has less influence on the decision maker's action as n becomes larger, since his conditional expected type is weighted at $1/n$. The expected reaction is weighted towards the expected type of any other agent, $1/2$.

Define

$$\begin{aligned} y^A(\theta_i) &\equiv \theta_i \\ y^{DM}(\theta_i, n) &\equiv \frac{1}{n}\theta_i + \frac{n-1}{n}\frac{1}{2}, \end{aligned}$$

where $y^A(\theta_i)$ denotes the agent's ideal action given his type θ_i, and $y^{DM}(\theta_i, n)$ is the decision maker's expected reaction given that θ_i is perfectly revealed to her.[11] If the agent's type is

[10] This also means that limiting sample size to less than n does not change the communicating agents' equilibrium strategies. Therefore, the decision maker strictly prefers to communicate with every agent.

[11] Note that $y_A(m_i)$ denotes the decision maker's expected reaction as a function of the agent's message, while $y^A(\theta_i)$ is the agent's ideal action as a function of his type.

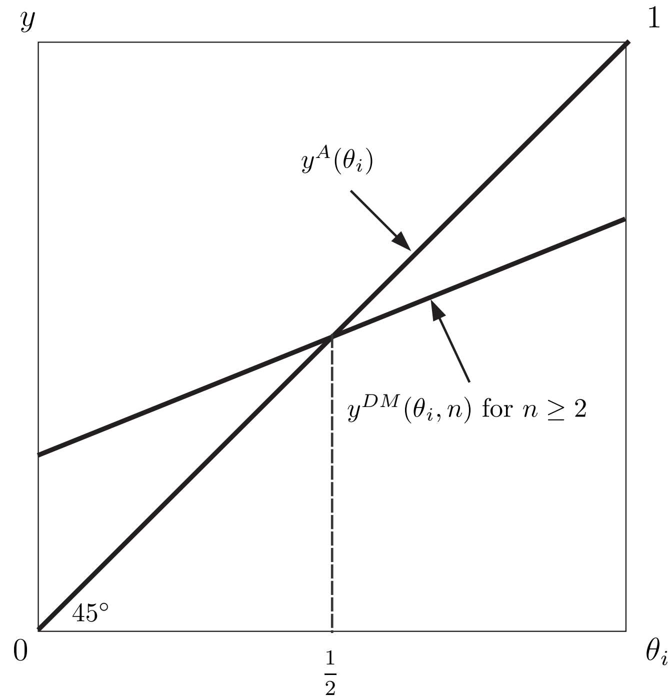

Fig. 1. *Agent's Ideal Action and Decision Maker's Reaction*

$\theta_i = 1/2$ we have $y^A(\theta_i) = y^{DM}(\theta_i, n)$ for any n. Except for $\theta_i = 1/2$, $y^A(\theta_i)$ and $y^{DM}(\theta_i, n)$ do not coincide when there are two or more agents.

The agent's ideal action and the decision maker's reaction for given θ_i are depicted in Figure 1, where the horizontal axis represents the agent's type θ_i and the vertical axis represents the decision maker's action y. When the decision affects only a single agent ($n = 1$), the decision maker's reaction given that the agent reports his type θ_i truthfully is $y^{DM}(\theta_i, 1) = \theta_i$, the 45 degree line, which implies $y^{DM}(\theta_i, 1) = y^A(\theta_i) = \theta_i$. In this case, the agent can induce his ideal action simply by revealing truthfully, because both parties' interests are perfectly aligned for all θ_i. However, when $n \geq 2$ the agent's ideal action may be higher or lower than the decision maker's reaction depending on his type. If $\theta_i < 1/2$, we have $y^A(\theta_i) < y^{DM}(\theta_i, n)$ so that the agent's ideal action is lower than the decision maker's expected reaction. On the other hand, if $\theta_i > 1/2$ then $y^A(\theta_i) > y^{DM}(\theta_i, n)$, which implies that the agent's ideal action is higher.

Figure 1 summarizes the nature of informational distortion we are considering. If the decision maker naively believes the agents, they have incentive to exaggerate their types, in the sense that an agent whose type is low (i.e. below 1/2) would report an even lower type than his type, and an agent whose type is high (above 1/2) would report an even higher type. Hence the following proposition holds:

Proposition 1. *For any $n \geq 2$ there does not exist a fully revealing equilibrium.*

1.1. *Equilibrium*

Let us examine the informative equilibria that take into account the agents' incentive to exaggerate.[12] We first derive an agent's equilibrium strategy given the decision maker's reaction (4). Let us introduce an alternative representation of the decision maker's reaction. Let $\underline{a}$ and $\overline{a}$ be two points in $[0,1]$ such that $\underline{a} < \overline{a}$. From (4) and the assumption that θ_i is uniformly distributed, we have

$$E[\theta_i \mid \theta_i \in [\underline{a}, \overline{a})] = \frac{\underline{a} + \overline{a}}{2}.$$

Define

$$\bar{y}_A(\underline{a}, \overline{a}) \equiv \frac{1}{n}\frac{\underline{a} + \overline{a}}{2} + \frac{n-1}{n}\frac{1}{2}. \tag{5}$$

$\bar{y}_A(\underline{a}, \overline{a})$ is the expected reaction from the agent's viewpoint, conditional on the decision maker's belief that the agent's type is such that $\theta \in [\underline{a}, \overline{a})$.[13] If $\theta_i = a$ then we write $\bar{y}_A(a, a)$. While $y_A(m)$ is defined as a function of the agent's message, $\bar{y}_A(\underline{a}, \overline{a})$ is a function of an interval although they both denote the decision maker's reaction. Note that the decision maker's action is a random variable from the agent's viewpoint. However, the randomness is caused only by messages from the other agents. Therefore, the variance of the decision maker's action is independent from the agent's strategy (message). The quadratic utility functions imply that we can focus our attention on the decision maker's expected reaction $\bar{y}_A$.

In any equilibrium partition, each boundary type $a_j \in (0,1)$ must satisfy the 'arbitrage' condition which says that the agent with $\theta_i = a_j$ is indifferent between inducing $\bar{y}_A(a_{j-1}, a_j)$ and $\bar{y}_A(a_j, a_{j+1})$. Solving the condition

$$-(\bar{y}_A(a_{j-1}, a_j) - a_j)^2 = -(\bar{y}_A(a_j, a_{j+1}) - a_j)^2 \tag{6}$$

by using (5) we obtain a second-order difference equation

$$\frac{1}{n}a_{j+1} - \left(4 - \frac{2}{n}\right) a_j + \frac{1}{n}a_{j-1} = \frac{2}{n} - 2. \tag{7}$$

From (7) we can easily construct the following example of an informative equilibrium:[14]

Example 1. The partitional strategy $\{[0, 1/2), [1/2, 1]\}$ supports a perfect Bayesian equilibrium for any n.

[12] To be precise, by informative equilibrium we mean an equilibrium where with strictly positive probability the decision maker's action is different from the action she chooses based only on her prior belief. The uninformative equilibrium refers to the equilibrium where the receiver's action is based only on her prior belief with probability 1.

[13] We drop the subscript i for y_A and boundary types a_j to simplify notation.

[14] Solve (7) for $a_0 = 0$ and $a_2 = 1$.

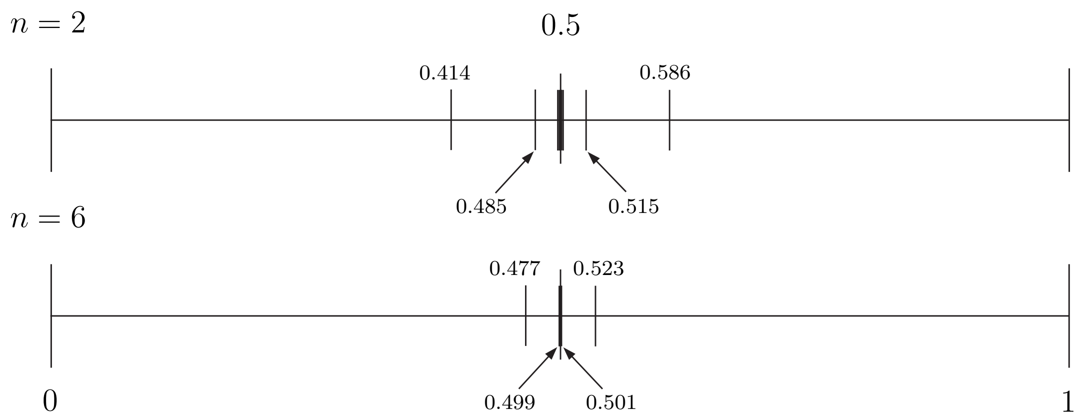

Fig. 2. *Equilibrium with Infinite Partition*

This example points to the 'robustness' of binary communication to exaggeration. As we have seen in Figure 1, the more agents there are, the stronger the incentive to exaggerate is. However, regardless of the number of agents, when an agent has the choice between two messages and is unable to send any other messages credibly, he has no room for exaggerating his preference.[15] Since the only source of informational distortion is exaggeration in the present setting, binary messages completely eliminate the incentive to misrepresent.

However, binary communication is not the unique informative equilibrium. In fact, it is easy to see that (6) can generate an infinite number of informative equilibria, by solving (6) for $a_0 = 0$ and $a_J = 1$. The following proposition states that, if we look for the equilibrium where the ex ante welfare is highest for given n, we should focus on the equilibrium that has the largest number of intervals.

PROPOSITION 2. *For any $n \geq 2$, both the decision maker and the agents are ex ante better off in an equilibrium with more intervals.*

Proof. See Appendix. ■

Let us consider the partition in the equilibrium with the largest number of intervals, which we also call the *most informative equilibrium.* Solving (7) with respect to a_j explicitly and letting $J \to \infty$, which give us the following sequences

$$a_j = \frac{1}{2} - \frac{1}{2}\left(-1 + 2n - 2\sqrt{n(n-1)}\right)^j \tag{8}$$

[15]This binary partitional equilibrium played with two messages is supported, for example, by assuming that the decision maker believes all out-of-equilibrium messages are sent by agents in one particular interval. Strictly speaking, as long as the agents in each interval induce the corresponding belief (expectation) on their types, the actual messages used in equilibrium are irrelevant to the outcome of the game. Therefore, in principle, the binary partition equilibrium can be played with more than two equilibrium messages. In relating the model to communication in practice, however, we posit that an equilibrium is played with the minimum number of messages for the equilibrium outcome.

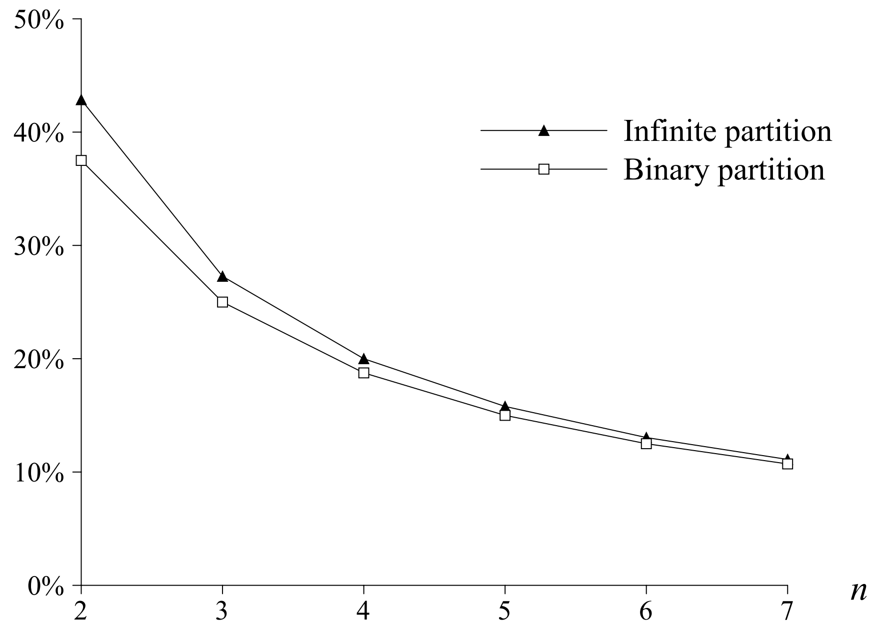

Fig. 3. *Percentage Increase in* EU^{DM} *and* EU^{A_i} *Relative to Uninformative Equilibrium*

and

$$a'_j = \frac{1}{2} + \frac{1}{2}\left(-1 + 2n - 2\sqrt{n(n-1)}\right)^j \tag{9}$$

where $a_1 = 0$ and $a'_1 = 1$. It is easy to show that (8) gives a strictly increasing sequence and (9) gives a strictly decreasing sequence, both of which converge to the average type 1/2. In this equilibrium an infinite number of messages are sent with positive probability but the messages are not fully revealing. The equilibrium partition is illustrated in Figure 2, where the horizontal lines denote the type space of an agent $\theta_i \in [0,1]$. We can see that the most informative equilibrium has an infinite number of intervals in the neighbourhood of 1/2. The length of intervals is longer as they are away from 1/2 and is narrower as they are closer to 1/2, which implies that agent types are more accurately inferred when they are closer to the average.

How beneficial is the most informative equilibrium compared to the others? Figure 3 indicates that the loss from playing the binary partition equilibrium we have seen in Example 1 as opposed to the most informative one (with the infinite partition) can be very small. In Figure 3 the number of agents is on the horizontal axis and the percentage increase in an agent's expected utility (= percentage increase in the decision maker's expected utility when every agent adopts the same strategy) relative to no communication is on the vertical axis. We can see that the difference between the expected utility in the most informative equilibrium and the expected utility in the binary communication equilibrium diminishes as n becomes larger. The diminishing difference implies that messages communicated in the most informative equilibrium on average become less informative, due to stronger incentive to exaggerate.

1.2. *Application to Multiple Choice Questionnaires*

When more than a few people are affected by a decision, multiple choice questions are commonly asked to find the preferences of the interested parties. Such questions can be simple 'Yes or No', or slightly elaborated as in 'Strongly Agree; Agree; Neutral; Disagree; Strongly Disagree' (five choices). The analysis so far indicates that i) an arbitrary number of informative messages can be used, but that ii) the benefit of having additional messages in equilibrium can be severely limited when the number of agents is large. The prevalent use of five or less choices may suggest that the informational benefit of allowing richer messages (e.g. free answer to a question) is indeed smaller as the number of people involved becomes larger.

Another observation from our analysis is that, when the question involves four or more choices the weight to be put on each choice must be adjusted to take into account the incentive to exaggerate. Let us consider the following example of informative equilibria with five intervals, which corresponds to the choice of 'Strongly Agree; Agree; Neutral; Disagree; Strongly Disagree':

EXAMPLE 2. The following partitional strategy[16] supports a perfect Bayesian equilibrium for any n:

$$\{[\underbrace{0}_{a_0},\underbrace{\frac{8n^2-8n+1}{16n^2-12n+1}}_{a_1}),[\underbrace{\frac{8n^2-8n+1}{16n^2-12n+1}}_{a_1},\underbrace{\frac{8n^2-6n}{16n^2-12n+1}}_{a_2}),[\underbrace{\frac{8n^2-6n}{16n^2-12n+1}}_{a_2},\underbrace{\frac{8n^2-6n+1}{16n^2-12n+1}}_{a_3}),$$
$$[\underbrace{\frac{8n^2-6n+1}{16n^2-12n+1}}_{a_3},\underbrace{\frac{8n^2-4n}{16n^2-12n+1}}_{a_4}),[\underbrace{\frac{8n^2-4n}{16n^2-12n+1}}_{a_4},\underbrace{1}_{a_5}]\}.$$

It is easy to see that the first boundary a_1 is increasing in n and the last boundary type a_4 is decreasing in n. In the case of a questionnaire with five choices, as the number of agents increases, a wider range of agent types report extreme messages, namely 'Strongly Agree' and 'Strongly Disagree'. Moreover, the first (last) boundary type increases (decreases) in n at a faster rate than the second boundary type a_2 (third boundary type a_3). Therefore when n is large, the informational content of those extreme messages may be very limited as most types fall into the first or the last intervals.

1.3. *Approximation to Binary Partition*

How do the characteristics of the most informative equilibrium change when the number of agents n increases? Since

$$-1+2n-2\sqrt{n(n-1)}$$

[16] Solve (7) for $a_0 = 0$ and $a_5 = 1$.

in (8) and (9) is decreasing in n for any $n \geq 2$, (8) and (9) imply that as n increases, every boundary type except for $a_0 = 0$ and $a'_0 = 1$ becomes closer to $1/2$. Intuitively, as the number of agents becomes larger the intervals in the most informative equilibrium are more concentrated around $1/2$, because the incentive to exaggerate is stronger and messages from agents whose types are away from $1/2$ become less and less informative. In particular, as $n \to \infty$, we have $a_1, a'_1 \to 1/2$: as the number of agents goes to infinity the most informative equilibrium converges to the equilibrium with two intervals. Even if the decision maker and the agents play the equilibrium with an infinite number of intervals, the probability that an agent induces either $\bar{y}(0, a_0)$ or $\bar{y}(a'_1, 1)$ may be close to 1, in which case communication may look as if the agents effectively send binary messages.

Note however that the agents' types are assumed to be drawn independently from a known identical (uniform) distribution. One important consequence of this iid assumption is that as the number of agents goes to infinity, by the law of large numbers the average type converges to $1/2$ with probability 1. This implies that $y = 1/2$ is almost surely a near-optimal action and communication is irrelevant when n is very large. In Section 2 we consider a non-iid setting in which the probability distribution of agent types is imperfectly known and communication does have a significant effect on the decision maker's action even with a large population.

1.4. *Individually Biased Agents*

Thus far we have assumed that there is no intrinsic preference divergence between the decision maker and each agent, so that if $n = 1$, both parties interests' perfectly coincide and perfect communication is possible. However, in public consultation, individuals may be biased towards a particular direction of policy. For example, there may be a fundamental conflict of interest between the government and people who live close to a large public project (incineration plant, airport, etc.) that can generate negative spillover effects. The government may want to know the local residents' opinions about the potential effects before implementing the project, but the residents would be biased against the project whatever their private information is. This type of intrinsic conflict of interest between the decision maker and an agent is known to give rise to informational distortion even when $n = 1$ (Crawford and Sobel, 1982). Here we examine the interaction between individual bias caused by an intrinsic conflict of interest, and the incentive to exaggerate due to the presence of multiple agents.

To formalize the idea, the utility of the decision maker is given by $-\sum_{i=1}^{n}(y-\theta_i)^2$ as above, but let agent i's utility be $-(y - \theta_i - b_i)^2$. We assume that $b_i \geq 0$ is common knowledge but θ_i is private information to agent i, and independently and uniformly distributed on $[0, 1]$. In the public project example above, the assumption that b_i is common knowledge

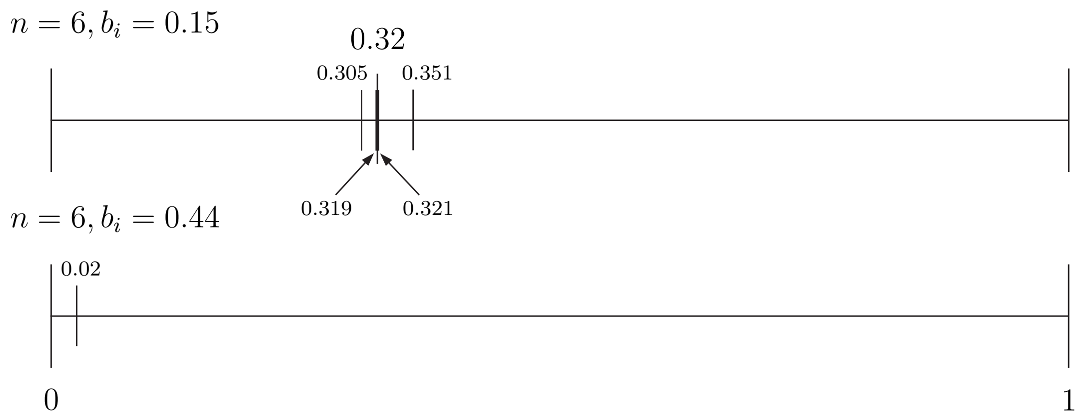

Fig. 4. *Communication with Individually Biased Agent*

would be appropriate if the government knows where the residents live. Each resident may have a different b_i, depending on how close their house is to the planned project. Note that $b_i > 0$ for at least one agent implies that the decision maker is no longer a utilitarian social welfare maximizer. The agent's ideal action and the decision maker's expected reaction when the agent reveals truthfully are

$$y^A(\theta_i, b_i) \equiv \theta_i + b_i$$

and

$$y^{DM}(\theta_i, n) \equiv \frac{1}{n}\theta_i + \frac{n-1}{n}\frac{1}{2},$$

respectively.

From the 'arbitrage' condition (6) we obtain the following second-order difference equation:

$$\frac{1}{n}a_{j+1} - \left(4 - \frac{2}{n}\right) a_j + \frac{1}{n}a_{j-1} = 4b_i + \frac{2}{n} - 2. \tag{10}$$

(10) determines the equilibrium partition for communication between the decision maker and agent i. Note that the equilibrium partitions obtained from (10) depend on each agent's individual bias b_i and thus differ among agents with different biases. When $b_i = 0$ the structure of informative equilibria is the same as the one we have seen earlier.

Figure 4 illustrates the equilibrium partitions with the largest number of intervals for $n = 6$ and $b_i > 0$. In the first case where $b_i = 0.15$, we have $y^A(\theta_i, b_i) = y^{DM}(\theta_i, n)$ at $\theta_i = 0.32$, so that there are an infinite number of intervals in its neighbourhood.[17] Note that the length of an interval is longer as it is farther from 0.32, which implies that the incentive to exaggerate *relative to* 0.32 is taken into account in equilibrium. In other words,

[17]We can show that the ex ante expected utilities of both the decision maker and agents are higher in an equilibrium with more intervals. The proof is very similar to that of Proposition 2.

an agent biased in one direction at the individual level may still be biased towards both extremes in the presence of multiple counterparts.

However, if individual bias is large, it dominates incentive to exaggerate. See the second partition in Figure 4 for $b_i = 0.44$, where at most two intervals can be supported in equilibrium. There the equilibrium partition accommodates only the upward bias due to a large intrinsic conflict of interest $b_i = 0.44$. As a result, messages from types in the upper interval $\theta_i \in [0.02, 1]$ are less informative than those from types the lower interval $\theta_i \in [0, 0.02)$. An informative equilibrium ceases to exist for $b_i > 11/24 \approx 0.458$. That is, unlike the case without individual bias, the binary partition equilibrium may not exist when b_i is very large.

2 Unknown Type Distribution

So far we have assumed that the agent types are independently and uniformly distributed. Generally speaking, if the type distribution is iid as in the previous section, then as the number of agents becomes larger, due to the law of large numbers, the prior expected type ($y = 1/2$) coincides with the (near) optimal policy and hence communication becomes irrelevant. However, the decision maker may not necessarily have enough information about the nature of the population, in which case neither she nor the agents may know the underlying probability distribution of preferences. This assumption seems appropriate especially when analyzing large-scale surveys or non-binding referendums.

In this section we consider the case where the probability distribution of preferences is uncertain and thus the law of large numbers does not apply. Then clearly, even if the number of agents is large, the decision maker would benefit from communicating with the agents and obtaining information about their preferences. In what follows we present a tractable model with unknown type distribution and show that binary communication emerges again as a robust mode of communication. We will also see that the decision maker cannot identify the first-best policy because she is unable to precisely infer the true distribution of preferences. The model developed in this section offers an explanation for the wide use of binary communication in large scale surveys, polls and referendums, but at the same time points to the impossibility of perfect information aggregation.

As in the previous section, let us consider communication between a welfare maximizing decision maker and n agents. We assume that their utility functions are the same as those in the previous section. If every agent reveals truthfully, the decision maker can simply choose the optimal action $y = \frac{1}{n}\sum_{i=1}^{n}\theta_i$, the average of the agents' (known) types. For tractability the type space is assumed to be discrete. The number of types is $H \geq 4$, and they are equally spaced on $[0, 1]$: $\theta^h < \theta^{h+1}$ and $\theta^h - \theta^{h-1} = \theta^{h+1} - \theta^h$, where $\theta^1 = 0$ and $\theta^H = 1$.

The location for each type θ^h is common knowledge. Let p^h be the probability mass of type θ^h, where $\sum_{h=1}^{H} p^h = 1$. We introduce the uncertainty about the probability distribution of preferences by assuming that the probability vector $p = \left(p^1, p^2, ..., p^H\right)$ follows the standard Dirichlet (multivariate beta) distribution with parameters $\alpha^1, \alpha^2, ..., \alpha^H > 0$. The distribution of the probability vector is also common knowledge. That is, both the decision maker and the agents share a common prior on the underlying probability distribution.

The Dirichlet is conjugate to the multinomial distribution and has several convenient properties for our analysis.[18] In particular the marginal prior distribution of p^h is the beta distribution $B(\alpha^h, \alpha^0 - \alpha^h)$, where $\alpha^0 \equiv \sum_{h=1}^{H} \alpha^h$. This implies that for any h the prior expectation of p^h is given by $E[p^h] = \frac{\alpha^h}{\alpha^0}$. Following the previous section, let the prior distribution be uniform, so that the probability vector is given by $\hat{p} = (1/H, 1/H, ..., 1/H)$.[19] It follows that $\alpha^1 = \alpha^2 =, ..., = \alpha^H \equiv \alpha$ and $\alpha^0 = H\alpha$. The prior expected type of the agents is therefore given by $\frac{1}{H} \sum_{h=1}^{H} \theta^h = 1/2$. We can interpret α as the strength of the prior belief about the underlying probability distribution. In particular, if $\alpha \to \infty$ then we have the uniform iid distribution such that $p = \hat{p}$ with probability 1. When α is low, then the prior and posterior are likely to be very different. We will shortly discuss α in more detail.

As in the previous section, suppose that every agent sends a message to the decision maker before she chooses her action.[20] From the strategic perspective, the main difference between the current setting and the iid setting in the previous section is that the players update their beliefs on the underlying probability distribution. Each agent does so according to his own type, and the decision maker updates her belief according to the messages she has received. Let us consider an agent's posterior after he has learnt that his type is θ^g. From this agent's viewpoint, the posterior marginal distribution of the other agents' types is $B(\alpha + 1, H\alpha - \alpha)$ for θ^g and $B(\alpha, H\alpha - \alpha + 1)$ for $\theta^h \neq \theta^g$. This implies $E[p^g \mid \theta_i = \theta^g] = \frac{\alpha+1}{H\alpha+1}$ and $E[p^h \mid \theta_i = \theta^g] = \frac{\alpha}{H\alpha+1}$. That is, any agent believes that the others are more likely to be of his own type and less likely to be of the other types, in comparison to the prior. In other words, the other agents' types are correlated with his own. The lower α is, the higher the correlation is.

Let us assume that all agents adopt a symmetric partitional strategy, where any agent in a certain type group $\{\theta^h, \theta^{h+1}, ..., \theta^{h+k-1}\}$ sends essentially the same message. Note that

[18] See DeGroot (1970), pp. 49-51, 174-175.

[19] The uniform prior assumption is not essential for our qualitative results, as we will discuss later.

[20] The assumption that the decision maker communicates with n agents is consistent with her inability to commit to a mechanism, because after she has received less than n messages the decision maker is tempted to communicate with more agents. Also, for some policies the decision maker may be required to consult all interested parties due to social/institutional reasons. Unlike the previous setting with a known distribution, however, if the decision maker can commit to a pre-determined sample size, then she may prefer to sample less than n agents since those agents may reveal more information. Kawamura (2010) studies random sampling when the population size is infinite.

k is also the number of types in a group. Under the Dirichlet assumption, the conditional expectation of an agent's type in a group is given by

$$\bar{\theta}(\theta^h, \theta^{h+k-1}) \equiv E[\theta_i \mid \theta_i \in \{\theta^h, \theta^{h+1}, ..., \theta^{h+k-1}\}] = \frac{1}{k} \sum_{t=h}^{h+k-1} \theta^t. \tag{11}$$

Note that (11) is independent from the realization of types. In other words, (11) is not affected by the number of agents who belong to the group or to any other groups. Thus the conditional expectation on the type of an agent in the group remains unchanged, before and after the decision maker receives messages.[21]

Let an agent's type be $\theta_i \in \{\theta^h, \theta^{h+1}, ..., \theta^{h+k-1}\}$ and consider from his perspective how the other agents' messages influence the decision maker's action. Let us focus on the expected influence of agent $-i$ such that $-i \neq i$. From agent i's viewpoint, $\theta_{-i} = \theta_i$ with probability $\frac{\alpha+1}{H\alpha+1}$ and $\theta_{-i} = \theta^h$ with probability $\frac{\alpha}{H\alpha+1}$ for any $\theta^h \neq \theta_i$. This means that, since there are k types in the group agent i belongs to, the probability that agent $-i$ sends the same message as agent i is $\frac{\alpha+1}{H\alpha+1} + (k-1)\frac{\alpha}{H\alpha+1} = \frac{1}{H\alpha+1} + k\frac{\alpha}{H\alpha+1}$. On the other hand, the probability that agent $-i$ sends the message that corresponds to any other group that consists of k' types is given by $k'\frac{\alpha}{H\alpha+1}$. By weighting the expected type of each group according to these probabilities, we obtain the decision maker's expectation of the type of agent $-i$, from agent i's perspective

$$\bar{\mu}(\theta^h, \theta^{h+k-1}) = \frac{H\alpha}{H\alpha+1}\frac{1}{2} + \frac{1}{H\alpha+1}\bar{\theta}(\theta^h, \theta^{h+k-1}), \tag{12}$$

which is a convex combination of the prior expected type 1/2 and the expected type of agent i's group $\bar{\theta}(\theta^h, \theta^{h+k-1})$.[22] From his perspective, $\bar{\mu}(\theta^h, \theta^{h+k-1})$ can also be thought of as the direction in which the other agents influence the decision maker's action through their messages. As the number of agents becomes larger, the decision maker's expected action is inclined towards (12), which gives rise to incentive to misreport.

Let us examine binary communication in light of (12). Suppose H is an even number, so that $\theta^h \neq 1/2$ for any h. Consider the binary partition of the type space $\{\theta^1, ..., \theta^{H/2}\}$, $\{\theta^{H/2+1}, ..., \theta^H\}$, where $\theta^{H/2} < 1/2 < \theta^{H/2+1}$. We then have

$$\bar{\mu}(\theta^1, \theta^{H/2}) \geq \theta^{H/2} \text{ and } \bar{\mu}(\theta^{H/2+1}, \theta^H) \leq \theta^{H/2+1} \tag{13}$$

for $\alpha \geq 1/2 - 1/H$.[23] If (13) holds, every agent below 1/2 expects the other agents to shift

[21] The important technical advantage of the independence of (11) from the realization of types is that the agents' choice of messages changes only the expectation, but not the variance of the distribution of the decision maker's action. This allows us to focus on the expected action throughout the analysis in this section.

[22] See Appendix for a formal derivation of $\bar{\mu}(\theta^h, \theta^{h+k-1})$.

[23] Since types are equally spaced on $[0, 1]$, if H is an even number we have $\theta^{H/2} = \frac{H-2}{2(H-1)}$, $\bar{\theta}(\theta^1, \theta^{H/2}) = \frac{H-2}{4(H-1)}$ and $\bar{\mu}(\theta^1, \theta^{H/2}) = \frac{H\alpha}{H\alpha+1}\frac{1}{2} + \frac{1}{H\alpha+1}\frac{H-2}{4(H-1)}$. Then $\bar{\mu}(\theta^1, \theta^{H/2}) \geq \theta^{H/2} \implies \alpha \geq 1/2 - 1/H$ and similarly for $\bar{\mu}(\theta^{H/2+1}, \theta^H) \leq \theta^{H/2+1}$.

the decision maker's action upwards to 1/2 relative to his ideal action, and every agent above 1/2 expects the other agents to shift the action downwards to 1/2. Intuitively, this implies that in order to countervail the other agents' influence, the best any agent can do is to stick to the binary strategy.[24]

As we have seen earlier, α is associated with the correlation of types. If $\alpha \to 0$ then the types are perfectly correlated in the sense that every agent has the same type. If $\alpha \to \infty$ then the agent types are independently distributed. It is common to interpret $\alpha = 1$ as representing no prior information about the underlying probability distribution, since then the probability vector $p = \left(p^1, p^2, ..., p^H\right)$ is uniformly distributed over its support (i.e. the unit $H-1$ simplex). When $\alpha < 1$ the distribution is likely to be sparse, in the sense that the majority of the probability mass is concentrated on a small subset of the p^h's, which is a direct consequence of strong correlation. From a practical viewpoint, α close to 0 would be of less interest because it means that most agents are likely to share the same type while the decision maker alone has little idea which type it is. This may not be realistic in the context of large scale public consultation, where some dispersion of preferences is normally expected. In what follows we assume $\alpha \geq 1/2 - 1/H$ so that (13) holds.

The decision maker's best response is given by (3). We have seen that the main difference from the known distribution (iid) setting in the previous section is the additional feature that each agent updates his belief on the underlying probability distribution of the other agents. The following proposition states that uncertainty in the probability distribution of types does not affect the robustness of binary communication.

Proposition 3. *There exists a binary communication equilibrium for any n.*

Proof. See Appendix. ■

The intuition is similar to the one we have discussed for Example 1 in the iid setting. As we have discussed above in relation to (13), if they could, the agents would have incentive to send extreme messages to render the decision maker's action closer to their ideal. However, binary communication rules out this possibility and hence supports an informative equilibrium. The following proposition is a non-iid analogue of our discussion in Section 1.3.

Proposition 4. *There exists $\bar{n}$ such that for $n \geq \bar{n}$, the most informative equilibrium is binary if $\theta^h \neq 1/2$ for any h, and ternary if there exists $\theta^h = 1/2$.*

Proof. See Appendix. ■

[24] If H is an odd number, for the ternary partition $\{\theta^1, ..., \theta^{(H+1)/2-1}\}, \{\theta^{(H+1)/2}\}, \{\theta^{(H+1)/2+1}, ..., \theta^H\}$ where $\theta^{(H+1)/2} = 1/2$, we obtain $\bar{\mu}(\theta^1, \theta^{(H+1)/2-1}) \geq \theta^{(H+1)/2-1}$ and $\bar{\mu}(\theta^{(H+1)/2+1}, \theta^H) \leq \theta^{(H+1)/2+1}$ when $\alpha \geq \frac{1}{4} - \frac{3}{4H}$, which is implied by $\alpha \geq 1/2 - 1/H$.

The main intuition behind the proposition is closely related to our convergence discussion for the iid setting in Section 1. Suppose that H is an even number so that $\theta^h \neq 1/2$ for any h. Consider any arbitrary partition with three or more groups and focus on a group whose expected type is neither lowest nor highest of all groups. From the viewpoint of an agent in the group $\{\theta^h, \theta^{h+1}, ..., \theta^{h+k-1}\}$, if the other agents adopt the partitional strategy, the expected action by the decision maker is inclined towards $\bar{\mu}(\theta^h, \theta^{h+k-1})$ and his own message has less influence as the number of agents n becomes larger. Therefore, unless his type coincides exactly with $\bar{\mu}(\theta^h, \theta^{h+k-1})$, the agent can render the expected action closer to his ideal by sending the message that corresponds to a lower or higher group, rather than his own group, when his influence is weak. This implies that for any partition with three or more groups, there always exists an agent type that deviates from the partitional strategy if n is large. Consequently, the most informative equilibrium must be binary.

When there is an odd number of types, the most informative equilibrium for large enough n has the type space partitioned into three groups where $\theta^{(H+1)/2} = 1/2$ reveals truthfully and separates itself from those in the other two groups below and above. This is similar to the iid setting with continuous types, in which agents closer to the average can send more precise messages (as reflected in the shorter intervals around 0.5 in Figure 2). Proposition 3 implies that a binary equilibrium also exists even if H is odd. The welfare gain from allowing ternary communication relative to binary communication decreases as the number of potential types H becomes larger.

Since our proofs (along with the intuitions) of Proposition 3 and 4 rely on the construction of binary equilibrium, but not directly on the uniform prior, we can extend the propositions to other prior distributions as long as we set the upper bound on the type correlation to ensure separation above and below the prior expectation. From this respect, the case where we have $\theta^h = 1/2$ is non-generic since the prior expectation does not have to coincide with any type in the discrete type setting.

The most important observation here is that, even if $n \to \infty$, communication does have a non-trivial welfare enhancing effect on the decision maker's action when the probability distribution of preferences is ex ante uncertain. Despite the large number of agents however, the decision maker (or the agents) cannot identify the true distribution of agent types since, as implied in Proposition 4, communication is imperfect. For instance, under binary communication the decision maker can precisely infer what proportion of agents belong to the group above or below the prior expectation, but she is unable to find out how exactly types are distributed within each group. This is in sharp contrast to previous results on efficient information aggregation through binary communication or voting with an arbitrarily large number of agents (e.g. Feddersen and Pesendorfer, 1997; Morgan and Stocken, 2008), where the distribution of preference distribution takes a much simpler form. In our model, type distribution within groups can be highly variable, and this gives rise to imperfect inference on the realized distribution of types and hence a sub-optimal action even when $n \to \infty$.

Finally, let us briefly discuss our assumptions on the preference distribution. Intuitively, an essential feature required for Proposition 3 and 4 is that every agent thinks that the others are, on average, the other agents bias the decision maker's action towards the prior expectation, which implies that the primary incentive to misreport is exaggeration. This is the case in our model under the Dirichlet distribution when the correlation of agent types are not too strong, as we have seen earlier.[25] However, more generally, if the preference distribution is such that some agents expect the others to shift the decision maker's action towards extremes, our argument for Proposition 4 is no longer valid. How people update their beliefs about others is an empirical question, but note that we observe incentive to exaggerate for the iid case in the previous section for exactly the same reason that every agent thinks the others are on average less extreme (as the prior mean is known and fixed). Therefore, it would be appropriate to think of our analysis in this section as a natural extension of the results and intuition we have developed in the preceding section.

3 Conclusion

This paper has developed a model of public consultation, where the decision maker communicates with multiple interested parties with heterogeneous preferences. We have shown that communication in such circumstances is subject to exaggeration, which becomes severer as the number of interested parties increases. Our model can shed new light on the nature of communication for public decision making. In particular, we have demonstrated that the concern for exaggeration may lead to the use of binary communication, so that reporting parties cannot possibly exaggerate their preferences. In other words, binary communication emerges endogenously as a 'robust' mode of information transmission, even when the preferences, actions, and potential message space are all non-binary. This could be thought of as a strategic foundation for the frequent use of binary questions/voting in practice especially when many interested parties are involved. In addition to the case where the agents have no individual conflict of interests with the decision maker, we have examined how incentive to exaggerate and individual bias interact in communication. In theoretical terms, this paper offers an analysis of communication for public good provision where the decision maker cannot commit to a mechanism and no transfers are available, which is relevant to a lot of practical situations, including political and regulatory decision

[25]If α is close to 0 (i.e. correlation is extremely strong) under a given partitional strategy we may for example have $\bar{\mu}(\theta^h, \theta^{h+k-1}) < \theta^{h+k-1} < 1/2$, which means that high types within the group have incentive to send messages closer to the prior expectation, or of the opposite extreme to their types. In fact, for relatively small n a binary communication equilibrium may not exist while there may be a fully revealing equilibrium. If n is very large, there may not exist an equilibrium in partitional strategy.

making as well as choice of action in classrooms and organizations.

A Appendix

A.1. *Proposition 2*

Before we prove the proposition, we provide some useful lemmas and outline how we construct the main proof. Let us call a sequence $(a_0, a_1, ..., a_j, ...a_J)$ that satisfies the 'arbitrage' condition (6) a '*solution*' to (6). We make use of the monotonicity condition (M) in Crawford and Sobel (1982, p.1444) which requires that, for given n, if we have two solutions a^+ and a^{++} with $a_0^+ = a_0^{++}$ and $a_1^+ > a_1^{++}$, then $a_j^+ > a_j^{++}$ for all $j = 2, 3, ...$ In other words, (M) says that starting from a_0, all solutions to (6) must move up or down together. Solving (7) with respect to a_j explicitly with $a_0 = 0$, we obtain[26]

$$a_j = \frac{1}{2} + \frac{a_1 - 1 + n - \sqrt{n(n-1)}}{4\sqrt{n(n-1)}} X^j - \frac{a_1 - 1 + n + \sqrt{n(n-1)}}{4\sqrt{n(n-1)}} Y^j, \qquad \text{(A.1)}$$

where $X = -1 + 2n + 2\sqrt{n(n-1)}$ and $Y = -1 + 2n - 2\sqrt{n(n-1)}$.

LEMMA 1. *Any solution to (6) satisfies (M).*

Proof. Since $X > Y > 0$, from (A.1) we have

$$\frac{da_j}{da_1} = \frac{X^j - Y^j}{4\sqrt{n(n-1)}} > 0,$$

which implies (M). ■

In order to show that the players' expected utility is higher in an equilibrium with more intervals, Crawford and Sobel (1982) deform the partition with J intervals to that with $J+1$ intervals, by continuously increasing the expected utility throughout the deformation. We follow this method, but we need to proceed with a two-step deformation, rather than one, because the deformation takes place towards the opposite directions for the right-hand and left-hand sides of 1/2 on $[0, 1]$.

Solving (7) with $a_0 = 0$ and $a_J = 1$ we obtain

$$a_j = \frac{1}{2} + \frac{X^j(1 + Y^J) - Y^j(1 + X^J)}{2(X^J - Y^J)}.$$

Using $XY = 1$, it is easy to check that a_j's are symmetrically located around 1/2. Let $a(J)$ be the equilibrium partition of size J. We show that $a(J)$ can be deformed to $a(J + 1)$ by

[26] (A.1) is used to obtain (8) and (9) in the main text.

two steps, continuously increasing the players' expected utility in each step. Let the sub-partition of $a(J)$ equal or below 1/2 be $\underline{a}(J) \equiv (a_0(J), a_2(J), ..., a_K(J))$ where $a_0(J) = 0$. In other words, K satisfies $a_K(J) \leq 1/2 < a_{K+1}(J)$. Now we will proceed in the following two steps:

1. We fix $a_K(J)$ and make the sub-partition $(a_K(J), a_{K+1}(J), ..., a_J(J))$ deform continuously to $(a_K(J), a_{K+1}(J+1), a_{K+2}(J+1), ..., a_{J+1}(J+1))$, increasing the expected utility.

2. We make the sub-partition $(a_0(J), a_1(J)..., a_K(J))$ deform continuously to $(a_0(J+1), a_2(J+1), ..., a_K(J+1))$, increasing the expected utility.

Lemma 2. *If $a(J)$ and $a(J+1)$ are two equilibrium partitions for the same n, then $a_{j-1}(J) < a_j(J+1) < a_j(J)$.*

Proof. See Lemma 3 in Crawford and Sobel (1982, p.1446). The proof follows directly from (M). ■

The first step of deformation is carried out as follows. Let $(a^x_K, a^x_{K+1}, ..., a^x_j, ..., a^x_{J+1})$ be the sub-partition that satisfies (6) for all $j = K+1, K+2, ..., J$ with $a^x_K = a_K(J)$, $a^x_J = x$ and $a^x_{J+1} = 1$. If $x = a_{J-1}(J)$ then $a^x_{K+1} = a^x_K = a_K(J)$. If $x = a_J(J+1)$ then we have $(a_K(J), a_{K+1}(J+1), ..., a_J(J+1))$, where (6) is satisfied for all $j = K+2, K+3, ..., J$. We are going show that, if $x \in [a_{J-1}(J), a_J(J+1)]$, which is again a non-degenerate interval by Lemma 2, then the agent's expected utility is strictly increasing in x.

In the second step, let $(a^z_0, a^z_1, ..., a^z_j, ..., a^z_K)$ be the sub-partition that satisfies (6) for $j = 1, 2, ..., K-1$, with $a^z_0 = 0$ and $a^z_K = z$. If $z = a_K(J)$ then $a^z_j = a_j(J)$ for all $j = 0, 1, ..., K$. If $z = a_K(J+1)$ then $a^z_j = a_j(J+1)$ for all $j = 0, 1, ..., K$. We will show that when $z \in [a_K(J+1), a_K(J)]$, which is again a non-degenerate interval by Lemma 2, the agent's expected utility is strictly decreasing in z.

Lemma 3. *Suppose that $(a_0, a_1, ..., a_j, ..., a_J)$ is a solution to (6). Then for all $j = 1, 2, ..., J-1$ if $a_j > (<)1/2$ then $a_j - a_{j-1} < a_{j+1} - a_j$ $(a_j - a_{j-1} > a_{j+1} - a_j)$. If $a_j = 1/2$ then $a_j - a_{j-1} = a_{j+1} - a_j$.*

Proof. The sequences that satisfy (6) are described by (7). Rearranging (7) we have

$$(a_{j+1} - a_j) - (a_j - a_{j-1}) = n\left(4a_j + \frac{2}{n} - 2\right) - 4a_j. \tag{A.2}$$

The left hand side $(a_{j+1} - a_j) - (a_j - a_{j-1}) = 0$ if

$$n\left(4a_j + \frac{2}{n} - 2\right) - 4a_j = 0 \Rightarrow a_j = \frac{1}{2}.$$

Since the right hand side of (A.2) is increasing in a_j, if $a_j > 1/2$ then $(a_{j+1} - a_j) - (a_j - a_{j-1}) > 0$, and if $a_j < 1/2$ then $(a_{j+1} - a_j) - (a_j - a_{j-1}) < 0$. ■

The above lemma says that an interval $[a_{j+1}, a_j)$ is longer (shorter) than the previous interval $[a_{j-1}, a_j)$ when $a_j > (<) 1/2$. The intuition is captured in Figure 2. The following lemma is similar but for a^x_K and a^z_{K+1}, which are fixed throughout the respective deformation so that the 'arbitrage' condition (6) is not satisfied at $a_j = a^x_{K+1}$ or $a_j = a^z_K$.

LEMMA 4. *$a^x_{K+1} - a^x_K < a^x_{K+2} - a^x_{K+1}$ and $a^z_K - a^z_{K-1} > a^z_{K+1} - a^z_K$.*

Proof. For $a^x_K < a^x_{K+1} < 1/2$, $a^x_{K+1} - a^x_K < a^x_{K+2} - a^x_{K+1}$ holds since from symmetry $1/2 - a^x_K = a_{K+1}(J) - 1/2$ and $a^x_{K+2} > a_{K+1}(J)$. For $a^x_{K+1} \geq 1/2$, from Lemma 3 we have $a^x_{K+1} - \tilde{a}_K \leq a^x_{K+2} - a^x_{K+1}$ where $\tilde{a}_K$ is defined such that $\{a_{j-1} = \tilde{a}_K,\ a_j = a^x_{K+1}, a_{j+1} = a^x_{K+2}\}$ satisfies (7). Since $a_K(J+1) < \tilde{a}_K < a_K(J) = a^x_K$ from Lemma 2, we have $a^x_{K+1} - a^x_K < a^x_{K+2} - a^x_{K+1}$. This proves the first part of the Lemma.

Similarly we have $a^z_K - a^z_{K-1} > \check{a}_{K+1} - a^z_K$ where $\check{a}_{K+1}$ is defined such that $\{a_{j-1} = a^z_{K-1}, a_j = a^z_K, a_{j+1} = \check{a}_{K+1}\}$ satisfies (7). Lemma 2 implies $a^z_{K+1} = a_{K+1}(J+1) < \check{a}_{K+1} < a_{K+1}(J)$. Hence we have $a^z_K - a^z_{K-1} > a^z_{K+1} - a^z_K$. ■

Proof of Proposition 2.

- Agent

The decision maker's action from an agent's viewpoint is a random variable, and since the utility functions are quadratic, we can separate them into the expected value term and the variance term. Let $y_i(m_i)$ be the decision maker's (random) action from the agent's viewpoint. The agent's utility in this separated form, conditional on his report, is given by

$$\begin{aligned} &E\left[-(y_i(m_i) - \theta_i)^2 \mid m_i\right] \\ &= -var(y_i(m_i)) - (Ey_i(m_i))^2 + 2\theta_i Ey_i(m_i) - {\theta_i}^2 \\ &= -var(y_i) - (Ey_i(m_i) - \theta_i)^2, \end{aligned} \tag{A.3}$$

where

$$Ey_i(m_i) \equiv y_A(m_i) = \frac{1}{n} E\left[\theta_i \mid m_i\right] + \frac{n-1}{n} \times \frac{1}{2}$$

from (4). The variance term $-var(y_i)$ is independent of the agent's message since the randomness is caused by the other agents' messages unobservable to the agent. Let agent i's expected type given his message be $\hat{a}_i(a_j, a_{j+1})$. If a message is sent from $\theta_i \in [a_j, a_{j+1})$, then

$$\hat{a}_i = \frac{a_j + a_{j+1}}{2}.$$

From (3) the decision maker's action is the mean of the posterior expected types. Hence, from agent i's viewpoint

$$var(y_i) = var\left(\frac{1}{n}\left(\sum_{l \neq i} \hat{a}_l + \hat{a}_i\right)\right) = \frac{1}{n^2} var\left(\sum_{l \neq i} \hat{a}_l + \hat{a}_i\right) = \frac{n-1}{n^2} var(\hat{a}_l),$$

where $var(\hat{a}_l)$ is the variance of the expected type of an agent given his equilibrium strategy. The last equality follows from the independent type distribution and symmetric strategies. In what follows we drop the subscript i and l.

The expected utility for the first part of deformation is given by

$$EU^A = -\sum_{j=1}^{K}\int_{a_{j-1}^x}^{a_j^x}\left(\frac{a_{j-1}+a_j}{2n}+\frac{n-1}{2n}-\theta\right)^2 d\theta - \sum_{j=K+1}^{J+1}\int_{a_{j-1}^x}^{a_j^x}\left(\frac{a_{j-1}^x+a_j^x}{2n}+\frac{n-1}{2n}-\theta\right)^2 d\theta$$
$$-\frac{n-1}{n^2}\left[\sum_{j=1}^{K}(a_j-a_{j-1})\left(\frac{a_{j-1}+a_j}{2}\right)^2+\sum_{j=K+1}^{J+1}(a_j^x-a_{j-1}^x)\left(\frac{a_{j-1}^x+a_j^x}{2}\right)^2-\frac{1}{4}\right].$$

It follows that

$$\begin{aligned}\frac{dEU^A}{dx} &= \sum_{j=K+1}^{J+1}\frac{da_j^x}{dx}\left\{-\left(\frac{a_{j-1}^x+a_j^x}{2n}+\frac{n-1}{2n}-a_j^x\right)^2+\left(\frac{a_j^x+a_{j+1}^x}{2n}+\frac{n-1}{2n}-a_j^x\right)^2\right.\\ &\quad -\frac{1}{n}\left[\int_{a_{j-1}^x}^{a_j^x}\left(\frac{a_{j-1}^x+a_j^x}{2n}+\frac{n-1}{2n}-\theta\right)d\theta+\int_{a_j^x}^{a_{j+1}^x}\left(\frac{a_j^x+a_{j+1}^x}{2n}+\frac{n-1}{2n}-\theta\right)d\theta\right]\\ &\quad \left.-\left[\frac{n-1}{2n^2}(a_{j+1}^x)^2-(a_{j-1}^x)^2+\frac{(a_{j-1}^x+a_j^x)^2-(a_j^x+a_{j+1}^x)^2}{2}\right]\right\}\\ &= \sum_{j=K+1}^{J+1}\frac{da_j^x}{dx}\left\{\frac{a_{j+1}^x-a_{j-1}^x}{2n}\left[\frac{a_{j-1}^x-2a_j^x+a_{j+1}^x}{2}\right]\right\}>0.\end{aligned}$$

The inequality follows because $\frac{da_j^x}{dx}>0$ by (M), and from Lemmas 3 and 4, we have $a_j-a_{j-1}<a_{j+1}-a_j \Rightarrow a_{j-1}^x-2a_j^x+a_{j+1}^x>0$ for all $j=K+1,K+2,...,J$.

We have the second part of deformation as follows:

$$\begin{aligned}\frac{dEU^A}{dz} &= \sum_{j=1}^{K}\frac{da_j^z}{dz}\left\{-\left(\frac{a_{j-1}^z+a_j^z}{2n}+\frac{n-1}{2n}-a_j^z\right)^2+\left(\frac{a_j^z+a_{j+1}^z}{2n}+\frac{n-1}{2n}-a_j^z\right)^2\right.\\ &\quad -\frac{1}{n}\left[\int_{a_{j-1}^z}^{a_j^z}\left(\frac{a_{j-1}^z+a_j^z}{2n}+\frac{n-1}{2n}-\theta\right)d\theta+\int_{a_j^z}^{a_{j+1}^z}\left(\frac{a_j^z+a_{j+1}^z}{2n}+\frac{n-1}{2n}-\theta\right)d\theta\right]\\ &\quad \left.-\frac{n-1}{2n^2}\left[(a_{j+1}^z)^2-(a_{j-1}^z)^2-\frac{(a_{j-1}^z+a_j^z)^2-(a_j^z+a_{j+1}^z)^2}{2}\right]\right\}\\ &= \sum_{j=1}^{K}\frac{da_j^z}{dz}\left\{\frac{a_{j+1}^z-a_{j-1}^z}{2n}\left[\frac{a_{j-1}^z-2a_j^z+a_{j+1}^z}{2}\right]\right\}<0.\end{aligned}$$

The inequality follows because $\frac{da_j^z}{dz}>0$ by (M), and from $a_0,a_1,...,a_K\leq 1/2$ and Lemmas 3 and 4 we have $a_j-a_{j-1}>a_{j+1}-a_j \Rightarrow a_{j-1}^z-2a_j^z+a_{j+1}^z<0$ for all $j=1,2,...,K$.

Since we have completed the deformation from $a(J)$ to $a(J+1)$ in two steps while increasing the expected utility, we conclude that the agent's expected utility is higher in an equilibrium with more intervals.

- Decision Maker

Since the decision maker's utility is the sum of the agents' utilities, we can apply the above result for an agent's expected utility directly to show that the decision maker's expected utility is higher in an equilibrium with more intervals. ■

A.2. Derivation of $\bar{\mu}(\theta^h, \theta^{h+k-1})$

Consider an arbitrary partition of the type space $\{\theta^1, \theta^2, ..., \theta^H\}$ into $J(\leq H)$ disjoint groups. Let $\theta^{(j,1)}$ be the lowest and $\theta^{(j,S_j)}$ be the highest type in the jth group that consists of S_j types (we count the groups from the left hand side). Let $\Pi : (j, \cdot) \mapsto \{1, 2, ..., H\}$ be a function from the identity of a group j and the order within the group to the original type ordering. Clearly we have $\theta^{(1,1)} = \theta^1$ and $\theta^{(J,S_J)} = \theta^H$. For a group with a single type $S_j = 1$ and $\theta^{(j,S_j)} = \theta^{(j,1)}$.

First, let us denote the expected type of an agent in the jth group by

$$\bar{\theta}(j) \equiv \frac{1}{S_j} \sum_{h=\Pi(j,1)}^{\Pi(j,S_j)} \theta^h, \tag{A.4}$$

where $\Pi(j, s)$ represents the original order (in $\{1, 2, ..., H\}$) of the sth type in the jth group. Suppose that an agent's type is one in the qth group. Conditional on his own type, he believes that, if all the other agents follow the partitional strategy, their expected type from the decision maker's viewpoint is

$$\bar{\mu}_q = \sum_{j=1, j\neq q}^{J} \frac{S_j \alpha}{H\alpha + 1} \bar{\theta}(j) + \frac{S_q \alpha + 1}{H\alpha + 1} \bar{\theta}(q) = \frac{H\alpha}{H\alpha + 1} \frac{1}{2} + \frac{1}{H\alpha + 1} \bar{\theta}(q). \tag{A.5}$$

Hence for any partition, we have $\bar{\mu}(\theta^h, \theta^{h+k-1}) = \frac{H\alpha}{H\alpha+1}\frac{1}{2} + \frac{1}{H\alpha+1}\bar{\theta}(\theta^h, \theta^{h+k-1})$.

A.3. *Proposition 3*

Proof. Suppose H is an even number and consider the binary partition of the type space, $\{\theta^1, ..., \theta^{H/2}\}, \{\theta^{H/2+1}, ..., \theta^H\}$, where $\theta^{H/2} < 1/2 < \theta^{H/2+1}$. Let us focus on the strategy of an agent whose type is $\theta_i \in \{\theta^1, ..., \theta^{H/2}\}$. If the agent follows the binary partitional strategy and sends the same message as the ones in the group he belongs to, then he induces the expected action $\frac{1}{n}\bar{\theta}(\theta^1, \theta^{H/2}) + \frac{n-1}{n}\bar{\mu}(\theta^1, \theta^{H/2})$. If the agent deviates and sends the same message as the ones in the other group $\{\theta^{H/2+1}, ..., \theta^H\}$ then he induces $\frac{1}{n}\bar{\theta}(\theta^{H/2+1}, \theta^H) + \frac{n-1}{n}\bar{\mu}(\theta^1, \theta^{H/2})$.

If $\theta_i > \frac{1}{n}\bar{\theta}(\theta^1,\theta^{H/2}) + \frac{n-1}{n}\bar{\mu}(\theta^1,\theta^{H/2})$ then we have[27]

$$\theta_i - \left[\frac{1}{n}\bar{\theta}(\theta^1,\theta^{H/2}) + \frac{n-1}{n}\bar{\mu}(\theta^1,\theta^{H/2})\right] < \left[\frac{1}{n}\bar{\theta}(\theta^{H/2+1},\theta^{H}) + \frac{n-1}{n}\bar{\mu}(\theta^1,\theta^{H/2})\right] - \theta_i, \quad \text{(A.6)}$$

which implies that he induces the expected action closer to his ideal θ_i by following the partitional strategy. This is because (A.6) can be rewritten

$$\theta_i < \frac{1}{n}\frac{\bar{\theta}(\theta^1,\theta^{H/2}) + \bar{\theta}(\theta^{H/2+1},\theta^{H})}{2} + \frac{n-1}{n}\bar{\mu}(\theta^1,\theta^{H/2}) = \frac{1}{n}\frac{1}{2} + \frac{n-1}{n}\bar{\mu}(\theta^1,\theta^{H/2}),$$

which follows readily from (13) as it implies $\theta_i \le \bar{\mu}(\theta^1,\theta^{H/2}) < 1/2$.

If $\theta_i \le \frac{1}{n}\bar{\theta}(\theta^1,\theta^{H/2}) + \frac{n-1}{n}\bar{\mu}(\theta^1,\theta^{H/2})$ we obtain

$$\theta_i \le \frac{1}{n}\bar{\mu}(\theta^1,\theta^{H/2}) + \frac{n-1}{n}\bar{\theta}(\theta^1,\theta^{H/2}) < \frac{1}{n}\bar{\theta}(\theta^{H/2+1},\theta^{H}) + \frac{n-1}{n}\bar{\mu}(\theta^1,\theta^{H/2})$$

since by definition $\bar{\theta}(\theta^1,\theta^{H/2}) < \bar{\theta}(\theta^{H/2+1},\theta^{H})$. Therefore, whether the expected action the agent induces is below or above θ_i, it is closer to the agent's ideal θ_i when he follows the binary partitional strategy. Likewise, if $\theta_i \in \{\theta^{H/2+1},...,\theta^{H}\}$ then he also prefers to follow the partitional strategy. Therefore, the binary partition of the type space supports a perfect Bayesian equilibrium.

If H is an odd number then we have $\theta^{(H+1)/2} = 1/2$. Let $\theta^{(H+1)/2}$ randomize equally between the message sent by all types below 1/2 and the message sent by all types above 1/2, where we have $\bar{\theta}(\theta^1,\theta^{(H+1)/2}) = \frac{2}{H}\left(\sum_{h=1}^{(H+1)/2-1}\theta^h + \frac{1}{2}\times\frac{1}{2}\right)$ and $\bar{\theta}(\theta^{(H+1)/2},\theta^{H}) = \frac{2}{H}\left(\sum_{h=(H+1)/2+1}^{H}\theta^h + \frac{1}{2}\times\frac{1}{2}\right)$. Then we can use the same binary equilibrium construction as above, except for the middle type $\theta^{(H+1)/2}$ who is indifferent between sending the two messages and randomizes between them. ■

A.4. *Proposition 4*

Proof. Consider an arbitrary partition of the type space $\{\theta^1,\theta^2,...,\theta^H\}$ into $J \ge 3$ disjoint groups. Following the notation in Section A.2, let an agent whose type is $\theta^{(2,1)}$ be the lowest in the second group and assume $\theta^{(2,1)} < 1/2$. From $\theta^{(2,1)} \le \bar{\theta}(2)$ by definition and (A.5) we have $\theta^{(2,1)} < \bar{\mu}_2$. In other words, from the agent's viewpoint, the decision maker's expectation of the other agents' type $\bar{\mu}_2$ is higher than his own type $\theta^{(2,1)}$, which implies that the other agents are likely to move the decision maker's expected action higher

[27]Note that, from (11), $\bar{\theta}(\cdot)$ depends only on the parameters of the prior and is independent from the ex post realization of agent types. This implies that the variance of the decision maker's action from the agent's viewpoint does not change according to his own message. This allows us to focus on which message induces the closest expected action to the agent's desirable action.

than his ideal. Suppose that all the agents except i follow the partitional strategy. Since $\bar{\theta}(1) < \bar{\theta}(2)$, there exists $\bar{n}^*$ such that for $n \geq \bar{n}^*$

$$\theta^{(2,1)} < \underbrace{\frac{1}{n}\bar{\theta}(1) + \frac{n-1}{n}\bar{\mu}_2}_{\text{exp. action if deviates}} < \underbrace{\frac{1}{n}\bar{\theta}(2) + \frac{n-1}{n}\bar{\mu}_2}_{\text{exp. action under partitional strategy}}.$$

This implies that for sufficiently large n, the decision maker's expected action is closer to the agent's ideal action if he mimics an agent in the first group.

Similarly, consider an agent whose type is $\theta^{(J-1,S_{J-1})}$, the largest in the $(J-1)$th group and $\theta^{(J-1,S_{J-1})} > 1/2$. From $\theta^{(J-1,S_{J-1})} \geq \bar{\theta}(J-1)$ and (A.5) we have $\theta^{(J-1,S_{J-1})} > \hat{\mu}_2$. Suppose again that all the agents except i follow the partitional strategy. Since $\bar{\theta}(J) > \bar{\theta}(J-1)$, there exists $\bar{n}^{**}$ such that for $n \geq \bar{n}^{**}$

$$\theta^{(J-1,S_{J-1})} > \frac{1}{n}\bar{\theta}(J) + \frac{n-1}{n}\bar{\mu}_{J-1} > \frac{1}{n}\bar{\theta}(J-1) + \frac{n-1}{n}\bar{\mu}_{J-1},$$

which implies that the decision maker's action is closer to his ideal action when he mimics an agent in the Jth group.

Therefore, for any arbitrary partition with three or more groups, if $\theta^h \neq 1/2$ for any h (i.e. H is an even number), there exists an agent type in the second or penultimate group that deviates for large enough n. Together with Proposition 3 we conclude that there exists $\bar{n}$ such that the most informative equilibrium is binary for $n \geq \bar{n}$.

By the same argument as above, if H is an odd number and we have $\theta^{(H+1)/2} = 1/2$, there exists $\bar{n}$ such that for all $n \geq \bar{n}$ the most informative equilibrium has the ternary partition such that $\theta^h = 1/2$ induces the expected action $\frac{1}{n}\bar{\theta}(2) + \frac{n-1}{n}\bar{\mu}_2 = 1/2$, while all types strictly below 1/2 induce $\frac{1}{n}\bar{\theta}(1) + \frac{n-1}{n}\bar{\mu}_1 = \frac{1}{n}\bar{\theta}(\theta^1, \theta^{(H+1)/2-1}) + \frac{n-1}{n}\frac{1}{n}\bar{\theta}(\theta^1, \theta^{(H+1)/2-1})$ and those strictly above 1/2 induce $\frac{1}{n}\bar{\theta}(3) + \frac{n-1}{n}\bar{\mu}_3 = \frac{1}{n}\bar{\theta}(\theta^{(H+1)/2+1}, \theta^H) + \frac{n-1}{n}\frac{1}{n}\bar{\theta}(\theta^{(H+1)/2+1}, \theta^H)$.
■

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