

# Computational and Geometric Approaches for Nonlinear Phenomena

Date: August 5–7, 2015

Location: Meeting Room 04, 05 (2nd floor) at Building No. 63  
Waseda University, 3-4-1 Okubo, Shinjuku-ku, Tokyo

## Wednesday, August 5

12:30–13:20

**Speaker:** Hiroyuki Ochiai (Kyushu University)

**Title:** Computer graphics and mathematics

**Abstract:** Computer graphics (CG) is increasingly transforming every aspect of image creation and processing, and is an integration of technology and science. In this talk, we introduce our mathematical approach to CG, especially for making animations directable. No prerequisite on CG is assumed in this talk. A part of this talk is based on an experience in the CREST project “Mathematics for Expressive Image Synthesis” (team leader: Ken Anjyo) approved by “Alliance for breakthrough between mathematics and sciences” (research director: Yasumasa Nishiura) supported by JST.

13:30–14:20

**Speaker:** Kenji Kajiwara (Kyushu University)

**Title:** Integrable deformations of discrete curves

**Abstract:** In this talk, we present some recent results on integrable discrete deformations of space/plane discrete curves. We first demonstrate the connection between deformation theory of curves and integrable systems by showing that the modified KdV equation and its hierarchy naturally arise in the isoperimetric deformation of plane curves. Then after a brief explanation of discretization of soliton equations preserving integrability, we apply this idea to deformation of curves to formulate the isoperimetric deformation of discrete plane curves described by the discrete modified KdV equation. In the second half of the talk, we consider discretization of the local induction equation (also known as the Heisenberg ferromagnet equation) describing the dynamics of vortex filaments in  $R^3$ , where the curvature and the torsion of vortex filaments are governed by the nonlinear Schrödinger equation through the Hasimoto transformation. We then formulate the discrete deformation of discrete space curves whose complex curvature is described by the discrete nonlinear Schrödinger equation. We finally construct an explicit formula for both smooth and discrete space curves in terms of the tau functions of two-component KP hierarchy. Some animations of both numerical simulations and exact solutions will be presented. This talk is based on the collaborated works with S. Hirose, J. Inoguchi, N. Matsuura and Y. Ohta.

14:30–15:20

**Speaker:** Tomohiro Yanao (Waseda University)

**Title:** Roles of geometric phases in conformational transitions of molecular systems

**Abstract:** Conformational changes of molecular systems are central to chemical reactions, phase transitions of materials, and functions of biological macromolecules. This study explores the roles of

geometric phases arising from conformational changes of molecular systems. These geometric phases are analogous to the ones that arise in the so-called “ falling cat ” phenomenon, where a falling cat can change its overall orientation by changing its shape even under conditions of vanishing total angular momentum. We will highlight the significance of such geometric phases in the functions of biological molecular motors.

15:30–16:20

**Speaker:** Mathilde Badoual (University of Paris-Sud)

**Title:** Modeling homotypic and heterotypic cell interactions in gliomas

**Abstract:** Diffuse low-grade gliomas (DLGG) are brain tumors that grow slowly, but that are incurable. Despite treatments, a DLGG always recurs. This recurrence is due to the fact that some glioma cells migrate within the parenchyma surrounding the tumor, where their density is too low to be detected by classical imaging modalities such as magnetic resonance imaging. It is thus necessary to develop new approaches, such as modeling, in order to predict what is the real extent of these tumors and to better understand their natural history. Here, we will present a stochastic approach, based on a cellular automaton, where the interactions between migrating cells are taken into account and the properties of the correlations between cells are studied in order to characterize the leading edge of the tumor. In order to study the influence of these cell-cell interactions at the scale of a real tumor, we have calculated the continuous limit of our cellular automaton, and we show that inhibiting the interactions between migrating cells could help to reduce the invasion of the tumors.

16:30–17:20

**Speaker:** Yoichi Nakata (University of Tokyo)

**Title:** Gaps on the flow of the simplified path-preference cellular automaton model

**Abstract:** The path-preference model is a cellular-automaton model to describe the dynamics of RNA polymerase II in transcription. We found that the number of particles is dominant to the dynamics of simplest version of this model and observed that there are not only expected phase shift but also several non-continuous gaps as the number of particles increases. By considering limit cycles, we discuss the condition where such gaps appear.

17:30–18:20

**Speaker:** Tatsuya Hayashi (University of Tokyo)

**Title:** Integrate and fire model with refractory period for synchronization of two cardiomyocytes

**Abstract:** A cardiomyocyte is a cardiac muscle cell which beats autonomously. In case cardiomyocytes are isolated, they will just beat independently, while if they come into contact and interact with each other, their beating rhythms become synchronized. It was conjectured for a long time that, in a network of cardiomyocytes, firing of one cardiomyocyte triggers induced firing of the adjacent cardiomyocytes and all the cardiomyocytes start beating synchronously and that the beating rate is tuned to the fastest one. However, recent experiments of Yasuda laboratory have revealed that other cells are not synchronized to the fastest one but the one with the least fluctuation of the beating rhythm. We have constructed a mathematical model for two cardiomyocytes reproducing the experimental results of Yasuda laboratory. In this presentation, we explain our model and discuss the comparison of the numerical simulations to the experimental results.

**Thursday, August 6**

9:20–10:10

**Speaker:** Mariusz Bialecki (Institute of Geophysics, Polish Academy of Sciences)

**Title:** Random Domino Automaton with short range interactions

**Abstract:** Random Domino Automaton is a completely discrete stochastic slowly driven system, which store energy in clusters and release it through avalanches. Rules of the time evolution determine two mechanisms of clusters grow: enlarging and coalescence. In the presentation we introduce a version of of Random Domino Automaton with two extra parameters which allow to control separately an efficiency of each of the two mechanisms of clusters grow. We will discuss general structure of the set of equations describing a stationary state of the automaton in a mean field approximation as well as investigate properties of produced distributions of clusters and avalanches for some specific choices of the parameters.

10:20–11:10

**Speaker:** Akiko Fukuda (Shibaura Institute of Technology)

**Title:** Computation of eigenpairs based on the discrete integrable systems of hungry type

**Abstract:** Some eigenvalue algorithms have been designed based on the discrete integrable systems. Based on the discrete hungry Lotka-Volterra (dhLV) system, an eigenvalue algorithm, named the dhLV algorithm, has been proposed for computing all complex eigenvalues of nonsymmetric banded matrices. Based on the discrete hungry Toda (dhToda) equation, an eigenvalue algorithm has also been proposed for computing all eigenvalues of totally nonnegative (TN) matrices. Here, a matrix is referred to as TN if all of its minors are nonnegative. In this talk, we first give a brief review of these algorithms. Though these algorithms can compute eigenvalues with high relative accuracy, they don't have features of computing eigenvectors. It is shown that some characteristics of the integrable systems help us to compute eigenvectors efficiently. Whereas there exists the Bäcklund transformation between the dhLV system and the dhToda equation, we present an interesting relationship between eigenvalues/eigenvectors of the banded and the TN matrices.

11:20 12:10 Narimasa Sasa (Japan Atomic Energy Agency)

**Title:** Momentum conservation law in symplectic integrators for PDEs

**Abstract:** The momentum conservation law for partial differential equations in a time evolution by symplectic integrators is investigated. By showing the equivalence between the integral form of the total momentum and a sum of areas of projections of a hypersurface, which is a Poincare invariant in the phase space of the system, the conservation of the total momentum in symplectic integrators is generally proven. We also discuss an approximation formula of the total momentum for numerical simulations of the spatially discretized system. In addition, we show the results of some numerical simulations to confirm our theoretical consideration.

12:10-13:30 Lunch Time

13:30–14:20

**Speaker:** G.R.W. Quispel (LaTrobe University)

**Title:** Geometric numerical integration of integrable and non-integrable differential equations

**Abstract:** A geometric numerical integration method is a (discrete) integration algorithm that preserves some mathematical property/properties of a class of (continuous) differential equations exactly. This talk will be in two parts. In the first part we give a brief introduction to geometric integration. In the second part we present a few examples of discretizations that preserve complete integrability.

14:30–15:20

**Speaker:** Takaharu Yaguchi (Kobe University)

**Title:** Numerical integrations that preserve energy behaviors using the variational principle

**Abstract:** In this talk we present two general approaches to deriving numerical schemes that preserve energy behaviors. One is based on the Lagrangian mechanics and the other one is on the Hamiltonian mechanics. Both approaches use the variational principle and symmetry to derive numerical methods. Although the target equations of these approaches are the equations of motion in mechanics, there are other differential equations that are derived from a certain kind of non-standard variational principles. We apply these approaches to the equations from such non-standard variational principles.

15:30–16:20

**Speaker:** Shigetoshi Yazaki (Meiji University)

**Title:** Structure-preserving numerical scheme for Hele-Shaw flows by the method of fundamental solutions combined with the uniform distribution method

**Abstract:** The solutions to the one-phase interior or the classical Hele-Shaw problem are discretized in space by means of the method of fundamental solutions (MFS) combined with the uniform distribution method (UDM), and then a system of ordinary differential equations is obtained, which is solved by the usual fourth order Runge-Kutta method. The one-phase interior Hele-Shaw problem has curve-shortening (CS), area-preserving (AP) and barycenter-fixed (BF) properties. Under our numerical scheme, a discrete version of CS-, AP- and BF-properties holds, while simple boundary element method does not satisfy these properties in general. The one-phase exterior Hele-Shaw problem and the one-phase interior Hele-Shaw problem with sink/source points can also be treated. In each problem, a non-trivial exact solution is constructed and an experimental order of convergence is shown.

16:30–17:20

**Speaker:** Nobito Yamamoto (University of Electro-Communications)

**Title:** How to construct Lyapunov functions in dynamical systems by validated computation

**Abstract:** We provide a systematic method to validate the domain of Lyapunov functions around hyperbolic fixed points both for continuous and for discrete dynamical systems with computer assistance. We also give the construction of Lyapunov functions around periodic orbits as fixed points of Poincaré maps as well as validation examples in our talk.

Friday, August 7

9:20–10:10

**Speaker:** Adam Doliwa (University of Warmia and Mazury)

**Title:** Non-commutative q-P-VI

**Abstract:** We propose a difference equation for totally non-commuting fields which in the commutative reduction goes to q-P-VI equation of Jimbo and Sakai. We present a complete derivation of the equation starting from non-commutative Hirota system. At the intermediate step of the reduction procedure we obtain a non-isospectral extension of the (non-commutative) non-autonomous discrete mKdV equation. That non-isospectral factor plays a crucial role in recovering all the parameters in q-P-VI equation.

10:20–11:10

**Speaker:** Anton Dzhamay (University of Northern Colorado)

**Title:** Higher-rank Schlesinger transformations and difference Painlevé equations

**Abstract:** We present explicit evolution equations for elementary (two-point) Schlesinger transformation of an arbitrary rank  $r$  (i.e., this transformation changes  $r$  of the characteristic indices by unit shifts). Such arbitrary rank transformations are needed to obtain examples of difference Painlevé equations with the higher symmetry groups affine  $E_7$  and  $E_8$  as reductions of Schlesinger transformations, since the corresponding Fuchsian systems have degeneracy of characteristic indices. We also consider some examples of such reductions.

11:20–12:10

**Speaker:** Folkert Müller-Hoissen (Max-Planck-Institute for Dynamics and Self-Organization)

**Title:** The combinatorial structure underlying simplex equations, and generalizations of the pentagon equation

**Abstract:** The Yang-Baxter (YB) equation is often visualized on a cube. The latter can be taken to carry the Bruhat order  $B(3,0)$ , and the two sides of the YB equation then correspond to two ways of deforming an initial chain of three adjacent edges to a final chain (which expresses "consistency around a cube"). These two ways represent the two maximal chains of the Bruhat order  $B(3,1)$ . Higher Bruhat orders were introduced by Manin and Schechtman in 1986 in order to display the combinatorics underlying generalizations of the YB equation, the simplex equations. We present a fresh view of these relations. Furthermore, a decomposition of higher Bruhat orders leads to (higher) Tamari orders, and associated with them is a new family of equations which we call "polygon equations". They generalize the pentagon equation (which is structurally closely related to the YB equation). These polygon equations share integrability features with the simplex equations and this is what makes them special. Induced by the abovementioned decomposition, there are relations between (solutions of) polygon and (solutions of) simplex equations. All this is based on joint work with Aristophanes Dimakis (SIGMA 11 (2015) 042).