News begets news:
A model of endogenously repeated costly consultation*

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Abstract

We examine a model of endogenously repeated (almost) cheap talk in a Markovian environment. Each period, the uninformed party (the receiver, i.e. the public) can consult the informed party (the sender, i.e. the media) at a cost. The sender, who is primarily driven by profit maximization, has an incentive to generate uncertainty about tomorrow's state in order to encourage consultation tomorrow. We find that the receiver may benefit from a higher consultation cost, because it dampens his responsiveness to messages with respect to future visits, thereby making him less manipulable, which in turn discourages information distortion by the sender.

Keywords: cheap talk, strategic information transmission, Markovian processes.
JEL classification: D81, D83.

1 Introduction

In contemporary liberal societies, citizens obtain their information to a large extent via news media. Three stylized facts of media markets stand out. First, news providers generate revenue from readers (or viewers) whether directly by selling access to news or indirectly by selling access to readers to advertisers (Doyle, 2013; Noam, 2013). Second, the average cost of accessing news contents has arguably decreased significantly over the years, in consequence of technological change as well as changing market conditions (e.g. increased competition), both of which relate closely to the increasing prominence of online news (Newman et. al., 2016). [1] Third, sensationalism, which we define as the manipulation of news contents for the sake of attracting more readership, is on the rise. Though the term (and the critique) emerged at the

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[1] While technology does allow news outlets to implement paywalls, most news media adopt either no paywall or a "soft" paywall on which key articles are still freely available. See Ingram, M. "The Guardian, Paywalls and the Death of Print Newspapers," Fortune, 27 February 2016. See also Arrese (2016) for the development of payment systems for online media.

same time as mass-media itself in the late 19th century (see Spencer, 2007), the last two decades have witnessed an intensification of the phenomenon, especially in online news.

Katherine Viner, Editor in Chief of the British newspaper the *Guardian*, summarizes the overall situation as follows: "*(T)he trouble is that the business model of most digital news organisations is based around clicks. News media around the world has reached a fever-pitch of frenzied binge-publishing, in order to scrape up digital advertising's pennies and cents.(...)The impact on journalism of the crisis in the business model is that, in chasing down cheap clicks at the expense of accuracy and veracity, news organisations undermine the very reason they exist: to find things out and tell readers the truth – to report, report, report.*"[2]

We study the relationship between the informativeness of media reports and the (exogenous or endogenous) cost of access to news in a simple dynamic game of information transmission. An informed sender (media firm) whose main concern is to obtain revenue from access to its report (e.g. through clicks or purchases) has an incentive to lie about today's state in order to create uncertainty about tomorrow's state and thereby induce the receiver (the representative reader, i.e. the public) to acquire its report tomorrow. In particular, we demonstrate that cheaper access to news may lower reporting accuracy in equilibrium, since the receiver's decision whether to read a news report becomes more manipulable when the cost of access is lower. The effect may also be strong enough to make the receiver worse off as access becomes cheaper.

Specifically, in our model the underlying state follows a binary Markov process, and each current state entails a different (high or low) level of uncertainty about the state in the next period. An informed sender (S) who perfectly observes the state produces a report in each period. An uninformed receiver (R) makes two decisions in each period, namely i) whether to access the report by incurring costs (including opportunity costs) ; and ii) which action to take in order to match the state of the world as closely as possible. While S's primary source of payoffs is visits by R, he also derives a positive payoff from truth-telling, but the latter concern is assumed to be small relative to the revenue generated from visits.

For an intermediate range of the expected cost of access, we find that equilibrium communication is partially but not fully informative. In equilibrium S occasionally wrongfully reports that the current state is the one that involves high future uncertainty, in order to induce access in the future. Equilibrium communication thus features a combination of uncertainty resolution and uncertainty generation. Reports are informative on average, which is why R is willing to acquire these at a cost. But S occasionally produces untruthful announcements aimed at generating uncertainty and readership tomorrow. A second main finding is that the accuracy of S's reporting (i.e. his tendency to lie) is *decreasing* in the expected cost of access since R's decision whether to access tomorrow's report becomes more responsive to the contents of the current report, which in turn gives S a higher incentive to manipulate information.

Assuming that the cost of access lies in an intermediate range has a natural interpretation. If the cost is very low, R will acquire tomorrow's news report no matter today's report (and the

[2]Viner, K. "How Technology Disrupted the Truth," Guardian, 12 July 2016. See also Murtha, J. "What It's Like to be Paid for Clicks," Columbia Journalism Review, 13 July 2015. See also Sherman, J. "Daily Mail 'Too Unreliable' for Wikipedia," The Times, 10 February 2017. See Rusbridger (2018) for a comprehensive description of fundamental changes that have occurred in the print media industry during the last decades. See also Kilgo et al. (2017) for the link between social media recommendations and sensationalism in online news.

implied uncertainty level concerning tomorrow's state).[3] If the cost is very high, R is unlikely to access future reports regardless of the current report. Whether costs are very low or very high, S is thus not tempted to strategically misreport in order to induce more consultation and accordingly reports truthfully.

Our key results hold both in a simple two-period model and in an infinite-horizon version of the model, which we analyze in sequence.

We conclude our analysis with an extension of the two-period model featuring an endogenous price paid by R to S for consulting, under respectively monopoly and duopoly. We show that competition cannot a priori be guaranteed to be welfare improving for readers as compared to monopoly, since it leads to a lower visiting price and hence lower revenue per access, which in turn reduces the informativeness of equilibrium reports by the mechanism described above.

Literature review A key distinction in the media literature, in regards to to incentives to misreport, is between political motives and profit motives, the case of mixed motives being an interesting third possibility (see Anderson and McLaren, 2012) .[4] While the partisan approach has provided fundamental insights (e.g. Duggan and Martinelli, 2011), our contribution relates more to the second strand. In Mullainathan and Shleifer (2005) and Bernhardt, Krasa, and Polborn (2008), a media firm biases news reports to target a subset of behavioural readers who, besides seeking information, value confirmatory news more. The empirical study of Ahern and Sosyura (2015) examines financial media reporting on speculated corporate mergers and finds that less accurate stories are more likely to feature well-known firms with broader readership appeal. This suggests that the primary source of inaccuracy is the incentive to attract a broader readership. An important question in the literature on profit seeking media is the effect of competition on media bias. Mullainathan and Shleifer (2005) find that competition generally reduces newspaper prices, but does not reduce and might in fact even exaggerate media bias, though conscientious readers who buy all papers are better off. In Anderson and McLaren (2012), under competition, readers who buy all papers learn more.

Our paper also relates to a theoretical literature on repeated communication with reputational concerns. These models feature uncertainty about either the competence or the preferences of S and the latter has an incentive to signal a good type by biasing reports. In Bénabou and Laroque (1982), Sobel (1985), Morris (2001), Ely and Valimaki (2003), the expert wants to signal benevolent preferences. In Ottaviani and Sorensen (2006) and Gentzkow and Shapiro (2006) (which focuses on a media application), the informed party biases its news towards consumers' priors to appear of high expertise type.

There also exists a large literature on repeated cheap talk or disclosure in the absence of reputational concerns, where the state is either fixed or changing over time (Aumann and Hart (2003), Krishna and Morgan (2004), Renault, Solan, and Vieille (2013), Golosov et al. (2014), Koessler and Forges (2008). The general insight is that in such dynamic settings, the use of jointly controlled lotteries and sophisticated punishment strategies generates equilibria with superior informational properties.

[3]However, our model rules out this case, since the cost of reading a news report includes an opportunity cost which is always positive in expectation.

[4]See e.g. Gentzkow, Shapiro and Stone (2015) and Puglisi and Snyder (2015) for excellent surveys.

Our paper also relates to a literature on the design of protocols for the sequential disclosure of information over multiple periods. Ely, Frankel and Kamenica (2015) consider the design of dynamic information disclosure policies (with commitment) aimed at maximizing variance ("suspense") or change ("surprise") in the receiver's beliefs over multiple periods. Hörner and Skrzypacz (2016) also consider dynamic revelation by an agent of verifiable information concerning his ability level ("gradual persuasion"). They find that gradualism can help mitigate the hold-up problem faced by the agent.

Our contribution also relates to a literature on seller's strategic behaviour when the buyer(s) may engage in repeat purchases. Bulow (1986) showed that a durable goods monopolist may reduce product durability in an earlier period in order to mitigate the time inconsistency problem of producing too much (and thus reduces the buyers' willingness to pay in an earlier period). Waldman (1993, 1995) also studies planned obsolescence, focusing on the introduction of new products. Other papers on strategic behaviour for repeat purchases mostly focus on pricing and the choice of product quality under technological change (see e.g. Fudenberg and Tirole 1998; Waldman, 2003). Klemperer (1987, 1995) studies how the presence of consumer switching costs affects the seller's strategy.

The overall relation between our model and existing contributions can be summarized as follows. The novelty of our approach is to shed light on information distortion caused by a rational sender's incentive to induce future revisit by a rational information seeking receiver in a setup featuring a serially correlated state. The payoff of the sender is primarily affected only by whether the receiver makes a costly visit, but not by whether the receiver's action matches the state, which distinguishes our model from most of the literature on cheap talk. There is no uncertainty regarding the sender's expertise or preferences. Finally, readers only care about being informed, are fully rational and have no behavioural preferences (e.g. preferring confirmatory news). We thus offer a rationality-based approach to noisy media communication (which could also be termed sensationalism in news production). Our scenario simply relies on the fact that in the market for news, there is an intrinsic linkage between the contents of today's product and tomorrow's demand for the product. Today's news has the ability to increase tomorrow's demand for news (hence the title "News begets news") by generating uncertainty about tomorrow's state.

We proceed as follows. Section 2 studies the two-period model, followed by Section 3 which develops an infinite-horizon version of the model. Section 4 presents some extensions for the two-period model and Section 5 concludes. Unless explicitly mentioned otherwise, proofs are relegated to the Appendix (which is divided into parts A, B and C).

2 Two-period Model

The periods are denoted by $t = 1, 2$. In each period t, the state ω_t is drawn from the binary state space $\{A, B\}$. The prior probability that the state is A in period 1 is $P(\omega_1 = A) = \theta$, where $\theta \in [\frac{1}{2}, 1)$. The process governing the state across periods is Markovian and transition probabilities are given by $P(\omega_{t+1} = A \mid \omega_t = A) = \gamma \in \left(\frac{1}{2}, 1\right)$ and $P(\omega_{t+1} = B \,|\omega_t = B\,) = \frac{1}{2}$. State B thus entails higher uncertainty with respect to the state in the next period. Note that the stationary probability of states A and B is given by respectively $\mu_A \equiv \frac{1}{3-2\gamma}$ and $\mu_B \equiv 1 - \mu_A$.

There are two players, a sender S and a receiver R. S observes ω_t at the beginning of period t and chooses a message for the period (we also use the term "report") denoted by $m_t \in \{A, B\}$. In period t, R observes neither ω_t nor ω_{t-1}.

Receiver's consultation and action Each period t is divided into three stages for R, namely beginning, middle and end. With probability $\alpha \in [0, 1)$, R exogenously observes message m_t at the beginning of period t. With probability $1 - \alpha$, R does not exogenously observem_t and instead has the option to consult (we also use the term "visit") S at a cost and thereby observe m_t at the beginning of t. Consultation comes at cost $w + v_t$, where $w \in (0, \frac{1}{2}]$ is a fixed cost that is constant across all periods, and v_t is a privately observed random cost drawn anew at each period from a uniform distribution on $[0, c]$. We assume $c \in \left[\frac{1}{2}, 1\right]$. R observes v_t before deciding whether or not to consult in period t. Let η_t be a random variable that takes value 1 if observing the message is costly in period t and 0 otherwise. The variable η_t is thus i.i.d across periods and as noted earlier, $P(\eta_t = 0) = \alpha$. Let d_t be a variable that takes value 1 if R consults in period t and 0 otherwise. In the middle stage of period t, R chooses an action $a_t \in \{A, B\}$, which is informed by m_t if R has already observed the latter. At the end of period t, R observes m_t even if he did not observe it earlier. This captures the notion that news reports are only excludable for a short amount of time, as information spreads fast through various communication channels.

Payoffs The payoff of R for period t depends on both his consultation choice and his action's match with the state. The cost incurred for consultation has already been described. In addition, if his chosen action matches the state, i.e. $a_t = \omega_t$, then R obtains an action-payoff of 1 in period t. If instead $a_t \neq \omega_t$, then R's action-payoff for the period is 0.

S's payoff for period t depends on two aspects. First, he receives a per visit revenue f if $\eta_t = 1$ and R makes a costly visit. Second, reporting $m_t = \omega_t$ yields a truth-telling reward of $z = x(1 - \alpha) f$, where $x \in (0, 1)$ and is assumed to be small, as we will describe shortly. [5]

If instead the report at t does not match the state ($m_t \neq \omega_t$) then this message-dependent payoff is 0. One can interpret z as S's intrinsic preference for truth-telling, or as an implication of unmodeled reputational concerns. Note that by construction $z < f$, so that S's primary concern is to induce consultation. Also, we shall focus on the case where the intrinsic preference for truth-telling is weak (i.e. x is small) since otherwise truthful communication trivially constitutes an equilibrium outcome. We assume no discounting for simplicity. We also assume that the fixed cost of consultation is in a middle range as follows, in order to focus on interesting cases:

Assumption 1

$$\max\left\{1 - \gamma, \frac{1}{2} - cx - \frac{\theta(2\gamma - 1)}{2}\right\} < w < \frac{1}{2} - cx. \tag{1}$$

[5]Throughout this paper, we make a distinction between *truthful* communication (or truth-telling) and *fully informative* communication. S communicates truthfully if $m_t = \omega_t$, regardless of the actual meaning of the message in equilibrium, as pinned down by Bayes' rule. In contrast, equilibrium communication is fully informative if R can always infer the state perfectly based on the message observed, using Bayes'rule. That is, fully informative communication does not necessarily imply truth-telling (consider an equilibrium where S always sends $m_t = \{A, B\} \backslash \omega_t)$. However, we will see later that in our model, fully informative equilibrium communication has to come in the form of truth-telling.'

We focus on x strictly positive but small, so that the range is well-defined ($1 - \gamma < \frac{1}{2} - cx$) where $\gamma > \frac{1}{2}$. As we will see shortly, the LHS of (1) ensures that if the state is known to be A in period 1, R never (i.e. even if $v_2 = 0$) consults in period 2 since the uncertainty is too low (γ high) relative to the fixed cost w. The case where the fixed cost w is located outside the above range will be discussed in Section 4.1.

Strategies A communication strategy of S for the whole game is given by the combination of communication strategies for both periods. A communication strategy of S for period t is given by a distribution over the two messages (A or B) given the history observed by S as of period t. A strategy for period t is informative if $P(m_t = A \mid \omega_t = A) \neq P(m_t = A \mid \omega_t = B)$ and it is otherwise uninformative. We define a *simple* communication strategy for period t as one that has the following features. First, the probability of sending $m_t \in \{A, B\}$ is only a function of the current state ω_t. Denote by τ_J^t the probability that message $m_t \in \{A, B\}$ is sent in period t given that $\omega_t = J$. The second feature of a simple strategy is $\tau_B^t = 1$, which means that S reports truthfully when the state is B. Intuitively, since state B is the one associated with high uncertainty about the future, there is no reason for S to misreport in period 1 if the state is B. If anything, misreporting should occur when the state is A, since reporting truthfully (if R takes $m_1 = A$ at face value) leads to no visit under Assumption 1. Given $\tau_B^t = 1$, a simple strategy for period t is *partially informative* if $\tau_A^t \in (0, 1)$, *fully informative* if $\tau_A^t = 1$ and *uninformative* if $\tau_A^t = 0$. We will show later that focusing on simple strategies is without loss of generality.

A strategy of R for the whole game is given by the combination of strategies for both periods. R's strategy for period t has two components, a consultation rule and an action rule. A consultation rule for period t specifies the probability of consulting in period t for each possible history observed by R until the beginning of period t (including the realization of the random cost v_t). An action rule for period t specifies a conditional distribution over actions for each possible history observed by R up to the middle stage of period t. In an equilibrium where R acquires the message with positive probability p conditional on $\eta_t = 1$ (i.e. m_t is costly to observe) and a given observed history h_t, we say that R consults or makes a costly visit with probability p at t given observed history h_t. We say that an equilibrium features a positive probability of consultation in period t if, conditional on $\eta_t = 1$, R consults with positive probability at t. A strategy profile together with a set of equilibrium beliefs constitutes a perfect Bayesian equilibrium if each player's strategy is sequentially rational given his beliefs and the other player's strategy, and beliefs are derived from Bayes' rule whenever possible.

Discussion The random cost of visit v_t captures an unobservable component of how difficult or easy it is to access and read a news report in period t (including opportunity costs). Parameter α captures exogenous information leakage. We often receive news for free, without actively pursuing it, from individuals or from free news sources (SNS, TV, etc). The observability of m_{t-1} at t echoes the role of α. This means that someone who did not actively acquire news in the morning of period t eventually learns about it by the end of the day. Such information diffusion would occur through social networks or word of mouth.

We can interpret f as the advertising revenue generated by a certified visit of R to S's website (a click), rather than as a direct transfer from R to S. Advertising revenue is typically the main source of income of media and many news websites do not charge for access. This interpretation of f furthermore implies that a subscription contract would not directly solve all incentive problems of S in our game. Indeed, a subscription does not mean a commitment to actually visiting the website regularly. Rather, the subscriber will only visit at any given moment if his uncertainty is sufficiently high to outweigh the various costs associated with visiting (including opportunity costs of time and cognitive effort).

The truth-telling reward $z = (1-\alpha)fx$ is a reduced-form representation of journalistic commitment to truthful reporting, reputational concerns or the threat of legal costs in case of formally inaccurate reporting. It would be natural to assume that the truth-telling reward is positively associated with the size of the market and generable revenue. Note that our formalization is equivalent to simply assuming $z = (1-\alpha)r$, where r is some positive constant. We will see later that key equilibrium quantities (in particular the truth-telling probabilities) are affected by z and f exclusively through the ratio $\frac{z}{f}$, which is why letting $z = (1-\alpha)fx$ is convenient for exposition. All of our results, except for those directly related to the effect of exogenous changes in f, are qualitatively unaffected if f does not enter the truth-telling reward.

We assume that there is only one receiver for expositional convenience, but our model can be reinterpreted as having a continuum of receivers. In this alternative interpretation, the distribution of v_t is i.i.d. across all receivers and periods, and α denotes the proportion of the receivers who freely observe m_t at the beginning of t before choosing their respective actions in period t.

We explicitly incorporate monetary transfers from R to S for each visit later in an extension presented in Section 4.

2.1 Equilibrium Behavior

Throughout our analysis of the two-period model, we make use of the feature that in period 2, S reports truthfully with probability one, as stated below.

Lemma 1. *In any equilibrium, S reports truthfully in period 2.*

The proof of the Lemma is immediate. In period 2, S has a strict incentive to report truthfully (i.e. to send $m_2 = \omega_2$) in any putative equilibrium. No matter m_2, R will not consult tomorrow (there is no period 3) and sending $m_2 = \omega_2$ yields an extra truth-telling reward. Lemma 1 allows us to focus on the informativeness of communication in period 1 and our next Lemma provides preliminary insights into this matter.

Lemma 2. *a) No equilibrium features uninformative communication in period 1.*
b) No equilibrium features fully informative communication in period 1.

The proof of point a) is by contradiction. Suppose S reports both $m_1 = A$ and $m_1 = B$ with positive probability in period 1. In this case, uninformative communication implies that messages $m_1 = A$ and $m_1 = B$ must yield the same probability of consultation in period 2. However, given that $R's$ consultation behavior in period 2 is independent of m_1, S is strictly better off truth-telling (sending $m_1 = \omega_1$) in period 1, as this yields the extra truth-telling reward. This case can thus be excluded. The second type of uninformative communication is the case where either $m_1 = A$ or $m_1 = B$ is sent with probability one in period 1. The proof that this cannot be an equilibrium is in the Appendix. The proof of point b) is also by contradiction and there are again two cases to consider (see the Appendix). The key mechanism is that when $\omega_1 = A$, S has a strict incentive to deviate to the period-1 message that implies that $\omega_1 = B$, this message being the only one to induce a positive probability of consultation in period 2.

Our next two Lemmas concern R's equilibrium behavior.

Lemma 3. *No equilibrium features zero probability of consultation in period 2.*

The above Lemma implies that some equilibrium message in period 1 has to lead to consultation with positive probability in period 2. This opens up the possibility that one of the messages available to S in period 1 makes consultation in period 2 more likely than the other, thereby creating an incentive to send the former message in order to maximize the probability of consultation in period 2. The next Lemma demonstrates that R's consultation decision in period 2 is affected significantly by the period-1 message, if S's communication strategy for period 1 is what we call a *simple* communication strategy.

Lemma 4. *In any equilibrium featuring a simple communication strategy in period 1, R consults with probability zero in period 2 if $m_1 = A$, and with strictly positive probability if $m_1 = B$.*

Since the fixed cost of visit w is in an intermediate range by Assumption 1, R never makes a costly visit in period 2 if he is sure that $\omega_1 = A$ (entailing low uncertainty about ω_2), which is the case here given $m_1 = A$. Our last Lemma establishes that the equilibrium communication strategy in period 1 must indeed be a simple communication strategy, which implies that focusing on simple strategies in both periods is without loss of generality.

Lemma 5. *In any equilibrium that features a positive probability of consultation in period 2, the communication strategy in period 1 is a simple communication strategy.*

We characterize the (unique) equilibrium in our next Proposition. The Lemmas presented so far considerably simplify the task of proving this result. We already know that if there exists an equilibrium, it satisfies the following description. The equilibrium must feature a simple partially informative communication strategy in period 1 (Lemmas 2 and5) and truth-telling in period 2 (Lemma 1). Furthermore, R consults with strictly positive probability in period 2 if $m_1 = B$ and with probability 0 if $m_1 = A$ (Lemmas 3 and 4). The Proposition derives a unique equilibrium satisfying the description, along with the value of τ_A^1 and the probability of consultation in period 2 given $m_1 = B$.

Proposition 1. *There exists a unique equilibrium. In the equilibrium, S reports truthfully in period 2 and uses a partially informative simple communication strategy in period 1, where*

$$\tau_A^{1*} = \frac{2w + \theta\,(2\gamma - 1) + 2cx - 1}{2\theta\,(\gamma + w + cx - 1)} \in (0,1). \tag{2}$$

Furthermore, R consults with probability x in period 2 if $m_1 = B$, and with probability zero if $m_1 = A$.

The conditions under which we can guarantee existence and uniqueness are not restrictive (the equilibrium prediction outside this parameter range is provided in the extensions section). For example, for x small enough and θ close enough to μ_A (i.e. when the period 1 prior is sufficiently close to the stationary distribution), we can guarantee the existence and uniqueness given $\frac{2(1-\gamma)}{(3-2\gamma)} < w \leq \frac{1}{2}$, which is not significantly more restrictive than simply assuming $1-\gamma < w \leq \frac{1}{2}$.[6]

Given that the equilibrium always involves truth-telling in period 2 and truth-telling in period 1 if the state is B, τ_A^{1*} is the only aspect of S's communication strategy that varies with exogenous parameters, and the informativeness of period 1 communication increases as τ_A^{1*} increases. Note first that τ_A^{1*} is independent of f (S's revenue per visit) and α (the probability that R exogenously receives a free report for the current period). In the identified equilibrium, S is indifferent between sending messages A and B in period 1 given $\omega_1 = A$, which is captured by the equality

$$(1-\alpha)fx = (1-\alpha)fP(d_2 = 1 \mid m_1 = B, \eta_2 = 1). \tag{3}$$

Above, the LHS corresponds to the period-1 truth-telling reward obtained when sending $m_1 = A$. Recall that this message induces consultation with probability zero in period 2. The RHS is the expected revenue from costly consultation in period 2 induced by sending $m_1 = B$ (thereby misreporting). The consultation probability in period 2 conditional on $m_1 = B$ is strictly increasing in the accuracy of the report τ_A^{1*}. Indeed, a higher τ_A^{1*} implies higher uncertainty about ω_2 conditional on $m_1 = B$, as this implies that $m_1 = B$ is discounted less by R (i.e. R is less sceptical). Clearly, $(1-\alpha)f$ cancels out on both sides of (3), so that the indifference condition reduces to a requirement that the probability of consultation in period 2 conditional on $m_1 = B$ equals x, the parameter for the truth-telling reward.

Our next Proposition characterizes the comparative statics of τ_A^{1*} (as given in 2).

Proposition 2. *τ_A^{1*} is constant in f, increasing in w, c, γ and x.*

The most important finding here is that an increase in the expected cost of consultation, originating in an increase in w or c, leads to an increase in the probability of truth-telling in equilibrium. This is in line with the discussion presented in the Introduction where we suggest that online news, which involve significantly lower costs of access relative to traditional paper-based media, may also be less reliable.

This result is an immediate implication of the indifference condition (3) that characterizes our equilibrium. First, as the expected cost of consulting increases, R's willingness to consult in period 2 after $m_1 = B$ decreases, holding fixed the informativeness of the report in period 1. Meanwhile, (3) indicates that in equilibrium, the probability of consulting in period 2 after $m_1 = B$ must remain equal to x in order to ensure that S is indifferent in period 1 between sending A and B conditional on $\omega_1 = A$. Therefore, as the expected cost of consultation increases, the truth-telling probability τ_A^{1*} must increase in order to ensure that R's visiting probability after B remains x. Indeed, an increase in τ_A^{1*} implies a higher degree of uncertainty about the state in period 2 given $m_1 = B$, since the true state in period 1 is more likely to be B, which entails higher future uncertainty.

We can also consider the equilibrium relationship between the cost of consultation and the informativeness of the period-1 report in terms of R's commitment power. Recall that when the state is A (low uncertainty about the future), S faces a trade-off between inducing a positive probability of consultation in period 2 by misreporting ($m_1 = B$) and instead reaping the

[6]Note first that cx tends to 0 for x tending to 0. Second, note that $\frac{1}{2} - \frac{\mu_A(2\gamma-1)}{2} = \frac{2(1-\gamma)}{(3-2\gamma)}$. Finally, note that $\frac{2(1-\gamma)}{(3-2\gamma)}$ is larger than $1-\gamma$ but very close to $1-\gamma$ for any $\gamma > \frac{1}{2}$.

immediate truth-telling reward by sending $m_1 = A$. A small reduction in the cost of consulting makes R's decision to visit more manipulable in the sense that his probability of visit in period 2 after $m_1 = B$ increases (while the probability of visit following $m_1 = A$ remains zero). This increase in the elasticity of R's demand for information in period 2 with respect to his uncertainty in turn gives rise to a larger temptation to misreport for S when $\omega_1 = A$ and, in fine, to a lower informativeness of the report in period 1. A higher cost of consultation thus de facto acts as a useful commitment device for R (not to overreact to uncertainty creating messages), which in turn allows to reduce the incentive to misreport.

Finally, the intuition for the positive effect of an increase in γ is similar to the intuition underlying the effect of w or c. For a given informativeness of the report in period 1, increasing γ reduces R's responsiveness to message $m_1 = B$ by reducing his uncertainty in period 2 conditional on $m_1 = B$, which benefits R.

We conclude our analysis of equilibrium behaviour with an observation on R's consultation decision in period 1.

Lemma 6. *There is some $\epsilon>0$ such that if $\theta \in (\frac{1}{2}, \frac{1}{2} + \epsilon)$ and $w \in \left(\frac{1}{2} - cx - \epsilon, \frac{1}{2} - cx\right)$, R consults with strictly positive probability in period 1.*

Whether R visits in period 1 depends on the prior θ as well as the cost of visit. Clearly, the higher the underlying uncertainty about the state in period 1 is (i.e. the closer θ is to $\frac{1}{2}$), the higher the incentive to consult in period 1. On the other hand, a high enough w will ensure that period-1 communication is sufficiently informative to warrant costly period-1 consultation (note that τ_A^{1*} is non-linear in w) . Note that the above is only a sufficient (and not a necessary) condition for a positive probability of visit in period 1. Note also that whether or not R consults with positive probability in period 1 does not affect S's reporting incentives in period 1, given that R observes the report for sure at the end of period 1.

2.2 Welfare

For our analysis of welfare, let us assume that the prior distribution of the state in period 1 coincides with the stationary distribution:

Assumption 2

$$\theta = \mu_A \equiv \frac{1}{3 - 2\gamma}.$$

Hereafter we focus on the case where Assumptions 1-2 are satisfied. We first consider R's equilibrium payoff. Recall that his total expected payoff has two components, namely the (fixed and random) costs of consultation and the payoff from whether his action matches the state.

Proposition 3. *a) If $\alpha > (<) \frac{cx^2}{2+cx^2}$, R's equilibrium expected payoff is strictly increasing (decreasing) in w.*

b) For any $\alpha \in [0, 1]$, the equilibrium expected payoff of R is strictly increasing in c and x.

An increase in w and c generates a trade-off between the increased informativeness of m_1 and the increased average cost of consultation in both periods. If observing m_1 is always costly (i.e. $\alpha = 0$), the adverse cost effect of an increase in w outweighs the benefit of more informative communication. As α increases, R becomes more likely to observe m_1 for free and there is thus a threshold level of α above which the informational benefit of an increase in w dominates the adverse cost effect. It is easy to see that for small x, as we have assumed throughout, this threshold value of α is also very low and much smaller than $\frac{1}{2}$.

In contrast, an increase in c is beneficial regardless of the size of α. An intuition is that the marginal effect on the expected cost of visit given τ_1^{A*} is small (recall that the variable cost is v_t is uniformly distributed on $[0, c]$).

An increase in x also benefits R, as it increases S's incentive to report truthfully and hence the informativeness of the report, without increasing the cost of access.

We now turn to S's expected payoff. We obtain the following characterization.

Proposition 4. *a) For any $\alpha \in [0, 1)$, the equilibrium expected payoff of S is increasing in f, w, c, x .*

b) The equilibrium expected payoff of S is decreasing in γ if $w > \frac{1}{4} + \gamma(1-\gamma) - cx$, and increasing in γ if $w < \frac{1}{4} + \gamma(1-\gamma) - cx$.

Given $\alpha < 1$, an increase in w or c generates two contrasting effects, both of which originate in an increase in the equilibrium accuracy of the report τ_A^{1*}. First, as τ_A^{1*} increases, the ex ante probability of consultation in period 2 decreases since the ex ante probability that $m_1 = B$ decreases, while the conditional probability of R consulting in period 2 given $m_1 = B$ stays equal to x. This effect reduces the expected payoff of S. The benefit of having a higher τ_A^{1*} for S is to increase the ex ante probability of reaping the truth-telling reward in period 1. Proposition 4 shows that this second effect outweighs the first effect.

On the other hand, an increase in x influences the payoff of S via three different channels. It increases the probability of consultation in period 2 after $m_1 = B$, which benefits S. It decreases the probability that $m_1 = B$ and thus the probability that R consults in period 2, which reduces the payoff of S. Finally, it increases the ex ante probability of reaping the truth-telling reward. Overall, the positive effects dominate the negative effect.

As to the effect of γ on S, observe first that for any $\gamma, c, x, \in (0, 1)$ we have

$$\left(\frac{1+4\gamma(1-\gamma)}{4} - cx\right) \in \left(\frac{1}{2} - cx - \frac{\mu_A(2\gamma-1)}{2}, \frac{1}{2} - cx\right),$$

so that the cut-off for w is meaningful in the light of the range of w implied by Assumptions 1 and 2. Recall that γ quantifies the future uncertainty in the low uncertainty state, A. Suppose that w is large and thus the frequency of truth-telling is already high. An increase in γ then only slightly increases τ_A^{1*} (, which yields a slightly higher probability of reaping truth-telling reward), while it in contrast significantly increases the probability that $\omega_1 = A$ since $\theta = \mu_A \equiv \frac{1}{3-2\gamma}$. This means, together with the increase in τ_A^{1*}, that m_1 is more likely to be A and hence the overall probability of visit in period 2 is lower. The loss of revenue from the latter effect in this (high w) case outweighs the positive effect of the higher probability of truth-telling, while the opposite is the case when w is low.

3 Infinite Horizon Model

In this section we extend our two-period model to an infinite horizon setup. A main motivation is to demonstrate that our key results and intuitions do not depend on the artificial feature of last period truth-telling that arises in the two period model. In reality, the underlying state (event) often evolves over time without a specific end date. In such a context, an infinite horizon model captures the fact that the incentive to affect future visits is always there and thereby describes the long-term interaction between a media firm and readers. The state ω_t in the infinite horizon model presented in this section can be thought of as a broad topic (politics, economy, sports, etc.) followed by readers over the long run, rather than a particular news event that develops over a short period of time. Technically, we make use of the recursive structure of the game given the Markovian environment.

3.1 Setup and strategies

Let the time period be $t = 0, 1, 2, ..., +\infty$. Both S and R discount their future payoffs at a common discount factor δ and they maximize the discounted future stream of payoffs in every period. The transition matrix of the state, the players, their action sets and per-period payoffs, are the same as in Section 2. Let $\tilde{x} \equiv x/\delta$. We modify Assumption 1 as follows:

Assumption 1′

$$1 - \gamma < w \leq \frac{1}{2} - c\tilde{x}.$$

Note that 1′ is simpler than Assumption 1, as it contains no equivalent of the inequality $\frac{1}{2} - cx - \frac{\theta(2\gamma-1)}{2} < w$. At the beginning of each period, S observes ω_t. In period t, R observes neither ω_t nor ω_{t-1} but instead observes ω_{t-2}, m_{t-1}. The assumption that R observes ω_{t-2} captures the notion that information about the state diffuses in society through various channels and eventually reaches those who have not actively searched for it, albeit with a lag. S chooses messages from period 1 onwards, and R chooses whether to visit and what action to take from period 2 onwards. The timing of payoff realizations is irrelevant, as long as it does not allow R to infer ω_t and ω_{t-1} during period t.

Strategy of S A communication strategy of S for the whole game is given by a sequence of communication strategies, one for each t. A stationary communication strategy for the whole game is such that S uses the same strategy for each period t. A strategy for period t is said to belong to class n if it conditions the message sent in period t on neither more nor less than

$$\{\omega_{t-n+1}, ..., \omega_{t-2}, \omega_{t-1}, \omega_t\}.$$

In other words, a class n strategy is conditioned on the history of the state up to $n-1$ periods ago. Note that a class n strategy is conditioned only on realized states, but not on S's own messages in the past. A *basic* strategy for period t is a strategy for period t such that S always sends $m_t = B$ if $\omega_t = B$.

In order to gain a concrete idea, as an example, let us illustrate a basic class 2 strategy for period t in some detail. First, the probability of sending $m_t \in \{A, B\}$ in period t is a function of (and only of) $\{\omega_{t-1}, \omega_t\}$. Such a strategy for period t is therefore described by four truth-telling probabilities at t and we denote by $\tau^t_{\omega_{t-1}\omega_t}$ the probability that $m_t = \omega_t$ given $\{\omega_{t-1}, \omega_t\}$. Second, a basic strategy for period t implies $\tau^t_{\omega_{t-1}B} = 1$ for any ω_{t-1}. A basic class 2 strategy for period t is *partially informative* if neither $\tau^t_{AA} = \tau^t_{BA} = 1$ nor $\tau^t_{AA} = \tau^t_{BA} = 0$. If $\tau^t_{AA} = \tau^t_{BA} = 1$ then the basic class 2 strategy for period t is *fully informative* (and also features truth-telling). Finally, if $\tau^t_{AA} = \tau^t_{BA} = 0$ then the basic class 2 strategy for period t is *uninformative* since $m_t = B$ regardless of the state. A stationary strategy of S for the whole game, involving the infinite repetition of a basic (class n) strategy for each period, is called a basic (class n) stationary strategy. In what follows, for the sake of simplicity, we restrict ourselves to basic class 1 and 2 stationary communication strategies.

Strategy of R A strategy of R for the whole game is given by a sequence of strategies, one for each t. R's strategy for each period has two components, a consultation rule and an action rule. A stationary strategy of R is such that R uses the same strategy for each period t. A strategy of R is said to belong to class n if R's choices are conditioned on neither more nor less than

$$\{\omega_{t-n+1}, ..., \omega_{t-2}, m_{t-1}, m_t\},$$

where we (abusively) denote $m_t = \emptyset$ if m_t was not observed. Given our exogenous restriction to class 1 and class 2 stationary communication strategies of S, it is without loss of generality to focus on stationary strategies of R of class no larger than 3. A class 3 strategy for period t, as is the case for a strategy for period t of any class, involves a consultation rule that takes the form of a set of threshold rules. For each possible observed history (ω_{t-2}, m_{t-1}), there is a threshold value of the random cost $v_t \in [0, c]$, denoted by $v(\omega_{t-2}, m_{t-1})$, such that R visits in the beginning of period t if and only if $v_t \leq v(\omega_{t-2}, m_{t-1})$. We denote by $\varphi_{\omega_{t-2}m_{t-1}}$ the conditional probability that R consults in period t given (ω_{t-2}, m_{t-1}), which is simply $P(v_t \leq v(\omega_{t-2}m_{t-1}))$. A class 3 strategy of R for period t involves an action rule which conditions R's chosen action at t on $\{\omega_{t-2}, m_{t-1}, m_t\}$, where we let $m_t = \emptyset$ if m_t was not observed.

In what follows, we focus on equilibria featuring stationary strategies for both S and R, and we call such equilibria stationary. In an equilibrium where R acquires m_t with positive probability p conditional on $\eta_t = 1$ (i.e. m_t is costly) and the observed $n-1$ elements history $\{\omega_{t-n+1}, ..., \omega_{t-2}, m_{t-1}, m_t\}$, we say that R consults with probability p at t given this $n-1$ elements observed history. We say that a stationary equilibrium featuring a class n strategy of R features a positive probability of consultation if there is some $(n-1)$ elements history $\{\omega_{t-n+1}, ..., \omega_{t-2}, m_{t-1}\}$ whose stationary probability is positive and which is such that R's stationary strategy specifies a positive probability of consultation in period t.

Discussion In our infinite horizon analysis, we do not consider non-stationary strategies (e.g. Grim-Trigger, Stick and Carrot) that might achieve more informative equilibrium reporting through punishment for lies (conditional on a sufficiently high discount factor). Such strategies would in principle be feasible since R observes past states (though with a lag), thereby ex post identifying lies, and could thus choose not to visit for a while after a misreport in order to punish S. Note that the simplest unilateral version of such strategies might not work. The punishment by R must be accompanied by less uninformative messaging by S during the punishment phase, since otherwise R might have an incentive to consult during the punishment phase. However, if S knows that R never consults over a time period, S has a strict incentive to report truthfully and reap the truth-telling reward. Equilibrium strategies would thus have to be more sophisticated, involving constructions of the Stick and Carrot type (i.e. punishing for not punishing).

Another concern is that such sophisticated equilibrium constructions are somewhat implausible in the context of communication between a media firm and a potentially large audience (many receivers). There is a public good aspect to punishing the sender, which generates a free-riding problem. Yet another aspect is that equilibria featuring sophisticated punishment strategies are typically not renegotiation proof, in contrast to stationary equilibria.

3.2 Incentives

3.2.1 Beliefs and decisions by R

Let us first examine R's beliefs and incentives, assuming that S uses a basic class 2 stationary communication strategy (the case where S uses a basic class 1 strategy can be thought of a special case of this analysis). We derive R's best response to such a strategy of S. In order to derive R's decision to make a costly visit in period t given ω_{t-2} and m_{t-1}, we need to consider the conditional joint distribution of the state in periods t and $t-1$, as of the beginning of period t. This is denoted $P(\omega_{t-1}, \omega_t \mid \omega_{t-2}, m_{t-1})$, since the accuracy of the report in period t (i.e. $\tau^t_{\omega_{t-1}\omega_t}$) potentially depends on both ω_{t-1} and ω_t.

Note that the joint distribution is given as follows:

$$P(\omega_{t-1} = A, \omega_t = A \,|\omega_{t-2}, m_{t-1}) = P(\omega_{t-1} = A \,|\omega_{t-2}, m_{t-1})\gamma$$

and

$$P(\omega_{t-1} = B, \omega_t = A \,|\omega_{t-2}, m_{t-1}) = P(\omega_{t-1} = B \,|\omega_{t-2}, m_{t-1})\frac{1}{2}.$$

Let us derive the benefit of consultation at each of the three possible information sets of R at the beginning of t, ignoring for now the cost of consultation. First, recall that whether or not R consults in period t, he observes m_t by the end of period t. Therefore, R only needs to consider the expected payoff for period t when he decides whether or not to make a costly visit in period t. Second, note that given $\gamma > \frac{1}{2}$, the more likely state in period t is A if R does not observe m_t, regardless of the history. Denote by $\pi^R_t(d_t \,|\omega_{t-2}, m_{t-1})$ R's (gross) expected payoff for period t given ω_{t-2}, m_{t-1} and the consultation decision $d_t \in \{0, 1\}$, ignoring the cost of consultation. Assuming that $m_t = B$ induces R to choose action B (as must be true if consultation is worthwhile), his gross expected payoff for period t if he visits is given by

$$\begin{aligned}\pi^R_t(d_t &= 1 \mid \omega_{t-2}, m_{t-1}) \\ &= P(\omega_t = B \mid \omega_{t-2}, m_{t-1}) + P(\omega_{t-1} = A, \omega_t = A \mid \omega_{t-2}, m_{t-1})\tau_{AA} \\ &\quad + P(\omega_{t-1} = B, \omega_t = A \mid \omega_{t-2}, m_{t-1})\tau_{BA}. \end{aligned} \tag{4}$$

Meanwhile, if R does not visit, he chooses A and his expected payoff is thus

$$\pi_t^R(d_t = 0 \mid \omega_{t-2}, m_{t-1}) = 1 - P(\omega_t = B \mid \omega_{t-2}, m_{t-1}). \tag{5}$$

Taking the difference between (4) and (5), the (gross) benefit of visit is given by

$$\Delta_t^R\left(\omega_{t-2}, m_{t-1}\right) \equiv \pi_{t,t}^R(d_t = 1\,|\omega_{t-2}, m_{t-1}\,) - \pi_{t,t}^R(d_t = 0\,|\omega_{t-2}, m_{t-1}\,).$$

In order for R to choose to visit at t given ω_{t-2} and m_{t-1}, it has to be the case that the above benefit is greater than the cost of visiting, namely

$$\Delta_t^R\left(\omega_{t-2}, m_{t-1}\right) \geq w + v_t. \tag{6}$$

We denote by $v\left(\omega_{t-2}, m_{t-1}\right)$ the threshold value of v_t such that given ω_{t-2} and m_{t-1}, R visits in period t if $v_t \leq v\left(\omega_{t-2}, m_{t-1}\right)$. Note that (6) rewrites as

$$\widetilde{v}(\omega_{t-2}, m_{t-1}) \equiv \Delta_t^R\left(\omega_{t-2}, m_{t-1}\right) - w \geq v_t.$$

That is, R visits at t given ω_{t-2} and m_{t-1} if the random consultation cost is lower than the threshold $\widetilde{v}(\omega_{t-2}, m_{t-1})$. Denote by $\varphi_{\omega_{t-2}m_{t-1}}$ the probability that R visits in period t given ω_{t-2} and m_{t-1}. For $\widetilde{v}(\omega_{t-2}, m_{t-1}) \in [0, c]$ we have

$$\varphi_{\omega_{t-2},m_{t-1}} \equiv P\left(v_t \leq \widetilde{v}(\omega_{t-2}, m_{t-1})\right) = \frac{\widetilde{v}(\omega_{t-2}, m_{t-1})}{c},$$

since v_t is uniformly distributed on $[0, c]$.

3.2.2 Reporting strategy of S

Let us now consider the strategy of S. Note first that the message sent in period t does not affect the payoff of S in period $t+2$ and beyond, since from $t+2$ onwards R knows the true state ω_t, which implies that m_t is irrelevant to decision making in and after $t+2$. The optimal choice of m_t thus takes into account only S's expected payoffs in periods t and $t+1$. Let $\pi_{t,t+1}^S\left(m_t\,|\omega_{t-2}, \omega_{t-1}, \omega_t, m_{t-1}\,\right)$ denote the *sum* of the expected payoffs of S for periods t and $t+1$, given $\omega_{t-2}, \omega_{t-1}, \omega_t, m_{t-1}$ and m_t. Let $I_t(\omega_t, m_t) = 1$ if S reports truthfully (i.e. $m_t = \omega_t$) and $I_t(\omega_t, m_t) = 0$ otherwise. We have

$$\begin{aligned}\pi_{t,t+1}^S\left(m_t \mid \{\omega_t\}_{s=t-2}^t, m_{t-1}\right) = (1-\alpha)P(d_t = 1\,|\omega_{t-2}, m_{t-1}, \eta_t = 1)f + I_t(\omega_t, m_t)z \\ +\delta(1-\alpha)P(d_{t+1} = 1 \mid \omega_{t-1}, m_t, \eta_{t+1} = 1)f \\ +\delta P(m_{t+1} = \omega_{t+1})z.\end{aligned}$$

Note that in the above expression, only the following terms

$$I_t(\omega_t, m_t)z + \delta(1-\alpha)P(d_{t+1} = 1\,|\omega_{t-1}, m_t, \eta_{t+1} = 1\,)f$$

depend on m_t. It follows that in period t, S prefers to send m to m' if and only if:

$$\begin{aligned} I_t(\omega_t, m)z + \delta(1-\alpha)P(d_{t+1} &= 1 \mid \omega_{t-1}, m, \eta_{t+1} = 1)f \geq \\ I_t(\omega_t, m')z + \delta(1-\alpha)P(d_{t+1} &= 1 \mid \omega_{t-1}, m', \eta_{t+1} = 1)f. \end{aligned} \quad (7)$$

In equilibrium, Swill always reports truthfully whenever $\omega_t = B$. In this state, reporting truthfully leads to both the truth-telling reward and the higher probability of visit in period $t+1$. In contrast, when $\omega_t = A$, S faces a choice between obtaining the truth-telling reward now and inducing higher probability of visit tomorrow (by sending $m_t = B$). If (7) holds with equality, S is willing to randomize between messages, which will be the case in the equilibrium that we study.

3.3 Equilibrium behavior and welfare

The following Lemma puts some structure on R's consultation behavior.

Lemma 7. *a) No stationary equilibrium features zero probability of consultation.*

b) No stationary equilibrium in which S uses a basic communication strategy is such that the probability of consultation at t is positive given $(\omega_{t-1} = \omega, m_t = A)$, for some $\omega \in A, B$.

The next Lemma concerns S's equilibrium reporting.

Lemma 8. *a) No stationary equilibrium features an uninformative or a fully informative communication strategy.*

b) No stationary equilibrium features a basic class 1 communication strategy.

An essential insight is that there exists no stationary equilibrium with fully informative communication given our assumption that x is not too large. In such a putative equilibrium, when $\omega_t = A$, S can significantly increase the probability of consultation in the next period by sending $m_t = B$ (i.e. misreporting). This deviation incentive breaks down the fully informative equilibrium.

The next Proposition presents our equilibrium characterization. We obtain existence and uniqueness of an equilibrium featuring a *basic* class 1 or 2 stationary communication strategy. The argument leading to the result builds on our Lemmas. By Lemma 7, any stationary equilibrium must feature a positive probability of consultation, as well as a probability zero of consultation at t if $m_{t-1} = B$. By Lemma 8, any stationary equilibrium features a partially informative communication strategy, which can furthermore not be a basic class 1 strategy. It follows that if we find a stationary equilibrium featuring either a basic class 1 or 2 communication strategy, then the equilibrium must feature a basic class 2 communication strategy and R must consult with positive probability at t if and only if $m_{t-1} = B$. The last part of the characterization is to prove that there exists an equilibrium satisfying the above description, and this is the focus of the proof provided in the Appendix.

Proposition 5. *There exists a unique stationary equilibrium featuring a basic communication strategy belonging to class 1 or 2. It features a class 2 communication strategy that satisfies $\tau^*_{AA}, \tau^*_{BA} \in (0,1)$ and $\tau^*_{AA} \neq \tau^*_{BA}$. R consults with probability $\tilde{x}$ if $m_{t-1} = B$ and with probability zero if $m_{t-1} = A$.*

The intuition behind the equilibrium is remarkably similar to that obtained for the analysis of the two-period model. We now describe key comparative statics properties of equilibrium communication.

Proposition 6. *In the equilibrium identified in Proposition 5, τ^*_{AA} and τ^*_{BA} are constant in f and α, increasing in w and c, decreasing in $\tilde{x}(= x/\delta)$ and independent of α.*

The comparative statics results are qualitatively the same as in the two-period model. An increase in w or c would lower the probability of visit at t given $m_t = B$, holding fixed τ^*_{AA}, τ^*_{BA}. This implies that the equilibrium informativeness of communication must increase to keep the consultation probability equal to $\tilde{x}$. Equivalently, decreasing R's drive to consult makes misreporting (i.e. sending $m_t = B$ when $\omega_t = A$) less attractive. As S becomes more patient (δ higher and $\tilde{x}$ lower) the incentive to deviate from truth-telling increases, since a future visit induced by misreporting becomes more valuable relative to the truth-telling reward achievable in the current period. Also, f and α do not enter τ^*_{AA} and τ^*_{BA}. Proposition 6 demonstrates that the main insights obtained in the two-periods setup do *not* depend on the feature that truth-telling occurs in the final period.

The next proposition establishes how the equilibrium payoff of R varies with underlying parameters, for α high enough. In what follows, the average per-period payoff of R is the latter's average payoff across periods 2 to T, for T tending to infinity. The empirical frequency of state A over this time interval thus coincides for sure with the stationary distribution.

Proposition 7. *For α high enough, the average per-period payoff of R in the informative equilibrium identified in Proposition 5 is constant in f, increasing in w and c and decreasing in $\tilde{x}$.*

These comparative statics results are also similar to those obtained in the two-periods model. When the probability of receiving the report for free is high enough, the effect of an increase in w or c becomes unambiguous. Recall that an increase in the latter two variables has two effects, one negative and one positive, on R's expected payoff. The first is a direct increase in the cost of consultation. The second (and indirect) effect is an improvement in the informativeness of reports. Clearly, when α is large enough (so that the report is free often enough), the second effect dominates the first.

4 Extensions

4.1 Non-intermediate visiting costs

In our analysis of the two-periods model, for expositional convenience, we focused on w satisfying

$$\max\left\{1-\gamma, \frac{1}{2} - cx - \frac{\theta(2\gamma-1)}{2}\right\} < w < \frac{1}{2} - cx.$$

Let us now consider equilibrium behavior in the two-period model outside of the parameter range. To make the exposition transparent, assume that

$$1-\gamma < \frac{1}{2} - cx - \frac{\theta(2\gamma-1)}{2}.$$

The inequality can be written as $cx < \frac{1}{2}(1-\theta)(2\gamma-1)$, and thus merely implies that we focus on small x, which we have maintained throughout the paper. The following proposition indicates that, echoing the comparative statics results we presented in Proposition 2, the equilibrium informativeness of the period-1 report diminishes and then reaches zero as w becomes lower, while there is a truth-telling equilibrium as w becomes high enough. In other words, the informativeness is weakly decreasing in w throughout.

Proposition 8. *a) If $w \leq \frac{1}{2} - cx - \frac{\theta(2\gamma-1)}{2}$, there is a unique equilibrium. It features uninformative communication in period 1 and truth-telling in period 2.*

b) If $w \in \left(\frac{1}{2} - cx, \frac{1}{2}\right)$, there exists an equilibrium featuring truth-telling in both periods and a strictly positive probability of consultation in period 2.

Point a) indicates that a very low fixed cost of consultation (w) makes it impossible to induce any informative communication by S in period 1. Intuitively, when w is very low, R becomes too manipulable for S to report informatively. Notice that in the current setup, $R's$ period-2 consultation behavior never becomes unresponsive even if w tends to 0. This is because R still bears a random cost of visiting $v_t \in [0, c]$, where $c \in \left[\frac{1}{2}, 1\right]$. Thus, as w becomes close to 0, the expected cost of consultation does not tend to 0 but to $c/2$. Note that if c were instead also close to 0, R would become completely unresponsive to the period 1 report as w tends to 0, since he would visit in period 2 with ex ante probability 1. This would generate an equilibrium with period-1 truth-telling.

Point b) shows that instead, for a sufficiently high fixed cost of consultation, fully informative period-1 communication combined with a positive probability of consultation in period 2 can be achieved in equilibrium. In this equilibrium, the benefit of truth-telling outweighs the benefit of the increased probability of visit induced by misreporting (i.e. sending $m_1 = B$ when $\omega_1 = A$) because this latter probability remains very low.

4.2 Pricing and competition

Thus far, we have assumed that the fixed cost of consultation w and the revenue per visit f are exogenously given. An exogenous decrease in w could in particular be interpreted as reflecting the easier access to news rendered possible by the internet (a change in technology). In reality, these variables are often at least partially determined endogenously in the market. As a first attempt to endogenize w and f, we let $w = f = p_t$, where p_t is thus a monetary transfer from R to S that occurs when the former purchases message m_t, and where the price p_t is chosen by S. In this section, we thus solve for communication strategies, consultation behavior and *prices* within the two-period model we developed in Section (2). Assume for simplicity that the marginal cost of producing a report is zero in any period. We start by analyzing a monopoly media firm, and subsequently offer a preliminary examination of a duopoly market. Assume as in the main analysis that $\alpha \in [0, 1)$.

Monopoly Under monopoly, let p_t denote the price charged in period t and $\bar{\tau}^t_\omega$ denote the probability that $m_t = \omega_t$ (i.e. truth-telling) given $\omega_t = \omega$. The expected payoff from truth-telling in period 1 is assumed to be $x(1-\alpha)p_2$, where p_2 is the equilibrium price in period 2 given $m_1 = B$ (under our parameter assumptions, the probability of period 2 consultation will be zero if $m_1 = A$). That is, the truth-telling benefit in period 1 is proportional to the profits in the future (period 2). This is consistent with our previous interpretation on the truth-telling reward as reputational concerns, since reputation is closely related to future stakes.

We study equilibria in which S uses a simple communication strategy in each period (i.e. $m_t = B$ when $\omega_t = B$) and call such equilibria *simple* equilibria. Suppose $w > 1-\gamma$ as in Section 2 and that the prior in period 1 is the stationary distribution $\theta = \mu_A \equiv \frac{1}{3-2\gamma}$. In addition let

$$w \in \left(\frac{1}{2} - 2cx - \frac{2\gamma-1}{6-4\gamma}, \frac{1}{2} - 2cx\right). \tag{8}$$

We do not have to specify the equilibrium price for period 1, as (in contrast to the price in period 2) it bears no relation to the informativeness of communication in period 2.

Proposition 9. *Under monopoly, any simple equilibrium features*

$$\bar{\tau}_A^{1*} = \frac{2\gamma(1-w-2cx)+3w+6cx-2}{\gamma+w+2cx-1} \in (0,1),$$

truth-telling in period 2, and the equilibrium price $p_2^* = cx$. R consults with probability zero in period 2 if $m_1 = A$ and with strictly positive probability if $m_1 = B$.

The equilibrium has a very similar structure to that obtained under an exogenous visiting cost and revenue. The comparative statics properties of $\bar{\tau}_A^{1*}$ with respect to exogenous parameters also echo those obtained under exogenous prices. Note that p_2 is increasing in x and c. This is because a higher x or c induces more equilibrium truth-telling, which in turn implies more uncertainty about ω_2 after $m_1 = B$ and thus a higher willingness to pay for the message in period 2, which the monopoly firm exploits.

Duopoly Let us now present a preliminary examination of duopoly. We make several strong assumptions to ensure tractability. There are two firms S_1, S_2 who both observe the state perfectly in each period. Let $m_t^{S_i}$denote the period-t message picked by firm S_i and let $p_t^{S_i}$ denote the price of $m_t^{S_i}$. The pricing and messaging strategy of sender i conditions freely on the history h_t^i that he has observed, which contains past prices, messages and states. At the start of period t, S_i simultaneously chooses $p_t^{S_i}$ and $m_t^{S_i}$. At the end of period 1, after choosing an action, R observes $\left\{m_1^{S_1}, m_1^{S_2}\right\}$ independently of what message he purchased earlier in this period.

We study equilibria in perfectly correlated symmetric strategies where both senders use the same communication strategy, always send the same message at t (i.e. perfectly correlate their randomization) and charge identical prices in each period. Let τ_t^ω denote the probability that $m_t^{S_i} = \omega_t$ given $\omega_t = \omega$, for $i = 1,2$. Suppose R has *sceptical* out-of-equilibrium beliefs of the following form: If R observes at the end of period 1 that $m_1^{S_1} \neq m_1^{S_2}$, he believes that $\omega_1{=}A$ with probability 1. One interpretation of this belief is that R considers any misalignment of reports as evidence that one of the firms was honest in reporting the state $\omega_t = A$, thereby foregoing a future visit. This is consistent with the notion that there is no reason to misreport when $\omega_t = B$ as truthful reporting yields both the truth-telling benefit and a higher probability of visit tomorrow. We define a *simple perfectly correlated symmetric* equilibrium as one that features identical reports across senders, simple communication strategies in each period and sceptical out-of-equilibrium beliefs.

An interpretation of the above type of (sunspot) equilibrium is that journalists socialize in the same environments, are subject to the same fashions, pressures, etc. But our main justification for focusing on symmetric and perfectly correlated communication strategies is analytical simplicity. Effectively, we restrict the direct effect of competition to the price of reports, and thereby aim at isolating the linkage between competition and informativeness that arises in consequence of the effect of competition on prices. Note that competition typically also directly affects informativeness by generating *endogenous certification* equilibria in which the receiver compares reports across senders and punishes all senders when reports do not match.

R's decision to purchase a message before choosing an action in each period is modeled as follows. R first chooses one of the news providers, and then decides whether or not to purchase from the provider. In each period, S_i is chosen with probability $\left(1 - \frac{p_2^{S_i}}{p_2^{S_1}+p_2^{S_2}}\right)$ and R is thus more likely to choose the firm that offers the lower price. Then, R purchases from the selected firm if and only if purchasing its report is worth it, given his uncertainty and the price of the message. If R does not purchase the message, he chooses an action uninformed. This choice procedure generates effects reminiscent of product differentiation. That is, setting a higher price than the competitor does not divert all demand, thereby leading firms to set prices above marginal cost. This in turn generates an incentive to encourage (profitable) visits in period 2 by misreporting in period 1.

The truth-telling reward for each firm is assumed to be $\chi(1-\alpha)\widetilde{p}_2 x$, where $\widetilde{p}_2 = p_2^{S_1} = p_2^{S_2}$ is the equilibrium price in period 2 if $m_1^{S_1} = m_2^{S_2} = B$, where χ parameterizes the size of the truth-telling benefit (reputational concerns) under competition. If χ is directly associated with market size, then the two firms splitting the market equally would imply $\chi = 1/2$. If on the other hand competition makes reputation concerns more prominent, χ would be higher.

Suppose $w > 1-\gamma$ and that $\theta = \mu_A \equiv \frac{1}{3-2\gamma}$. In addition, let

$$w \in \left(\frac{1}{2} - 3\chi c x - \frac{2-\gamma}{6-4\gamma}, \frac{1}{2} - 3\chi c x\right). \tag{9}$$

As before, we do not specify the equilibrium price in period 1, since it bears no relation to the informativeness of communication in period 2, which we focus on.

Proposition 10. *Under duopoly, there exists a simple perfectly correlated symmetric equilibrium and any such equilibrium satisfies the following. There is truth-telling in period 2. Furthermore,*

$$\widetilde{\tau}_A^{1*} = 3 - 2\gamma - \frac{(1-\gamma)(2\gamma-1)}{\gamma + w + 3\chi c x - 1} \in (0,1)$$

and $\widetilde{p}_2^* = \chi c x$. *$R$ consults with strictly positive probability if* $m_1^{S_1} = m_1^{S_2} = B$, *and with probability zero otherwise. Also,* $\widetilde{p}_2^*$ *and* $\widetilde{\tau}_A^{1*}$ *are increasing in* χ.

Note that the ranges of w assumed for the monopoly and duopoly setups, namely (8) and (9), converge as x tends to zero. Let us assume that x is small as we have maintained throughout our analysis so that given the same relevant parameter values, the equilibria described by Propositions 9 and 10 both exist. The following remark compares the monopoly and duopoly setups with respect to the equilibrium prices in period 2 and the informativeness of the messages in period 1, for different values of χ.

Remark. For $\chi = 1$, $\widetilde{p}_2^* = p_2^*$ and $\widetilde{\tau}_A^{1*} > \bar{\tau}_A^{1*}$. For $\chi = \frac{2}{3}$, $\widetilde{p}_2^* < p_2^*$ and $\widetilde{\tau}_A^{1*} = \bar{\tau}_A^{1*}$. For $\chi = \frac{1}{2}$, $\widetilde{p}_2^* < p_2^*$ and $\widetilde{\tau}_A^{1*} < \bar{\tau}_A^{1*}$.

Note that for χ close enough to $\frac{1}{2}$, transition from monopoly to duopoly involves a trade-off (in terms of R's expected utility) between lower prices and lower informativeness. While competitive pressure decreases the price in period 2, this in turn diminishes the informativeness of reports in period 1. This is because a lower price in period 2 increases R's responsiveness to uncertainty in period 1, thereby increasing the senders' incentive to send $m_1 = B$ when $\omega_1 = A$. For χ close enough to $\frac{1}{2}$, it is thus ambiguous whether competition is beneficial R.

5 Conclusion

The central mechanism at work in our model is the media firm's incentive to generate uncertainty about tomorrow's state in order to encourage future clicks or news purchases by potential readers. We have demonstrated that a higher cost of consultation can increase the informativeness of communication by dampening a potential reader's responsiveness to uncertainty generating messages and thereby discouraging the media firm's incentive to distort its private information. We argue that this may explain why many journalists, commentators, and media researchers have expressed concerns about diminishing accuracy of media reports in light of the development of online news, which has significantly reduced the costs of access for the last two decades or so.

Intuitively, this novel explanation would extend beyond the context of media. Similar incentives to misreport private information may well be present in many repeated relationships between a uninformed party and an expert who sells his reports over time (for example doctor-patient relations, or financial advisers, etc.). In those circumstances, a lower cost of consultation also makes the receiver more manipulable, which may lead to less informative communication. However, a key difference between our model and expert consultation is that in the latter, the receiver remains uninformed unless he consults with the expert, which poses a technical challenge because we need to specify how the receiver's belief evolves over time given not only the informativeness of the message but also the receiver's decision whether to obtain the message in each period. We could avoid the issue, thanks to the assumption that the receiver knows the message sent in the previous period, which seems plausible and appropriate if we consider media reports that spread in society quickly.

6 Appendix A: Two-period model

6.1 Lemma 2

Proof. **Step 1** The proof of part a) is by contradiction. There are two cases of uninformative period-1 communication to consider. The first is the case where both $m_1 = A$ and $m_1 = B$ are sent with strictly positive probability. In this case, both $m_1 = A$ and $m_1 = B$ yield the same probability of consultation in period 2 since both are uninformative. However if it is the case, since $R's$ consultation behavior in period 2 is independent of m_1, S is strictly better off reporting truthfully with probability 1, as this yields the extra truth-telling benefit. Therefore, this case is excluded.

The second case is such that either $m_1 = A$ or $m_1 = B$ is sent with ex ante probability one in period 1. Assume that S always sends the same message $\widetilde{m}_1$=$J \in \{A, B\}$ in period 1. Now, the out-of-equilibrium belief in period 1 that maximizes S's incentive to choose $\widetilde{m}_1$ in period 1 is that, given $m_1' \neq \widetilde{m}_1$, R assigns probability one to $\omega_1 = A$. Consider thus a putative equilibrium in which there is some $J \in A, B$ such that S always sends message $\widetilde{m}_1 = J$ in period 1 and if R observes $m_1' = -J = \{A, B\} \setminus J$ in period 1, he assigns probability 1 to $\omega_1 = A$. We now show that in such a putative equilibrium, S has a deviation incentive in period 1 when $\omega_1 = -J$. Let $\omega_1 = -J$. Note that sending m_1'= $-J$ yields the expected payoff $2(1-\alpha)fx$ across the two periods if $w > 1-\gamma$ as implied by Assumption 1. Indeed, S obtains the truth-telling payoff in period 1, and anticipates that he will receive it as well in period 2 though R will not consult in period 2. On the other hand, sending $\widetilde{m}_1 = J$ yields

$$(1-\alpha)\, f \left(\frac{\frac{1}{2} + \frac{1}{2}\theta - \theta\gamma - w}{c} \right) + (1-\alpha)\, fx.$$

Note that $\left(\frac{\frac{1}{2}+\frac{1}{2}\theta-\theta\gamma-w}{c} \right)$ is positive. After receiving message $\widetilde{m}_1 = J$ in period 1, R's belief about ω_1 remains unchanged, i.e. R considers that he has not learned anything concerning the period 1 state. In consequence, he visits with probability $\left(\frac{\frac{1}{2}+\frac{1}{2}\theta-\theta\gamma-w}{c} \right)$ in period 2. The second element $(1-\alpha)\, fx$ in the above expression is the expected truth-telling payoff in period 2. Now, note that

$$(1-\alpha)fx > (1-\alpha)\, f \left(\frac{\frac{1}{2} + \frac{1}{2}\theta - \theta\gamma - w}{c} \right)$$

rewrites as

$$w > \frac{1}{2} - cx - \frac{\theta(2\gamma-1)}{2},$$

as assumed in Assumption 1. It follows that in the considered equilibrium, S has a strict incentive to send $m_1' = -J$ in period 1 if $\omega_1 = -J$. We may conclude that case 2 is excluded, i.e. there exists no equilibrium such that either $m_1 = A$ or $m_1 = B$ is sent with ex ante probability one in period 1.

Step 2 The proof of part b) is also by contradiction and there are again two cases to consider. Consider first the case in which S's equilibrium strategy in period 1 is such that $m_1 = \omega_1$ for any realization of ω_1. In such an equilibrium, Assumption 1 implies that R consults with probability zero in period 2 after $m_1 = A$ and with positive probability after $m_1 = B$. Let us now show that S then strictly prefers to deviate to sending $m_1 = B$ given $\omega_1 = A$. Let $\omega_1 = A$. Message $m_1 = A$ leads to a visiting probability of zero in period 2 conditional on $\eta_2 = 1$, and thus yields an expected payoff of $(1-\alpha)fx$. In contrast, $m_1 = B$ leads to a strictly positive probability of visit in period 2 conditional on $\eta_2 = 1$, namely probability $\frac{\frac{1}{2}-w}{c}$. Message $m_1 = B$ thus yields expected payoff $(1-\alpha)f\left(\frac{\frac{1}{2}-w}{c}\right)$. Now, note that $(1-\alpha)f\left(\frac{\frac{1}{2}-w}{c}\right) > (1-\alpha)fx$ is equivalent to $\frac{1}{2} - cx > w$, as assumed in Assumption 1. S thus has a strict incentive to deviate to $m_1 = B$ when $\omega_1 = A$, so the equilibrium breaks down.

Consider next the case in which S's equilibrium strategy in period 1 is such that $m_1 = A$ if $\omega_1 = B$ and $m_1 = B$ if $\omega_1 = A$. In this case, the incentive to deviate given $\omega_1 = A$ is further reinforced, relative to the first case. That is, deviating to $m_1 = A$ given $\omega_1 = A$ indeed not only yields a positive probability of consultation tomorrow conditional on $\eta_2 = 1$ (as opposed to $m_1 = B$) , but the deviation also yields the extra truth-telling benefit. So the considered equilibrium breaks down. Hence we may conclude that there cannot be an equilibrium with fully informative communication in period 1. □

6.2 Lemma 3

Proof. The proof is by contradiction. Assume first that the probability of consultation in period 2 is zero, and that both messages ($m_1 = A$ and B) are sent with strictly positive probability in equilibrium. In this case, m_1 does not affect R's consultation decision on-the-equilibrium path, since the probability of visit remains zero. This implies that S is always strictly better off sending $m_1 = \omega_1$ in period 1, as this yields the truth-telling benefit. Given truth-telling in period 1, the probability of visit in period 2 after $m_1 = B$ is $\frac{\frac{1}{2}-w}{c} > 0$, which contradicts the assumption that the probability of consultation in period 2 is zero.

Second, assume that the probability of consultation in period 2 is zero, and that either $m_1 = A$ or B is sent with probability one in period. However, this implies an equilibrium with uninformative communication, which we have ruled out in Lemma 2.

Thus we conclude that the probability of visit is strictly positive in equilibrium. □

6.3 Lemma 4

Proof. Consider a putative equilibrium in which S's communication strategy in period 1 is a simple communication strategy. From Lemma 2, any equilibrium features an informative communication strategy in period 1. Therefore, we must have $\tau_A^1 > 0$. This implies that both $m_1 = A$ and $m_1 = B$ are sent with strictly positive probability in period 1. We also know from Lemma 3 that any equilibrium must feature a positive probability of consultation in period 2. In the following we consider R's consultation behavior in period 2 when $m_1 = A$ and $m_1 = B$, respectively.

First, if $m_1 = A$ under a simple communication strategy, R believes $P(\omega_1 = A \mid m_1 = A) = 1$. Note that if R does not consult in period 2, then he chooses action A in the period. Consulting in period 2 guarantees the correct action (as S reports truthfully) but comes at the cost $w + v_2$. Thus R consults in period 2 if and only if $1 - w - v_2 \geq P(\omega_2 = A \mid m_1 = A) = \gamma$. Assumption 1 implies $w + \gamma > 1$, which implies that R consults in period 2 with probability zero after $m_1 = A$.

Let $m_1 = B$. We know from Lemma 3 that any equilibrium must feature a strictly positive probability of consultation in period 2. We furthermore know that R consults with probability zero after $m_1 = A$. It follows that R must consult with strictly positive probability after $m_1 = B$. □

6.4 Lemma 5

Proof. The proof is organized as follows. We show by contradiction that in equilibrium, $m_1 = B$ must induce a weakly higher probability of consultation in period 2 than $m_1 = A$. In turn, we show that this immediately implies that S reports $m_1 = B$ with probability 1 when $\omega_1 = B$.

Note that the report is truthful in period 2 in any equilibrium. Lemma 2 indicates that any equilibrium involves informative communication in period 1. If the report is informative in period 1, R is (weakly) more likely to make a costly visit in period 2, if the message $m_1 \in \{A, B\}$ shifts R's posterior towards $\omega_1 = B$ than the other message.

Assume that $m_1 = A$ induces a weakly higher probability of consultation in period 2 than $m_1 = B$. If this is the case, then S reports $m_1 = A$ with probability 1 when $\omega_1 = A$, since by doing so S obtains both the truth-telling reward and induces a weakly higher probability of consultation in period 2 than by sending $m_1 = B$. However, if an equilibrium is such that S sends $m_1 = A$ for sure whenever $\omega_1 = A$, then the report $m_1 = B$ induces a strictly higher posterior that the state is $\omega_1 = B$ than $m_1 = A$. Indeed, note that if $\tau_A^1 = 1$, then

$$P(\omega_1 = B \mid m_1 = A) = \frac{(1-\theta)(1-\tau_B^1)}{(1-\theta)(1-\tau_B^1)+\theta} < 1$$

while

$$P(\omega_1 = B \mid m_1 = B) = \frac{(1-\theta)\tau_B^1}{(1-\theta)\tau_B^1+\theta(0)} = 1.$$

Consequently, the report $m_1 = B$ must induce a higher probability of consultation in period 2 than $m_1 = A$, but this contradicts the assumption that $m_1 = A$ induces a weakly higher probability of consultation. We have thus shown by contradiction that $m_1 = B$ must induce a strictly higher probability of consultation in period 2 than $m_1 = A$. Note furthermore that this fact immediately implies that S reports $m_1 = B$ with probability 1 when $\omega_1 = B$, since by doing so S obtains both the truth-telling reward and induces a strictly higher probability of consultation in period 2 than by sending $m_1 = A$. □

6.5 Proposition 1

Proof. We know from previous Lemmas that if there exists an equilibrium, it satisfies the following description. It features a simple partially informative communication strategy in period 1 and the fully informative simple communication strategy in period 2. Furthermore, R consults with strictly positive probability in period 2 if $m_1 = B$ and with probability 0 if $m_1 = A$. In the following proof, we show that there exists a unique equilibrium satisfying the above description, and characterize the value of τ_A^1 as well as the consultation probability in period 2 given $m_1 = B$. We first derive the probability of visit by R in period 2 given $m_1 = B$ and given that visiting is costly. Given this probability, we consider S's choice between reporting truthfully ($m_1 = A$) or sending m_1 =B when $\omega_1 = A$. We then examine the parameter conditions under which there exists an equilibrium of the above described type. We show that it must be unique, if it exists, and obtain the explicit solution for τ_A^1.

Assume in what follows an equilibrium in which S uses a simple strategy in both periods, the strategy in period 2 being fully informative. Given $m_1 = B$, costly visit by R in period 2 requires

$$1 - w - v_2 > P(\omega_1 = A \mid m_1 = B)\gamma + P(\omega_1 = B \mid m_1 = B)\frac{1}{2},$$

where

$$P(\omega_1 = B \mid m_1 = B) = \frac{1-\theta}{1-\theta+\theta(1-\tau_A^1)}.$$

Hence, when $m_1 = B$, R makes a costly visit in period 2 if and only if

$$1 - w - v_2 > \left(1 - \frac{1-\theta}{1-\theta+\theta(1-\tau_A^1)}\right)\gamma + \frac{1-\theta}{1-\theta+\theta(1-\tau_A^1)}\frac{1}{2},$$

which rewrites as

$$v_2 < \frac{\theta - 2\theta\gamma - 2\theta\tau_A^1 + 2\theta\gamma\tau_A^1 + 1}{2\left(1-\theta\tau_A^1\right)} - w.$$

Thus, given $m_1 = B$, R makes a costly visit in period 2 with positive probability if and only if

$$\frac{\theta - 2\theta\gamma - 2\theta\tau_A^1 + 2\theta\gamma\tau_A^1 + 1}{2\left(1-\theta\tau_A^1\right)} - w > 0. \tag{10}$$

Let

$$1 - w - v_2 > \left(1 - \frac{1-\theta}{1-\theta+\theta(1-\tau_A^1)}\right)\gamma + \frac{1-\theta}{1-\theta+\theta(1-\tau_A^1)}\frac{1}{2},$$

which rewrites as

$$v_2 < \frac{\theta - 2\theta\gamma - 2\theta\tau_A^1 + 2\theta\gamma\tau_A^1 + 1}{2\left(1-\theta\tau_A^1\right)} - w.$$

Thus, given $m_1 = B$, R makes a costly visit in period 2 with positive probability if and only if

$$\frac{\theta - 2\theta\gamma - 2\theta\tau_A^1 + 2\theta\gamma\tau_A^1 + 1}{2\left(1-\theta\tau_A^1\right)} - w > 0. \tag{11}$$

Let

$$\varphi_B(\tau_A^1, w, c, \gamma) \equiv \frac{\frac{\theta-2\theta\gamma-2\theta\tau_A^1+2\theta\gamma\tau_A^1+1}{2\left(1-\theta\tau_A^1\right)} - w}{c}. \tag{12}$$

If (10) holds and $m_1 = B$, R makes a costly visit with probability

$$\max\left\{\min\left\{\varphi_B(\tau_A^1, w, c, \gamma), 1\right\}, 0\right\}.$$

Next, let us examine the incentives of S in period 1. Recall that S reports truthfully when $\omega_1 = B$. Suppose that instead $\omega_1 = A$. Sending $m_1 = A$ yields the truth-telling payoff $(1-\alpha)xf$ but no visit for sure in period 2. Therefore, given $\omega_1 = A$, S is indifferent between $m_1 = A$ and

$m_1 = B$ in period 1 if and only if

$$fx(1-\alpha) + 0 = 0 + f(1-\alpha)\varphi_B(\tau_A^1, w, c, \gamma),$$

which is satisfied when

$$\varphi_B(\tau_A^1, w, c, \gamma) = x. \tag{13}$$

Note that $\varphi_B(\tau_A^1, w, c, \gamma)$ is strictly increasing in τ_A^1, since from (10)

$$\frac{\partial \varphi_B(\tau_A^1, w, c, \gamma)}{\partial \tau_A^1} = \frac{\theta\,(1-\theta)\,(2\gamma-1)}{2\left(1-\theta\tau_A^1\right)^2 c} > 0,$$

which implies that there exists a unique solution to (13) such that $\tau_A^1 \in (0,1)$ if

$$\varphi_B(1, w, c, \gamma) > x$$

and

$$\varphi_B(0, w, c, \gamma) < x.$$

Note that $\varphi_B(1, w, c, \gamma) > x$ rewrites as $w < \frac{1}{2} - cx$, which is consistent with Assumption 1. On the other hand, $\varphi_B(0, w, c, \gamma) < x$ is rewritten

$$w > \frac{1}{2} - cx - \frac{\theta(2\gamma-1)}{2}.$$

Thus, there exists a unique equilibrium in which S uses a simple strategy in both periods, the strategy in period 2 being fully informative, if w satisfies

$$\frac{1}{2} - cx - \frac{\theta(2\gamma-1)}{2} < w < \frac{1}{2} - cx.$$

The interval is well-defined since $\frac{\theta(2\gamma-1)}{2} > 0$. The left hand side inequality is as stated in the Proposition, while Assumption 1 implies the right hand side inequality.

Solving the equality (13) for τ_A^1, we obtain the unique solution

$$\tau_A^{1*} = \frac{2w + \theta\,(2\gamma-1) + 2cx - 1}{2\theta\,(\gamma + w + cx - 1)}$$

as stated in the Proposition. □

6.6 Proposition 2

Proof. Recall that $\gamma > \frac{1}{2}$ and that the right hand side inequality in Assumption 1 ($w < \frac{1}{2} - cx$) is equivalent to $1 - 2cx - 2w > 0$. We have

$$\frac{\partial \tau_A^{1*}}{\partial f} = 0,$$
$$\frac{\partial \tau_A^{1*}}{\partial w} = \frac{2\gamma(1-\theta)+\theta-1}{2\theta(cx+\gamma+w-1)^2} > 0,$$
$$\frac{\partial \tau_A^{1*}}{\partial c} = \frac{(2\gamma-1)(1-\theta)x}{2\theta(cx+\gamma+w-1)^2} > 0,$$
$$\frac{\partial \tau_A^{1*}}{\partial \gamma} = \frac{(1-\theta)(1-2cx-2w)}{2\theta(cx+\gamma+w-1)^2} > 0,$$
$$\frac{\partial \tau_A^{1*}}{\partial x} = \frac{c(2\gamma-1)(1-\theta)}{2\theta(cx+\gamma+w-1)^2} > 0.$$

□

6.7 Lemma 6

Proof. **Step 1** Given the expression obtained for τ_A^{1*}, we now check conditions under which consultation in period 1 is worthwhile given that visiting is costly in period 1 (i.e $\eta_1 = 1$) but $v_1 = 0$ (i.e. the variable part of the consultation cost is 0). Note that R, in deciding whether or not to consult in period 1, only considers his expected payoff in period 1, as by assumption he observe m_1 before the beginning of period 2. The expected payoff of R for period 1 if he does not receive m_1 is the prior, in which case R's optimal action is trivially A since we assume $\theta \geq \frac{1}{2}$. If R consults in period 1 his expected payoff is for the period is$\theta\tau_A^{1*} + (1-\theta) - w - v_1$.

Step 2 If R consults in period 1, his expected payoff for the period is $\theta\tau_A^{1*} + (1-\theta) - w - v_1$. The probability of costly visit in period 1 is strictly positive if R prefers to visit when $v_1 = 0$. This implies $\theta\tau_A^{1*} + (1-\theta) - w > \theta$, or equivalently

$$\left(\theta\tau_A^{1*} + (1-\theta) - w\right) - \theta > 0$$
$$\Leftrightarrow \quad \left(\theta\frac{2w+\theta\,(2\gamma-1)+2cx-1}{2\theta\,(\gamma+w+cx-1)} + (1-\theta) - w\right) - \theta > 0.$$

If $\theta = \frac{1}{2}$ the inequality simplifies to

$$\frac{4cx+2\gamma+4w-3}{4(cx+\gamma+w-1)} - w > 0. \qquad (14)$$

Let $w = \frac{1}{2} - cx - g$. Then (14) rewrites as

$$cx + g + \frac{g}{1+2g-2\gamma} > 0.$$

Note that the left hand side of the above reduces to cx (a positive number) for g tending to 0. We may thus conclude that for θ close to $\frac{1}{2}$ and w close to $\frac{1}{2} - cx$, the probability of visit in period 1 conditional on $\eta_1 = 1$ must be positive. $\square$

6.8 Proposition 3

Proof. **Overview** Observe first that given our assumption $\theta = \mu_A$, R does not visit in period 1 if information is costly. Note that R, when deciding whether or not to consult in period 1, only considers his expected payoff in period 1, since whether or not he makes a visit in period 1, he will observe m_1 at the beginning of period 2. R's expected payoff for period 1 when he does not visit is simply μ_A since $\mu_A = \frac{1}{3-2\gamma} > \frac{1}{2}$ and thus his action is A. If R consults and $v_1 = 0$, then his expected payoff for period 1 is $\mu_A \tau_A + \mu_B - w$. This expression is bounded above by $1 - w$. Now note that $1 - w \leq \mu_A$ if $w \geq \frac{(2\gamma-2)}{(2\gamma-3)}$, which we have assumed. It follows that R never consults in period 1.

For the expected payoff in period 2, there are four possible events to take into consideration, which are defined by whether obtaining the message before choosing an action is costly or free in each period, that is, the realizations of η_1 and η_2. In the next following steps, we compute the expected payoff of R conditional on each of these four possible events. The final part of the proof derives the unconditional expected payoff of R.

Step 1: $\eta_1 = 1, \eta_2 = 1$. This event occurs with ex ante probability $(1-\alpha)^2$. We here compute the expected payoff of R conditional on this event. Let K_J denote the expected visiting cost incurred in period 2, conditional on actually visiting, $m_1 = J$ and $\eta_2 = 1$. Let r_J denote the probability that R visits in period 2, conditional on $m_1 = J$ and $\eta_2 = 1$. We have

$$r_B = \frac{(\gamma-1)\frac{\tau_A^{1*}-2}{\tau_A^{1*}+2\gamma-3} - w}{c} = x$$

and

$$\begin{aligned} K_B &= w + E\left(v_2 \middle| v_2 \leq (\gamma-1)\frac{\tau_A^{1*}-2}{\tau_A^{1*}+2\gamma-3} - w\right) \\ &= w + \frac{(\gamma-1)\frac{\tau_A^{1*}-2}{\tau_A^{1*}+2\gamma-3} - w}{2} = w + 2cx. \end{aligned}$$

Recalling that $r_A = 0$ in the simple noisy equilibrium, it follows that R's expected action-payoff is given by

$$\begin{aligned} P(\omega_1 &= A, \omega_2 = A)2\left[\tau_A + \left(1 - \tau_A^{1*}\right)\right] \\ +P(\omega_1 &= A, \omega_2 = B)\left[\tau_A^{1*} + \left(1 - \tau_A^{1*}\right)(1 + r_B)\right] \\ +P(\omega_1 &= B, \omega_2 = A) \\ +P(\omega_1 &= B, \omega_2 = B)r_B. \end{aligned}$$

Note that

$$P(\omega_1 = J, \omega_2 = J') = \mu_J P(\omega_2 = J' | \omega_1 = J).$$

The expected action-payoff of R is thus given by

$$\mu_A \gamma \tau_A^{1*} 2 + \mu_A \gamma 2(1-\tau_A^{1*}) + \mu_A(1-\gamma)\tau_A^{1*}$$
$$+\mu_A(1-\gamma)(1-\tau_A^{1*})(1+x) + \frac{1}{2}\mu_B + \frac{1}{2}\mu_B x.$$

The expected consultation cost incurred by R in the simple noisy equilibrium is

$$\underbrace{\left[P(\omega_1 = A)\tau_A^{1*}\right]}_{P(m_1=A)} r_A K_A + \underbrace{\left[P(\omega_1 = A)(1-\tau_A^{1*}) + P(\omega_1 = B)\right]}_{P(m_1=B)} r_B K_B.$$

This in turn rewrites

$$x\left(w + \frac{cx}{2}\right)\left(\mu_A(1-\tau_A^{1*}) + \mu_B\right).$$

We may now compute the expected payoff of R conditional on ($\eta_1 = 1$, $\eta_2 = 1$), by summing the expected action-payoff and the expected consultation cost as follows:

$$\begin{aligned}
\Pi_R^1(w,c,x,\gamma,\alpha) \;=\; & \left(\frac{1}{3-2\gamma}\right)\gamma\tau_A^{1*}2 + \left(\frac{1}{3-2\gamma}\right)\gamma 2(1-\tau_A^{1*}) \\
& + \left(\frac{1}{3-2\gamma}\right)(1-\gamma)\tau_A^{1*} + \left(\frac{1}{3-2\gamma}\right)(1-\gamma)(1-\tau_A^{1*})(1+x)) \\
& + \left(\frac{2-2\gamma}{3-2\gamma}\frac{1}{2}\right) + \left(\frac{2-2\gamma}{3-2\gamma}\frac{1}{2}\right)x \\
& - x\left(w + \frac{cx}{2}\right)\left(\left(\frac{1}{3-2\gamma}\right)(1-\tau_A^{1*}) + \frac{2-2\gamma}{3-2\gamma}\right).
\end{aligned}$$

We then have

$$\frac{\partial \Pi_R^1(w,c,x,\gamma)}{\partial w} = \frac{-cx^2((3-2\gamma)\gamma - 1)}{2(3-2\gamma)(cx+\gamma+w-1)^2}.$$

Note that $(3-2\gamma)\gamma - 1 > 0$ and thus the above is negative under our parameter assumptions. Also,

$$\frac{\partial \Pi_R^1(w,c,x,\gamma)}{\partial c} = \frac{x^2(1-\gamma)(2\gamma-1)(\gamma+w-1)}{2(3-2\gamma)(cx+\gamma+w-1)^2}.$$

Note that the above is positive under our parameter assumptions.

Step 2: $\eta_1 = 1$, $\eta_2 = 0$. This event happens with ex ante probability $\alpha(1-\alpha)$. Here, R for sure does not obtain information in period 1 and obtains information for sure in period 2. He thus chooses action A in period 1 and will learn the state perfectly in period 2. The expected payoff of R in this event is given as follows:

$$\Pi_R^2(w,c,q,\gamma) = \frac{1}{3-2\gamma} + 1.$$

Step 3: $\eta_1 = 0$, $\eta_2 = 1$. This event occurs with ex ante probability $\alpha(1-\alpha)$. R observes m_1 for free in period 1 and obtains information with positive probability in period 2. The expected action-payoff of R in this case is given by

$$\begin{aligned} P(\omega_1 &= A, \omega_2 = A)\left[2\tau_A^{1*} + \left(1-\tau_A^{1*}\right)\right] \\ +P(\omega_1 &= A, \omega_2 = B)\left[\tau_A^{1*} + \left(1-\tau_A^{1*}\right) r_B\right] \\ +P(\omega_1 &= B, \omega_2 = A) \times 2 \\ +P(\omega_1 &= B, \omega_2 = B)\,(1+r_B)\,. \end{aligned}$$

The above rewrites as

$$\begin{aligned} &\mu_A \gamma \tau_A^{1*} 2 + \mu_A \gamma (1-\tau_A^{1*}) \\ &+\mu_A(1-\gamma)\tau_A^{1*} + \mu_A(1-\gamma)(1-\tau_A^{1*})x \\ &+\left(\mu_B \frac{1}{2}\right) \times 2 \\ &+\mu_B \frac{1}{2}\,(1+x)\,. \end{aligned}$$

The expected consultation cost incurred by R in this event is on the other hand:

$$\underbrace{\left[P(\omega_1 = A)\tau_A^{1*}\right]}_{P(m_1=A)} r_A K_A + \underbrace{\left[P(\omega_1 = A)(1-\tau_A^{1*}) + P(\omega_1 = B)\right]}_{P(m_1=B)} r_B K_B.$$

This in turn rewrites as follows:

$$x\left(w + \frac{cx}{2}\right)\left(\mu_A(1-\tau_A^{1*}) + \mu_B\right).$$

The expected payoff of R in event 3 is thus given by

$$\begin{aligned} &\Pi_R^3(w,c,x,\gamma,\alpha) \\ = &\left(\frac{1}{3-2\gamma}\right)\gamma\tau_A^{1*}2 + \left(\frac{1}{3-2\gamma}\right)\gamma(1-\tau_A^{1*}) \\ &+\left(\frac{1}{3-2\gamma}\right)(1-\gamma)\tau_A^{1*} + \left(\frac{1}{3-2\gamma}\right)(1-\gamma)(1-\tau_A^{1*})x \\ &+2\left(\frac{2-2\gamma}{3-2\gamma}\frac{1}{2}\right) \\ &+\left(\frac{2-2\gamma}{3-2\gamma}\frac{1}{2}\right)(1+x) \\ &-x\left(w+\frac{cx}{2}\right)\left(\left(\frac{1}{3-2\gamma}\right)(1-\tau_A^{1*}) + \frac{2-2\gamma}{3-2\gamma}\right). \end{aligned}$$

Note that

$$\frac{\partial \Pi_R^3(w,c,x,\gamma)}{\partial w} = \frac{(1-\gamma)(2\gamma-1)\left(2-cx^2\right)}{2(3-2\gamma)(cx+\gamma+w-1)^2},$$

which is always positive, as $2-cx^2$ is positive under our assumptions. Also,

$$\frac{\partial \Pi_R^3(w,c,x,\gamma)}{\partial c} = \frac{(1-\gamma)(2\gamma-1)x(x(\gamma+w-1)+2)}{2(3-2\gamma)(cx+\gamma+w-1)^2},$$

is also positive.

Step 4: $\eta_1 = 0$ and $\eta_2 = 0$. This event occurs with ex ante probability α^2. Here, R receives messages for free in periods 1 and 2 before making the action for each period. The expected payoff of R conditional on this event is given by

$$\Pi_R^4(w,c,x,x,\alpha) = \left(\frac{1}{3-2\gamma}\right)\tau_A^{1*} + \left(1 - \frac{1}{3-2\gamma}\right) + 1.$$

The expression is increasing in parameters w and c, as τ_A^{1*} is increasing in w and c. Note that

$$\frac{\partial \Pi_R^4(w,c,x,\gamma)}{\partial w} = \frac{(1-\gamma)(2\gamma-1)}{(3-2\gamma)(cx+\gamma+w-1)^2}$$

and

$$\frac{\partial \Pi_R^4(w,c,x,\gamma)}{\partial c} = \frac{x(1-\gamma)(2\gamma-1)}{(3-2\gamma)(cx+\gamma+w-1)^2}.$$

Step 5 We may now derive R's unconditional expected payoff, which is denoted by $\Theta(w,c,x,\gamma,\alpha)$. We have

$$\begin{aligned}\Theta_R(w,c,x,\gamma,\alpha) &= (1-\alpha)^2\Pi_R^1(w,c,x,\gamma) + (1-\alpha)\alpha\Pi_R^2(w,c,x,\gamma) \\ &\quad +(1-\alpha)\alpha\Pi_R^3(w,c,x,\gamma) + \alpha^2\Pi_R^4(w,c,x,\gamma).\end{aligned}$$

We thus obtain

$$\begin{aligned}&\frac{\partial \Theta_R(w,c,x,\gamma,\alpha)}{\partial w} \\ =\ & (1-\alpha)^2\frac{\partial \Pi_R^1(w,c,x,\gamma)}{\partial w} + (1-\alpha)\alpha\frac{\partial \Pi_R^2(w,c,x,\gamma)}{\partial w} \\ & +(1-\alpha)\alpha\frac{\partial \Pi_R^3(w,c,x,\gamma)}{\partial w} + \alpha^2\frac{\partial \Pi_R^4(w,c,x,\gamma)}{\partial w} \\ =\ & \frac{(1-\gamma)(2\gamma-1)\left(2\alpha-(1-\alpha)cx^2\right)}{2(3-2\gamma)(cx+\gamma+w-1)^2}.\end{aligned}$$

The above expression is positive if and only if $\alpha > \frac{cx^2}{2+cx^2}$. The derivative of Θ_R with respect to c is instead clearly positive given that $\frac{\partial \Pi_R^i(w,c,x,\gamma)}{\partial c} > 0$ for $i \in \{1,3,4\}$ and $\frac{\partial \Pi_R^2(w,c,x,\gamma)}{\partial c} = 0$. Finally, note that from Proposition 2 τ_A^{1*} is strictly increasing in x. It follows immediately that for any α, Θ_R is also strictly increasing in x. □

6.9 Proposition 4

Proof. **Step 1** Let r_i denote the probability that R visits in period 2, conditional on $m_1 = J \in \{A,B\}$ and $\eta_2 = 1$. Let us define $E[\sharp]$ as the expected number of times that S reports truthfully over the two periods. The ex ante expected payoff of S is given by

$$\begin{aligned}\Pi_S(f,w,c,x,\gamma,\alpha) &= (1-\alpha)\left[fr_A P(\omega_1 = A)\tau_A^{1*} + fr_B\left(P(\omega_1 = A)(1-\tau_A^{1*}) + P(\omega_1 = B)\right)\right] \\ &\quad +E[\sharp]\, fx(1-\alpha).\end{aligned}$$

Note that $r_A = 0$ and $r_B = x$. Note furthermore that

$$\begin{aligned} E[\sharp] &= \left(\frac{\gamma}{3-2\gamma}\right)(2\left(\tau_A^{1*}\right)^2 + 2\tau_A^{1*}(1-\tau_A^{1*})) + \left(\frac{1-\gamma}{3-2\gamma}\right)(2\tau_A^{1*} + (1-\tau_A^{1*})) \\ &+ \left(\frac{2-2\gamma}{3-2\gamma}\right)\frac{1}{2}(2) + \left(\frac{2-2\gamma}{3-2\gamma}\right)\frac{1}{2}(2\tau_A^{1*} + (1-\tau_A^{1*})) \\ &= \frac{2}{3-2\gamma}\left(\tau_A^{1*} - 2\gamma + 2\right). \end{aligned} \tag{15}$$

We may thus write:

$$\begin{aligned} \Pi_S(f,w,\gamma,x,c,\alpha) &= (1-\alpha)fx\left(\left(\frac{1}{3-2\gamma}\right)(1-\tau_A^{1*}) + \frac{2-2\gamma}{3-2\gamma}\right) \\ &\quad + fx(1-\alpha)\left(\frac{2}{3-2\gamma}\left(\tau_A^{1*} - 2\gamma + 2\right)\right). \end{aligned}$$

Step 2 We have

$$\frac{\partial \Pi_S(f,w,c,x,\gamma,\alpha)}{\partial f} = x(1-\alpha)\left(\left(\frac{1}{3-2\gamma}\right)(1-\tau_A^{1*}) + \frac{2-2\gamma}{3-2\gamma}\right) + x(1-\alpha)E[\sharp]$$

Note that τ_A^{1*} is not a function of f. Thus, $\Pi_S(f,w,\gamma,x,c,\alpha)$ is a linear function of f, whose coefficient positive.

Step 3 We now examine the derivative of $\Pi_S(f,w,\gamma,x,c,\alpha)$ with respect to w. We have

$$\frac{\partial \Pi_S(f,w,c,x,\gamma,\alpha)}{\partial w} = (1-\alpha)\left(\frac{\partial\left(fx\left(\frac{1}{3-2\gamma}(1-\tau_A^{1*}) + \frac{2-2\gamma}{3-2\gamma}\right)\right)}{\partial w} + \frac{\partial\left(E[\sharp]fx\right)}{\partial w}\right).$$

Note that

$$\frac{\partial\left(fx\left(\frac{1}{3-2\gamma}(1-\tau_A^{1*}) + \frac{2-2\gamma}{3-2\gamma}\right)\right)}{\partial w} = -\frac{fx((3-2\gamma)\gamma-1)}{(3-2\gamma)(cx+\gamma+w-1)^2},$$

which is negative since $(3-2\gamma)\gamma > 1$ for any $\gamma \in (1/2, 1)$.

On the other hand, $E[\sharp]fx$ is clearly increasing in w given that the probability of truth-telling in period 1 (τ_A^{1*}) is increasing in w while the probability of truth-telling in period 2 is fixed and equal to 1. Now, we obtain

$$\begin{aligned} &\frac{\partial\left(fx\left(\frac{1}{3-2\gamma}(1-\tau_A^{1*}) + \frac{2-2\gamma}{3-2\gamma}\right)\right)}{\partial w} + \frac{\partial\left(\frac{2fx}{3-2\gamma}\left(\tau_A^{1*} - 2\gamma + 2\right)\right)}{\partial w} \\ = &\frac{fx(1-\gamma)(2\gamma-1)}{(3-2\gamma)(cx+\gamma+w-1)^2}. \end{aligned}$$

The above expression is positive under our assumptions.

Step 4 We now examine the derivative of $\Pi_S(f, w, \gamma, x, c, \alpha)$ with respect to c. We have

$$\begin{aligned}\frac{\partial \Pi_S(f,w,c,x,\gamma,\alpha)}{\partial c} &= (1-\alpha)\left(\frac{\partial\left(fx\left(\frac{1}{3-2\gamma}(1-\tau_A^{1*})+\frac{2-2\gamma}{3-2\gamma}\right)\right)}{\partial c}+\frac{\partial\left(\frac{2fx}{3-2\gamma}\left(\tau_A^{1*}-2\gamma+2\right)\right)}{\partial c}\right)\\ &= (1-\alpha)\left(\frac{(1-\gamma)(2\gamma-1)fx^2}{(3-2\gamma)(cx+\gamma+w-1)^2}\right),\end{aligned}$$

and the above expression is always positive.

Step 5 We now examine the derivative of $\Pi_S(f, w, \gamma, x, c, \alpha)$ with respect to x. We have

$$\frac{\partial \Pi_S(f,w,c,x,\gamma,\alpha)}{\partial x} = (1-\alpha)\left(\frac{\partial\left(fx\left(\frac{1}{3-2\gamma}(1-\tau_A^{1*})+\frac{2-2\gamma}{3-2\gamma}\right)\right)}{\partial x}+\frac{\partial\left(E\left[\sharp\right]fx\right)}{\partial x}\right).$$

Now, note that

$$\frac{\partial\left(fx\left(\frac{1}{3-2\gamma}(1-\tau_A^{1*})+\frac{2-2\gamma}{3-2\gamma}\right)\right)}{\partial x} = \frac{f(1-\gamma)(2\gamma-1)(\gamma+w-1)}{(3-2\gamma)(cx+\gamma+w-1)^2} > 0.$$

Also, from (15), $E\left[\sharp\right] fx$ is clearly increasing in x given that the truth-telling probability τ_A^{1*} is increasing in x as we saw in Proposition 2. It follows that the above expression is positive given our assumptions.

Step 6 We now examine the derivative of $\Pi_S(f, w, \gamma, x, c, \alpha)$ with respect to γ. We have:

$$\begin{aligned}&\frac{\partial \Pi_S(f,w,c,x,\gamma,\alpha)}{\partial \gamma} =\\ &(1-\alpha)\left(\frac{\partial\left(fx\left(\frac{1}{3-2\gamma}(1-\tau_A^{1*})+\frac{2-2\gamma}{3-2\gamma}\right)\right)}{\partial \gamma}+\frac{\partial\left(\frac{2fx}{3-2\gamma}\left(\tau_A^{1*}-2\gamma+2\right)\right)}{\partial \gamma}\right)\\ =\ &\frac{fx(cx+w)(4(1-\gamma)\gamma-4cx-4w+1)}{(3-2\gamma)^2(cx+\gamma+w-1)^2}\end{aligned}$$

The latter expression is positive if

$$w < \frac{1}{4}+\gamma(1-\gamma)-cx$$

and otherwise negative. □

7 Appendix B: Infinite horizon model

7.1 Lemma 7

Proof. Part a) Consider a putative stationary equilibrium with no consultation. In such an equilibrium, S would have a strict incentive to send m_t=ω_t in every period. So such an equilibrium would have to feature a fully informative basic communication strategy of S. But then, given our parameter assumptions, R would have a strict incentive to consult with positive probability after m_t= B.

Part b) In an equilibrium where $m_{t-1} = A$ implies that $\omega_{t-1} = A$ for sure, a necessary condition for R to choose to consult with positive probability in period t given $m_{t-1} = A$ is $1 - w > \gamma$, or equivalently $w < 1 - \gamma$ since the gross benefit of consultation (ignoring consultation costs) is at most $1 - \gamma$. This benefit corresponds to a scenario where S's communication is fully informative at t. A fortiori, if S's report is not fully informative in period t, R never visits in period t given $m_{t-1} = A$ if $w > 1 - \gamma$. Now, simply note that Assumption 1′ states $w > 1 - \gamma$. □

7.2 Lemma 8

Proof. **Step 1** Proof of a). Consider a putative stationary equilibrium featuring an uninformative communication strategy of S. Such an equilibrium must feature no consultation. But in a putative stationary equilibrium featuring no consultation, S would strictly favor sending $m_t = \omega_t$ for sure for any value of ω_t, thereby deviating from his assumed uninformative stationary strategy.

Step 2 Proof of a) continued. Consider a putative stationary equilibrium featuring a fully informative stationary communication strategy of S. Consider first the case where S's strategy is such that S always sends m_t=ω_t. Assume now that $\omega_t = A$. Message $m_t = A$ leads to a visiting probability of 0 in period $t+1$ conditional on $\eta_{t+1} = 1$ and thus yields a period $t+1$ expected payoff over t and $t+1$ of $(1-\alpha)fx$. In contrast, $m_t = B$ leads to a strictly positive probability of visit of $\left(\frac{\frac{1}{2}-w}{c}\right)$ in period $t+1$ given $\eta_{t+1} = 1$. It thus yields an expected payoff over t and $t+1$ of $\delta(1-\alpha)f\left(\frac{\frac{1}{2}-w}{c}\right)$. Now, note that $\delta(1-\alpha)f\left(\frac{\frac{1}{2}-w}{c}\right) \leq (1-\alpha)fx$ is equivalent to $\frac{1}{2} - c\widetilde{x} \leq w$, which contradicts Assumption 1′.

Consider now the case where S's strategy is such that S always sends $m_t \neq \omega_t$. Assume now that $\omega_t = A$. Message $m_t = B$ leads to a visiting probability of 0 in period $t+1$ conditional on $\eta_{t+1} = 1$ and thus yields an expected payoff of 0 over t and $t+1$. In contrast, $m_t = A$ yields the truth-telling reward $(1-\alpha)fx$ in period t and also leads to a strictly positive probability of visit of $\left(\frac{\frac{1}{2}-w}{c}\right)$ in period $t+1$ conditional on $\eta_{t+1} = 1$. It follows trivially that in this putative equilibrium, S strictly prefers to deviate to $m_t = A$ given $\omega_t = A$.

Step 3 Proof of b). Consider a putative stationary equilibrium featuring a stationary basic communication strategy of S that belongs to class 1. Consider two different cases, 1 and 2. In case 1, the stationary strategy of S builds on a pure period-t strategy. It is thus either fully informative or uninformative. We have already shown in a) and b) that this cannot be true in equilibrium. The second case to consider is that the stationary strategy of S builds on a randomized strategy of S in each period (this being identical in all periods by the stationarity assumption). By definition it must be true that S sends $m_t = B$ whenever $\omega_t = B$. So the randomization must take place when the state is A. Recall that $m_t = \omega_t$ yields the immediate truth-telling reward $(1-\alpha)fx$. Note also that if $m_{t-1} = A$, then R is thus sure that $\omega_{t-1} = A$. Given our assumptions on parameter values, it follows immediately that R consults with probability zero at t if $m_{t-1} = A$. Note furthermore that if given ω_t, S randomizes between messages A and B at t, then it must be true that both messages yield the same expected payoff for Sover t and $t+1$. Using the fact that R consults with probability zero at t if $m_{t-1} = A$, the required indifference condition is equivalent to stating that it needs to be true that

$$P(d_t = 1\,|m_{t-1} = B, \omega_{t-2} = A, \eta_t = 1) = P(d_t = 1\,|m_{t-1} = B, \omega_{t-2} = B, \eta_t = 1) = \tilde{x}.$$

But this in turn requires that

$$P(\omega_t = B\,|m_{t-1} = B, \omega_{t-2} = A) = P(\omega_t = B\,|m_{t-1} = B, \omega_{t-2} = B),$$

which guarantees that the benefit of consulting at t after $m_{t-1} = B$ is the same, no matter the observed value of ω_{t-2}. This equality can however not be satisfied given the Markov process faced. □

7.3 Proposition 5

Proof. **Step 1** Let us first show that there is no equilibrium that features $\tau_{AA} = 1$ or $\tau_{BA} = 1$ (a fortiori there is no truth-telling equilibrium, where $\tau_{AA} = \tau_{BA} = 1$). If there is such an equilibrium, it has to be that S is better off reporting $\omega_t = A$ truthfully, namely either

$$x + \delta P(d_t = 1 \mid m_{t-1} = A) f \geq 0 + \delta P(d_t = 1 \mid \omega_{t-2} = A, m_{t-1} = B) f \tag{16}$$

or

$$x + \delta P(d_t = 1 \mid m_{t-1} = A) f \geq 0 + \delta P(d_t = 1 \mid \omega_{t-2} = B, m_{t-1} = B) f. \tag{17}$$

Point b) of Lemma 7 states that $\varphi_{AA} = \varphi_{BA} = 0$ while Point a) of the Lemma states that $\varphi_{AB} > 0$ or $\varphi_{BB} > 0$. Meanwhile, $\tau_{AA} = 1$ or $\tau_{BA} = 1$ implies $\widetilde{v}(A, B) = \frac{1}{2} - w$ or $\widetilde{v}(B, B) = \frac{1}{2} - w$; and hence $\varphi_{AB} = \frac{\frac{1}{2}-w}{c}$ or $\varphi_{BB} = \frac{\frac{1}{2}-w}{c}$, respectively. Recall $\tilde{x} \equiv x/\delta$. From (16) and (17), if $\varphi_{JB} = \frac{\frac{1}{2}-w}{c}$, S reports truthfully when $\omega_{t-1} = J$ and $\omega_t = A$ if

$$(1-\alpha)\tilde{x} f \geq (1-\alpha)\left(\frac{\frac{1}{2}-w}{c}\right) f. \tag{18}$$

However, (18) simplifies to $w \geq \frac{1}{2} - c\tilde{x}$, which contradicts Assumption 1′. Therefore, neither $\tau_{AA} = 1$ nor $\tau_{BA} = 1$ can be supported in equilibrium.

Step 2 Let us prove the existence of an equilibrium that features $\tau_{AA} \in (0, 1)$ and $\tau_{BA} \in (0, 1)$. First let us consider R's decision to consult. In order to calculate his expected payoffs when he consults and when he does not consult, we need to consider his conditional expectations about the previous state ω_{t-1} and the current state ω_t, given ω_{t-2} and m_{t-1}. Note that

$$\begin{aligned} P(\omega_t = A \mid \omega_{t-2} = A, m_{t-1} = B) &= 1 - P(\omega_t = B \mid \omega_{t-2} = A, m_{t-1} = B) \\ &= \underbrace{\frac{1-\gamma}{\gamma(1-\tau_{AA}) + (1-\gamma)}}_{P(\omega_{t-1}=B|\omega_{t-2}=A, m_{t-1}=B)} \times \frac{1}{2} + \underbrace{\left(1 - \frac{1-\gamma}{\gamma(1-\tau_{AA}) + (1-\gamma)}\right)}_{P(\omega_{t-1}=A|\omega_{t-2}=A, m_{t-1}=B)} \times \gamma \end{aligned}$$

and

$$P(\omega_t = A \mid \omega_{t-2} = B, m_{t-1} = B) = 1 - P(\omega_t = B \mid \omega_{t-2} = B, m_{t-1} = B)$$
$$= \underbrace{\frac{\frac{1}{2}}{\frac{1}{2}(1-\tau_{BA}) + \frac{1}{2}}}_{P(\omega_{t-1}=B|\omega_{t-2}=B,m_{t-1}=B)} \times \frac{1}{2} + \underbrace{\left(1 - \frac{\frac{1}{2}}{\frac{1}{2}(1-\tau_{BA}) + \frac{1}{2}}\right)}_{P(\omega_{t-1}=A|\omega_{t-2}=B,m_{t-1}=B)} \times \gamma.$$

Basic sender strategies imply $P(\omega_t = A \mid \omega_{t-2}, m_{t-1} = A) = 1$ and $P(\omega_t = B \mid \omega_{t-2}, m_{t-1} = A) = 0$ for $\omega_{t-2} \in \{A, B\}$.

The conditional joint distribution of the present and the previous state satisfies

$$P(\omega_{t-1} = A, \omega_t = A \,|\omega_{t-2}, m_{t-1}) = P(\omega_{t-1} = A \,|\omega_{t-2}, m_{t-1})\gamma$$

and

$$P(\omega_{t-1} = B, \omega_t = A \,|\omega_{t-2}, m_{t-1}) = P(\omega_{t-1} = B \,|\omega_{t-2}, m_{t-1})\frac{1}{2}.$$

Since R chooses A when $m_t = A$ and he chooses B when $m_t = B$, his expected payoff for period t conditional on consultation at t (exclusive of the visiting costs) is given by

$$\begin{aligned}\pi_t^R(d_t = 1 \mid \omega_{t-2} = A, m_{t-1} = B) = {} & P(\omega_t = B \mid \omega_{t-2} = A, m_{t-1} = B)\\ & + P(\omega_{t-1} = A, \omega_t = A \mid \omega_{t-2} = A, m_{t-1} = B)\tau_{AA}\\ & + P(\omega_{t-1} = B, \omega_t = A \mid \omega_{t-2} = A, m_{t-1} = B)\tau_{BA},\end{aligned}$$

where $P(\omega_t = B \mid \omega_{t-2} = A, m_{t-1} = B)$ represents his payoff (of 1) multiplied by the conditional probability that $\omega_t = B$. If he does not consult, his expected payoff for period t is given by

$$\pi_t^R(d_t = 0 \mid \omega_{t-2} = A, m_{t-1} = B) = P(\omega_t = A \mid \omega_{t-2} = A, m_{t-1} = B)$$

since he chooses A. Thus the gross benefit of visiting given $\omega_{t-2} = A$ and $m_{t-1} = B$ is given by

$$\Delta_t^R\,(\omega_{t-2} = A, m_{t-1} = B) \equiv \pi_t^R(d_t = 1 \mid \omega_{t-2} = A, m_{t-1} = B) - \pi_t^R(d_t = 0 \mid \omega_{t-2} = A, m_{t-1} = B).$$

Similarly, for $\omega_{t-2} = B$ and $m_{t-1} = B$ we have

$$\begin{aligned}\pi_t^R(d_t = 1 \mid \omega_{t-2} = B, m_{t-1} = B) = {} & P(\omega_t = B \mid \omega_{t-2} = B, m_{t-1} = B)\\ & + P(\omega_{t-1} = A, \omega_t = A \mid \omega_{t-2} = B, m_{t-1} = B)\tau_{AA}\\ & + P(\omega_{t-1} = B, \omega_t = A \mid \omega_{t-2} = B, m_{t-1} = B)\tau_{BA},\end{aligned}$$

and

$$\pi_t^R(d_t = 0 \mid \omega_{t-2} = B, m_{t-1} = B) = P(\omega_t = A \mid \omega_{t-2} = B, m_{t-1} = B).$$

Thus the gross benefit of consulting given $\omega_{t-2} = B$ and $m_{t-1} = B$ is given by

$$\Delta_t^R\,(\omega_{t-2} = B, m_{t-1} = B) \equiv \pi_t^R(d_t = 1 \,|\omega_{t-2} = B, m_{t-1} = B\,) - \pi_t^R(d_t = 0 \,|\omega_{t-2} = B, m_{t-1} = B\,).$$

The above gross benefits, together with w and the realized value of v_t, determine R's best response given τ_{AA} and τ_{BA}.

Step 3 An equilibrium featuring a basic communication strategy as well as $\tau_{AA} \in (0,1)$ and $\tau_{BA} \in (0,1)$ requires two indifference conditions to hold simultaneously (one for $\omega_{t-2} = A$ and the other for $\omega_{t-2} = B$), namely

$$\underbrace{(1-\alpha)\tilde{x}f}_{\text{truth-telling benefit + no visit}} = \underbrace{(1-\alpha)\left(\frac{\Delta_t^R(\omega_{t-2}=A, m_{t-1}=B) - w}{c}\right)f}_{\text{no truth-teling benefit + positive prob of visit in } t+1} \tag{19}$$

and

$$(1-\alpha)\tilde{x}f = (1-\alpha)\left(\frac{\Delta_t^R(\omega_{t-2}=B, m_{t-1}=B) - w}{c}\right)f. \tag{20}$$

Solving simultaneously for τ_{AA} and τ_{BA}, we obtain three pairs of solutions.

The first pair is given by $\tau_{AA} = \frac{c\tilde{x}+w+2\gamma-1}{\gamma}$ and $\tau_{BA} = 2(c\tilde{x}+w)$. The second pair of solutions is given by

$$\tau_{AA} = \frac{(2c\tilde{x}+2w+6\gamma-1)\gamma + \sqrt{8\gamma^2(1-2c\tilde{x}-2w-4\gamma^2) + (6\gamma^2+2c\tilde{x}\gamma-\gamma+2w\gamma)^2}}{4\gamma^2},$$

and

$$\tau_{BA} = \frac{(3+2c\tilde{x}+2w-2\gamma)\gamma + \sqrt{8\gamma^2(1-2c\tilde{x}-2w-4\gamma^2) + (6\gamma^2+2c\tilde{x}\gamma-\gamma+2w\gamma)^2}}{4\gamma(1-\gamma)}.$$

The third pairs is given by:

$$\tau_{AA}^* = \frac{(2c\tilde{x}+2w+6\gamma-1)\gamma - \sqrt{8\gamma^2(1-2c\tilde{x}-2w-4\gamma^2) + (6\gamma^2+2c\tilde{x}\gamma-\gamma+2w\gamma)^2}}{4\gamma^2} \tag{21}$$

and

$$\tau_{BA}^* = \frac{(3+2c\tilde{x}+2w-2\gamma)\gamma - \sqrt{8\gamma^2(1-2c\tilde{x}-2w-4\gamma^2) + (6\gamma^2+2c\tilde{x}\gamma-\gamma+2w\gamma)^2}}{4\gamma(1-\gamma)}. \tag{22}$$

For $\tau_{AA} \in (0,1)$ and $\tau_{BA} \in (0,1)$, the first solution requires $w < 1-\gamma$, which is at odds with Assumption 1′. Similarly, the second solution requires $\tilde{x} < 0$ while we have assumed $\tilde{x} > 0$. The third solution gives $\tau_{AA}^* \in (0,1)$ and $\tau_{BA} \in (0,1)$ for $\gamma \in (1/2, 1)$ and

$$w \in \left(-c\tilde{x}, \frac{1}{2} - c\tilde{x}\right),$$

which is satisfied under our assumptions. Thus we conclude that the third pair of solutions (21) and (22) pins down the unique equilibrium in stationary basic strategies. □

7.4 Proposition 6

Proof. We use the closed form expressions for τ^*_{AA} and τ^*_{BA} appearing in (21) and (22). We obtain

$$\frac{\partial \tau^*_{AA}}{\partial w} = \frac{2 - \frac{\partial}{\partial w}F(w,c,\tilde{x},\gamma)}{4\gamma}, \tag{23}$$

$$\frac{\partial \tau^*_{BA}}{\partial w} = \frac{2 - \frac{\partial}{\partial w}F(w,c,\tilde{x},\gamma)}{4(1-\gamma)}, \tag{24}$$

$$\frac{\partial \tau^*_{AA}}{\partial c} = \frac{2\tilde{x} - \frac{\partial}{\partial c}F(w,c,\tilde{x},\gamma)}{4\gamma}, \tag{25}$$

$$\frac{\partial \tau^*_{BA}}{\partial c} = \frac{2\tilde{x} - \frac{\partial}{\partial c}(w,c,\tilde{x},\gamma)}{4(1-\gamma)}, \tag{26}$$

$$\frac{\partial \tau^*_{AA}}{\partial \tilde{x}} = \frac{2c - \frac{\partial}{\partial x}F(w,c,\tilde{x},\gamma)}{4\gamma} \tag{27}$$

$$\frac{\partial \tau^*_{BA}}{\partial \tilde{x}} = \frac{2c - \frac{\partial}{\partial x}F(w,c,\tilde{x},\gamma)}{4(1-\gamma)} \tag{28}$$

where

$$F(w,c,\tilde{x},\gamma) = \sqrt{8\gamma^2\left(1-2c\tilde{x}-2w-4\gamma^2\right) + \left(6\gamma^2+2c\tilde{x}\gamma-\gamma+2w\gamma\right)^2}.$$

Partially differentiating F, we have

$$\frac{\partial F(w,c,\tilde{x},\gamma)}{\partial w} = \frac{2\gamma^2(2c\tilde{x}+6\gamma+2w-5)}{\sqrt{\gamma^2\left(4c^2\tilde{x}^2+4c\tilde{x}(6\gamma+2w-5)+(3-2\gamma)^2+4w^2+4(6\gamma-5)w\right)}},$$

$$\frac{\partial F(w,c,\tilde{x},\gamma)}{\partial c} = \frac{2\tilde{x}\gamma^2(2c\tilde{x}+6\gamma+2w-5)}{\sqrt{\gamma^2\left(4c^2\tilde{x}^2+4c\tilde{x}(6\gamma+2w-5)+(3-2\gamma)^2+4w^2+4(6\gamma-5)w\right)}},$$

and

$$\frac{\partial F(w,c,\tilde{x},\gamma)}{\partial \tilde{x}} = \frac{2c\gamma^2(2c\tilde{x}+6\gamma+2w-5)}{\sqrt{\gamma^2\left(4c^2\tilde{x}^2+4c\tilde{x}(6\gamma+2w-5)+(3-2\gamma)^2+4w^2+4(6\gamma-5)w\right)}}.$$

The partial derivatives above are all non-positive when $2c\tilde{x}+6\gamma+2w-5 \leq 0$. The inequality can be written for $\gamma \in (1/2,1)$ as $w \leq \frac{1}{2} - c\tilde{x}$, which is consistent with Assumption 1. Therefore, from (23) to (28) we obtain $\frac{\partial \tau^*_{AA}}{\partial w} > 0$, $\frac{\partial \tau^*_{BA}}{\partial w} > 0$, $\frac{\tau^*_{AA}}{\partial c} > 0$, $\frac{\partial \tau^*_{BA}}{\partial c} > 0$, $\frac{\partial \tau^*_{AA}}{\partial \tilde{x}} > 0$, and $\frac{\partial \tau^*_{BA}}{\partial \tilde{x}} > 0$. □

7.5 Proposition 7

Proof. Denote by $\Pi^0_R(w,c,\tilde{x},\gamma)$ (resp. $\Pi^1_R(w,c,\tilde{x},\gamma)$) the average per-period payoff of R given respectively $\alpha = 1$ (no consultation cost) and $\alpha = 0$ (positive consultation costs). The average per-period payoff $\Pi_R(w,c,\tilde{x},\gamma,\alpha)$ of R is thus by definition given by

$$\alpha\Pi^0_R(w,c,\tilde{x},\gamma) + (1-\alpha)\,\Pi^1_R(w,c,\tilde{x},\gamma).$$

Note that

$$\Pi_R^0(w,c,\tilde{x},\gamma) = \sum_{\omega_{t-1}}\sum_{\omega_t} P(\omega_{t-1},\omega_t)P(m_t = \omega_t \mid \omega_{t-1},\omega_t),$$

where $P(\omega_{t-1},\omega_t) = \mu_{\omega_{t-1}} P(\omega_t \mid \omega_{t-1})$.

Note also that $P(m_t = \omega_t \mid \omega_{t-1},\omega_t)$ is constant in w, c and $\widetilde{x}$ for every $(\omega_{t-1},\omega_t) \notin \{(B,A),(A,A)\}$ and instead strictly increasing in w, c and decreasing in $\widetilde{x}$ for $(\omega_{t-1} = B, \omega_t = A)$ and $(\omega_{t-1} = A, \omega_t = A)$. Recall here that $P(m_t = \omega_t \mid \omega_{t-1} = J, \omega_t = A) = \tau_{JA}$, which we have shown to be strictly increasing in w,c and decreasing in $\widetilde{x}$ for $J \in A, B$. It follows that $\frac{\partial \Pi_R^0(w,c,\tilde{x},\gamma)}{\partial w} > 0$, $\frac{\partial \Pi_R^0(w,c,\tilde{x},\gamma)}{\partial c} > 0$ and $\frac{\partial \Pi_R^0(w,c,\tilde{x},\gamma)}{\partial \tilde{x}} < 0$.

Let us now focus on the derivative of $\Pi_R(w,c,\tilde{x},\gamma,\alpha)$ with respect to w. We have:

$$\frac{\partial \Pi_R(w,c,\tilde{x},\gamma,\alpha)}{\partial w} = \alpha \frac{\partial \Pi_R^0(w,c,\tilde{x},\gamma)}{\partial w} + (1-\alpha)\frac{\partial \Pi_R^1(w,c,\tilde{x},\gamma)}{\partial w}.$$

It follows trivially that for α large enough, $\Pi_R(w,c,\tilde{x},\gamma,\alpha)$ is increasing in w. The argument used to study the effect of changes in c and $\widetilde{x}$ is identical. □

8 Appendix C: Extensions

8.1 Proposition 8

Proof. **Outline:** Recall that throughout, as in the main text, we keep the assumption that $cx < \left(\frac{1}{2} - \frac{\theta(2\gamma-1)}{2}\right) - (1-\gamma)$ so that $1-\gamma < \frac{1}{2} - \frac{\theta(2\gamma-1)}{2} - cx$. Steps 1-5 prove part a) and thus study the case of $w \leq \frac{1}{2} - cx - \frac{\theta(2\gamma-1)}{2}$. In what follows, Steps 0-5 prove point 1 and thus study the case of $w \leq \frac{1}{2} - cx - \frac{\theta(2\gamma-1)}{2}$.

Step 1 proves the following three useful properties of equilibrium behavior of both S and R. First, any equilibrium must feature truth-telling in period 2 regardless of the state. Second, any equilibrium must feature a positive probability of consultation in period 2. Third, any equilibrium with informative communication in period 1 must feature a *simple* communication strategy, that is, if $m_1 = B$ then $\omega_1 = B$ (truth-telling when the state is B).

Step 2 shows that under the conditions imposed on part a), there exists no equilibrium featuring truth-telling in both periods.

Step 3 examines the case where $w < 1-\gamma$ and shows that there exists no equilibrium that features an informative simple communication strategy in period 1.

Step 4 examines the case where $w \in \left(1-\gamma, \frac{1}{2} - cx - \frac{\theta(2\gamma-1)}{2}\right)$ and shows the same result as that of Step 3 holds.

Step 5 shows that there exists an equilibrium that features uninformative communication in period 1 and truth-telling in period 2.

Step 6 confirms the statement of part a).

Step 7 focuses on part b) of the proposition and studies the case of $w > \frac{1}{2} - cx$.

Step 1 The fact that any equilibrium must feature truth-telling in period 2 regardless of the state follows from the same argument as that of Lemma 1. The fact that any equilibrium must feature a positive probability of consultation in period 2 follows from the same argument as in the proof of Lemma 1.

We now show that any equilibrium with informative communication in period 1 must feature a simple communication strategy, that is, truth-telling in period 1 if $\omega_1 = B$. We invoke the same argument as in the proof of Lemma 5 and we repeat it for convenience in what follows. We show by contradiction that in equilibrium, $m_1 = B$ must induce a weakly higher probability of consultation in period 2 than $m_1 = A$. In turn, we show that this immediately implies that S reports $m_1 = B$ with probability 1 when $\omega_1 = B$.

Suppose that the message in period 1 is informative in period 1. Clearly, R is (weakly) more likely to make a costly visit in period 2, if the message $m_1 \in \{A, B\}$ shifts R's posterior towards $\omega_1 = B$ relative to the other message.

Assume that $m_1 = A$ induces a weakly higher probability of consultation in period 2 than $m_1 = B$. If this is the case, then S reports $m_1 = A$ with probability 1 when $\omega_1 = A$, since by doing so S obtains both the truth-telling benefit and induces a weakly higher probability of consultation in period 2 than by sending $m_1 = B$. However, if an equilibrium is such that S sends $m_1 = A$ for sure whenever $\omega_1 = A$, then the report $m_1 = B$ induces a strictly higher posterior that the state is $\omega_1 = B$ than $m_1 = A$. Indeed, note that if $\tau_A^1 = 1$, then

$$P(\omega_1 = B \mid m_1 = A) = \frac{(1-\theta)(1-\tau_B^1)}{(1-\theta)(1-\tau_B^1)+\theta} < 1$$

while

$$P(\omega_1 = B \mid m_1 = B) = \frac{(1-\theta)\tau_B^1}{(1-\theta)\tau_B^1+\theta(0)} = 1.$$

Consequently, if communication in period 1 is informative, then $m_1 = B$ must induce a higher probability of consultation in period 2 than $m_1 = A$, but this contradicts the assumption that $m_1 = A$ induces a weakly higher probability of consultation. We conclude then by contradiction that if communication in period 1 is informative, then $m_1 = B$ must induce a strictly higher probability of consultation in period 2 than $m_1 = A$. Note furthermore that this fact immediately implies that, if communication in period 1 is informative, then S reports $m_1 = B$ with probability 1 when $\omega_1 = B$, since by doing so S obtains both the truth-telling benefit and induces a strictly higher probability of consultation in period 2 than by sending $m_1 = A$.

Step 2 Let us show that there exists no equilibrium that features truth-telling in period 1. Assume that there is an equilibrium with truth-telling in both periods. The visiting probabilities of R in period 2 satisfy the following. We have

$$P(d_2 = 1 \mid m_1 = A, \eta_2 = 1) = \frac{1-\gamma-w}{c}$$

if $w \leq 1-\gamma$ and $P(d_2 = 1 \,|m_1 = A, \eta_2 = 1) = 0$ if $w > 1-\gamma$. On the other hand,

$$P(d_2 = 1 \mid m_1 = B, \eta_2 = 1) = \frac{\frac{1}{2}-w}{c}.$$

We now consider the incentive of S in period 1. Let $\omega_1 = B$, in which case S strictly prefers to report truthfully in period 1, as this yields the truth-telling benefit and maximizes as well the visiting probability in period 2. Let$\omega_1 = A$. At period 1, the expected payoff of sending $m_1 = A$ is given by $(1-\alpha)fx$ if $w \in (1-\gamma, \frac{1}{2} - x - \frac{\theta(2\gamma-1)}{2}]$ since there is no visit in period 2, and the corresponding expected payoff for $w \leq 1-\gamma$ is given by

$$(1-\alpha)fx + \left(\frac{1-\gamma-w}{c}\right)(1-\alpha)f.$$

Meanwhile, the expected payoff when sending $m_1 = B$ is given by $\left(\frac{\frac{1}{2}-w}{c}\right)(1-\alpha)f$.

Therefore, when $w \leq 1-\gamma$,S strictly prefers to report $m_1 = B$ when $\omega_1 = A$ (i.e. deviating from truth-telling) if and only if

$$(1-\alpha)fx + \left(\frac{1-\gamma-w}{c}\right)(1-\alpha)f < \left(\frac{\frac{1}{2}-w}{c}\right)f(1-\alpha),$$

which reduces to $cx < \gamma - \frac{1}{2}$. This inequality is readily implied by

$$cx < \left(\frac{1}{2} - \frac{\theta(2\gamma-1)}{2}\right) - (1-\gamma),$$

for $\theta \in [\frac{1}{2}, 1)$..

If on the other hand $w \in (1-\gamma, \frac{1}{2} - cx - \frac{\theta(2\gamma-1)}{2}]$,$S$ strictly prefers sending $m_1 = B$ when $\omega_1 = A$ if and only if

$$(1-\alpha)fx < \left(\frac{\frac{1}{2}-w}{c}\right)(1-\alpha)f,$$

which is equivalent to $cx < \frac{1}{2} - w$. The inequality rewrites as $w < \frac{1}{2} - cx$, which holds true given $w \in (1-\gamma, \frac{1}{2} - cx - \frac{\theta(2\gamma-1)}{2}]$.

Thus from this step we conclude that no equilibrium can feature truth-telling in both periods.

Step 3 Let us focus on the case where $w < 1-\gamma$. Consider a putative equilibrium with a simple communication strategy ($m_1 = B$ when $\omega_1 = B$). R's visiting probability in period 2 after $m_1 = A$ is given by

$$P(d_2 = 1 \mid m_1 = A, \eta_2 = 1) = \frac{1-\gamma-w}{c}.$$

If $m_1 = B$, a positive probability of visiting in period 2 requires

$$1 - w - v_2 > P(\omega_1 = A \mid m_1 = B)\gamma + P(\omega_1 = B \mid m_1 = B)\frac{1}{2}.$$

Note that

$$P(\omega_1 = B \,|m_1 = B\,) = \frac{1-\theta}{1-\theta+\theta(1-\tau_A^1)}.$$

Thus given $m_1 = B$, R visits in period 2 if and only if

$$1 - w - v_2 > \left(1 - \frac{1-\theta}{1-\theta+\theta(1-\tau_A^1)}\right)\gamma + \frac{1-\theta}{1-\theta+\theta(1-\tau_A^1)}\frac{1}{2},$$

which is equivalent to $\frac{1}{2(1-\theta\tau_A^1)}\left(\theta - 2\theta\gamma - 2\theta\tau_A^1 + 2\theta\gamma\tau_A^1 + 1\right) - w > v_2$. Thus we have

$$P(d_2 = 1 \mid m_1 = B, \eta_2 = 1) = \frac{\frac{\left(\theta - 2\theta\gamma - 2\theta\tau_A^1 + 2\theta\gamma\tau_A^1 + 1\right)}{2(1-\theta\tau_A^1)} - w}{c},$$

provided that the numerator in the above expression is positive (note that it is never larger than c).

We now examine S's choice of message in period 1. Assume that $\omega_1 = A$. Sending A yields the truth-telling benefit $(1-\alpha)fx$, and R never makes a costly visit in period 2. If $\frac{1}{2(1-\theta\tau_A^1)}\left(\theta - 2\theta\gamma - 2\theta\tau_A^1 + 2\theta\gamma\tau_A^1 + 1\right) - w > 0$, then R visits with probability

$$\varphi_B(\tau_A^1, w, c, \gamma) = \frac{\frac{1}{2(1-\theta\tau_A^1)}\left(\theta - 2\theta\gamma - 2\theta\tau_A^1 + 2\theta\gamma\tau_A^1 + 1\right) - w}{c} \tag{29}$$

after $m_1 = B$, given $\eta_2 = 1$. If $\omega m_{11} = A$, S is thus indifferent between $m_1 = A$ and $m_1 = B$ in period 1 if and only if

$$(1-\alpha)fx + (1-\alpha)f\left(\frac{1-\gamma-w}{c}\right) = (1-\alpha)f\frac{\frac{1}{2(1-\theta\tau_A^1)}\left(\theta - 2\theta\gamma - 2\theta\tau_A^1 + 2\theta\gamma\tau_A^1 + 1\right) - w}{c}. \tag{30}$$

Note first that $\varphi_B(\tau_A^1, w, c, \gamma) \leq 1$ for any $\tau_A^1 \in [0,1]$. This is because the marginal value of consulting, ignoring the cost of consultation, is at most $\frac{1}{2}$. Note furthermore that

$$\frac{\partial\varphi_B(\tau_A^1, w, c, \gamma)}{\partial\tau_A^1} = \frac{1}{c}\left(\frac{1}{2}\theta\,(1-\theta)\,\frac{2\gamma-1}{\left(\theta\tau_A^1 - 1\right)^2}\right) > 0 \tag{31}$$

so that $\varphi_B(\tau_A^1, w, c, \gamma)$ is strictly increasing in τ_A^1. Note also that $\varphi_B(\tau_A^1, w, c, \gamma)$ is a continuous function of τ_A^1.

For the rest of this step, let us use a monotonicity argument to examine the existence of an equilibrium in simple strategies. Because of the continuity and monotonicity in (31), there exist a unique solution to (30) and hence an equilibrium if

$$\varphi_B(1, w, c, \gamma) = \frac{\frac{1}{2} - w}{c} \geq x + \left(\frac{1-\gamma-w}{c}\right)$$

and

$$\varphi_B(0, w, c, \gamma) = \frac{1 + \theta - 2w - 2\theta\gamma}{2c} < x + \left(\frac{1-\gamma-w}{c}\right),$$

since (30) requires

$$\varphi_B(\tau_A^1, w, c, \gamma) = x + \left(\frac{1-\gamma-w}{c}\right) \tag{32}$$

in equilibrium. However, our assumption $w < 1-\gamma < \frac{1}{2} - cx$ implies

$$\varphi_B(1, w, c, \gamma) = \frac{\frac{1}{2} - w}{c} < x + \frac{1-\gamma-w}{c},$$

which together with the monotonicity shown in (31) also implies (32) never holds since

$$\varphi_B(\tau_A^1, w, c, \gamma) < x + \left(\frac{1-\gamma-w}{c}\right)$$

for any $\tau_A^1 \in [0,1]$. Thus we conclude that if $w < 1-\gamma$ there is no informative equilibrium.

Step 4 In this step, we consider $w \in \left[1-\gamma, \frac{1}{2} - \frac{\theta(2\gamma-1)}{2} - cx\right]$. We refer to our equilibrium characterization (in Proposition 1) that assumed

$$w \in \left(\frac{1}{2} - cx - \frac{\theta(2\gamma-1)}{2}, \frac{1}{2} - cx\right).$$

The proof shows that given $w \geq 1-\gamma$, an informative equilibrium that features a simple communication strategy exists only if

$$w > \frac{1}{2} - cx - \frac{\theta(2\gamma-1)}{2}.$$

The inequality clearly does not hold for the range of w we consider and thus there is no informative equilibrium.

Step 5 We here show that if

$$cx < \left(\frac{1}{2} - \frac{\theta(2\gamma-1)}{2}\right) - (1-\gamma),$$

there always exists an equilibrium that features uninformative communication in period 1 and truth-telling in period 2. The equilibrium is constructed as follows. S always sends $m_1 = B$ in period 1, and if R receives the off-the-equilibrium message $m_1 = A$ in period 1, he assigns probability one to $\omega_1 = A$. We now check that S has no incentive to deviate from $m_1 = B$ in period 1.

Let us focus on $\omega_1 = A$. Given the off-the-equilibrium belief, $m_1 = A$ yields the expected payoff

$$(1-\alpha)\, fx + (1-\alpha)\, f\left(\frac{1-\gamma-w}{c}\right)$$

if $w < 1-\gamma$, while $m_1 = A$ yields $(1-\alpha) fx$ for $w \geq 1-\gamma$.

On the other hand, whether $w < 1-\gamma$ or $w \geq 1-\gamma$, $m_1 = B$ yields

$$(1-\alpha)\, f\left(\frac{\frac{1}{2} + \frac{1}{2}\theta - \theta\gamma - w}{c}\right).$$

Recall that given $m_1 = B$ in the putative equilibrium, R does not update his belief about the state. Therefore, whether $w < 1-\gamma$ or $w \geq 1-\gamma$, R sends $m_1 = B$ when $\omega_1 = A$ if

$$(1-\alpha)\, fx + (1-\alpha)\, f\left(\frac{1-\gamma-w}{c}\right) < f\,(1-\alpha)\left(\frac{\frac{1}{2} + \frac{1}{2}\theta - \theta\gamma - w}{c}\right), \tag{33}$$

where the left hand side represents the expected payoff from truth-telling when $w < 1-\gamma$

which is larger than that when $w \geq 1 - \gamma$. Note that (33) reduces to

$$cx < \left(\frac{1}{2} - \frac{\theta(2\gamma - 1)}{2}\right) - (1 - \gamma),$$

as assumed. Thus we conclude that there exists an uninformative equilibrium where S reports $m_1 = B$ regardless of the state.

Step 6 Using the observation that any informative equilibrium communication in period 1 must feature a simple communication strategy (Step 1), we have now shown that if $w \leq \frac{1}{2} - cx - \frac{\theta(2\gamma-1)}{2}$ and $cx < \left(\frac{1}{2} - \frac{\theta(2\gamma-1)}{2}\right) - (1 - \gamma)$, there exists no equilibrium featuring truth-telling in both periods or no equilibrium that features informative communication in period 1. We have then shown that under these same conditions, there exists an equilibrium with uninformative communication ($m_1 = B$ regardless of the state) in period 1.

Step 7 Assume that $w \in \left(\frac{1}{2} - cx, \frac{1}{2}\right)$ and $cx < \left(\frac{1}{2} - \frac{\theta(2\gamma-1)}{2}\right) - (1 - \gamma)$. Consider a putative equilibrium with truth-telling in both periods. The visiting probability in period 2 given $m_1 = B$ and $\eta_2 = 1$ is given by $\frac{\frac{1}{2} - w}{c}$, which is clearly positive. Given our assumptions, the visiting probability given $m_1 = A$ and $\eta_2 = 1$ is on the other hand is zero. We now examine the incentive of S. If $\omega_1 = A$, sending $m_1 = A$ yields the expected payoff $(1 - \alpha) f x$ whereas $m_1 = B$ yields $\frac{\frac{1}{2} - w}{c}(1 - \alpha) f$. Note that the truth-telling condition in this case

$$(1 - \alpha) f x \geq \frac{\frac{1}{2} - w}{c}(1 - \alpha) f$$

is equivalent to $cx \geq \frac{1}{2} - w$. Note that we are considering the case where $w > \frac{1}{2} - cx$, which implies $\frac{1}{2} - w < \frac{1}{2} - (\frac{1}{2} - cx) = cx$, which in turn implies $cx \geq \frac{1}{2} - w$. Thus we have established that there is an equilibrium that features truth-telling in period 1 (and period 2) strictly positive probability of consultation in period 2. □

8.2 Proposition 9

Proof. **Preliminary step** It is immediate that any equilibrium must feature truth-telling in period 2. Note that given $w > 1 - \gamma$, R strictly prefers to abstain from consulting in period 2 if he knows that $\omega_1 = A$,even if the price of consultation is 0 (recall that the costs $w + v_2$ are still present). In what follows, when studying the price in period 2, we focus on the price conditional on $m_1 = B$. Let $\Delta(\bar{\tau}_A^1)$ be the increase in the probability that R's action matches the state in period 2 by visiting, given perfect communication in period 2, m_1=B under a simple communication strategy in period 1. Recall from (10) in Appendix A and Assumption 2 ($\theta = \mu_A = \frac{2}{3-2\gamma}$)

$$\Delta(\bar{\tau}_A^1) = \frac{\theta - 2\theta\gamma - 2\theta\bar{\tau}_A^1 + 2\theta\gamma\bar{\tau}_A^1 + 1}{2\left(1 - \theta\bar{\tau}_A^1\right)} = \frac{(1 - \gamma)(2 - \bar{\tau}_A^1)}{3 - 2\gamma - \bar{\tau}_A^1}$$

Note that $\Delta(\bar{\tau}_A^1)$ is strictly increasing in $\bar{\tau}_A^1$.

Step 1 Let us study the price of the report in period 2. Consider a putative equilibrium in simple strategies in period 1. Then the probability of visiting in period 2 given price p_2 is $\frac{\Delta(\bar{\tau}_A^1) - w - p_2}{c}$. Therefore the profit-maximizing price for period 1 is determined by the following first order condition

$$\frac{\partial \left(\frac{\Delta(\bar{\tau}_A^1)-w-p_2}{c}\right) p_2}{\partial p_2} = 0.$$

The unique solution to the above is given by $p_2^* = \frac{1}{2}\left(\Delta(\bar{\tau}_A^1) - w\right)$.

Step 2 Now let us consider the reporting strategy in period 1. Suppose $\omega_1 = A$. Anticipating $p_2^* = \frac{1}{2}\left(\Delta(\bar{\tau}_A^1) - w\right)$, S is indifferent between $m_1 = A$ and $m_1 = B$ in the mixed strategy equilibrium if

$$\frac{\Delta(\bar{\tau}_A^1) - w - p_2^*}{c} p_2^* = p_2^* x.$$

Using $p_2^* = \frac{1}{2}\left(\Delta(\bar{\tau}_A^1) - w\right)$, the above simplifies to

$$\frac{\Delta(\bar{\tau}_A^1) - w}{2c} = x. \tag{34}$$

Step 3 Note that the LHS of (34) is increasing in $\bar{\tau}_A^1$. It follows that (34) has a unique solution with respect to $\bar{\tau}_A^1$ if and only if the LHS when $\bar{\tau}_A^1 = 0$ is smaller than x while the LHS when $\bar{\tau}_A^1 = 0$ is larger than x. Thus $\bar{\tau}_A^1 \in (0, 1)$ requires

$$\frac{2 - 2\gamma + 2\gamma w - 3w}{6c - 4c\gamma} < x \Leftrightarrow w > 1 - 2cx - \frac{1}{3 - 2\gamma} = \frac{1}{2} - 2cx - \frac{2\gamma - 1}{6 - 4\gamma} \tag{35}$$

$\bar{\tau}_A^1 = 0$ and

$$\frac{1 - 2w}{4c} > x \Leftrightarrow w < \frac{1}{2} - 2cx \tag{36}$$

for $\bar{\tau}_A^1 = 1$. We see that (8) is a condition for the existence of the unique equilibrium described by (35) and (36).

Step 4 Solving (34)

$$\bar{\tau}_A^{1*}(w, c, x, \gamma) = \frac{2\gamma(1 - w - 2cx) + 3w + 6cx - 2}{\gamma + w + 2cx - 1}.$$

Substituting the above into $p_2^* = \frac{1}{2}\left(\Delta(\bar{\tau}_A^1) - w\right)$, the price in period 2 is given by

$$p_2^* = \frac{1}{2}\left[\Delta(\bar{\tau}_A^{1*}(w, c, x, \gamma)) - w\right] = cx.$$

□

8.3 Proposition 10

Proof. **Step 1** Let us consider, the senders' choice of their prices in period 2. Consider a putative equilibrium in symmetric simple strategies, given truth-telling in period 2. With such a strategy profile, the probability of visiting firm i given the price profile $\{p_2^{S_1}, p_2^{S_2}\}$ in period 2 is given by

$$\frac{\Delta(\tilde{\tau}_A^1) - w - p_2^{S_i}}{c}\left(1 - \frac{p_2^{S_i}}{p_2^{S_1} + p_2^{S_2}}\right).$$

Thus the probability of making a costly visit to a firm in period 2 is given by respectively

$$\frac{\Delta(\tilde{\tau}_A^1) - w - a}{c} \left(1 - \frac{p_2^{S_1}}{p_2^{S_1} + p_2^{S_2}}\right)$$

for sender 1 and

$$\left(\frac{\Delta(\tilde{\tau}_A^1) - w - b}{c}\right) \left(1 - \frac{p_2^{S_2}}{p_2^{S_1} + p_2^{S_2}}\right)$$

for sender 2. The profit-maximizing prices for both firms in the second period are determined by the following first order conditions

$$\begin{aligned} \frac{\partial \left(\frac{\Delta(\tilde{\tau}_A^1)-w-a}{c}\left(1-\frac{p_2^{S_1}}{p_2^{S_1}+p_2^{S_2}}\right)p_2^{S_1}\right)}{\partial p_2^{S_1}} &= 0 \\ \frac{\partial \left(\frac{\Delta(\tilde{\tau}_A^1)-w-b}{c}\left(1-\frac{p_2^{S_2}}{p_2^{S_1}+p_2^{S_2}}\right)p_2^{S_2}\right)}{\partial b} &= 0. \end{aligned}$$

The unique solution to this system of two equations with two unknown variables is given by $p_2^{S_1*} = p_2^{S_2*} = \frac{1}{3}\left(\Delta(\tilde{\tau}_A^1) - w\right)$.

Step 2 Each individual firm anticipates that the profile of prices set in period 2 will be given by $p_2^{S_1*} = p_2^{S_2*} = \frac{1}{3}\left[\Delta(\tilde{\tau}_A^1) - w\right]$, and let us denote the latter by $\tilde{p}_2^*$. The informativeness of the report in period 1 $\tilde{\tau}_A^1$ is thus determined by the following indifference condition between reporting A and B truthfully

$$\frac{\Delta(\tilde{\tau}_A^1) - w - \tilde{p}_2^*}{c}\frac{1}{2}\tilde{p}_2^* = \chi x \tilde{p}_2^*.$$

Substituting $\tilde{p}_2^* = \frac{1}{3}\left(\Delta(\tilde{\tau}_A^1) - w\right)$ into the above, we obtain

$$\frac{\left(1-\frac{1}{3}\right)\left(\Delta(\tilde{\tau}_A^1) - w\right)}{c}\frac{1}{2} = \chi x. \tag{37}$$

Step 3 Clearly the LHS in (37) is strictly increasing in $\tilde{\tau}_A^1$. It follows that (37) has a unique solution with respect to $\tilde{\tau}_A^1$ if and only if the LHS when $\tilde{\tau}_A^1 = 0$ is strictly smaller than χx and the LHS when $\tilde{\tau}_A^1 = 0$ is strictly larger than χx. For $\tilde{\tau}_A^1 = 0$, the requirement is represented by

$$\frac{1}{3\chi}\frac{\frac{1}{2}\left(\frac{1-\gamma}{3-2\gamma}\right) - w}{c} < x$$

which rewrites as

$$w > \frac{1}{2} - 3\chi c x - \frac{2-\gamma}{6-4\gamma},$$

which is incorporated into (9) as assumed for Proposition 10. Likewise, for $\tilde{\tau}_A^1 = 1$ the requirement is given by

$$\frac{1}{3\chi}\left(\frac{\frac{1}{2} - w}{c}\right) \geq x.$$

It rewrites as $w < \frac{1}{2} - 3\chi cx$, which is also consistent with (9).

Step 4 Solving (37) $\tilde{\tau}_A^1$ with $p_2^* = \frac{1}{3}\left(\Delta(\tilde{\tau}_A^1) - w\right)$ yields a unique solution given by

$$\tilde{\tau}_A^{1*}(w, c, x, \gamma, \chi) = 3 - 2\gamma - \frac{(1-\gamma)(2\gamma - 1)}{\gamma + w + 3\chi cx - 1}$$

Finally, note that the price in period 2 implied by the above is

$$\frac{1}{3}\left[\Delta(\tilde{\tau}_A^{1*}(w, c, x, \gamma, \chi)) - w\right] = \chi cx.$$

□

8.4 Remark 1

Proof. Comparing the indifference conditions which determine the truth-telling probabilities $\bar{\tau}_A^{1*}$ (for monopoly) and $\tilde{\tau}_A^{1*}$ (for duopoly), we see that these truth-telling probabilities are identical if $\chi = \frac{2}{3}$. We have $\bar{\tau}_A^{1*}(w, c, x, \gamma) < \tilde{\tau}_A^{1*}(w, c, x, \gamma, \chi)$ if $\chi > \frac{2}{3}$; and $\bar{\tau}_A^{1*}(w, c, x, \gamma) > \tilde{\tau}_A^{1*}(w, c, x, \gamma, \chi)$ if $\chi < \frac{2}{3}$. □

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