Security Dilemma Subgame: Cooperation Stage

Consider the following cooperation game:

<table>
<thead>
<tr>
<th>State B</th>
<th>Cooperate</th>
<th>Defect</th>
</tr>
</thead>
<tbody>
<tr>
<td>State A</td>
<td>r, r</td>
<td>0, t + β</td>
</tr>
<tr>
<td>Cooperate</td>
<td>t + α, 0</td>
<td>w + α, w + β</td>
</tr>
</tbody>
</table>

where we assume \( r > t > w > 0 \).

- The parameters \( α \) and \( β \) indicate the continuation value for the shadow of coercive diplomacy, reflecting the willingness to engage in noncooperative or conflictual policies.
- If \( α = β = 0 \), the cooperation stage becomes the Stag Hunt game.

Prior Distribution

For simplicity, we assume that Nature draws (independently) both \( α \) and \( β \) from a uniform distribution with the support of \([0, x]\).

- State A believes that \( β \) takes a certain value is \( p_A(β) = 1/x \).
- State B believes that \( α \) takes a certain value is \( p_B(α) = 1/x \).

Prior Beliefs

What matters in the Security Dilemma subgame is whether the other state is of a moderate or hawkish type:

- State A believes that State B is of a moderate type with \( p_A(β < -w) = F_A(-w) = -w \).
- State B believes that State B is of a moderate type with \( p_B(α < -w) = F_A(-w) = -w \).

We look for a Bayesian equilibrium in which

- Hawkish State A, for which \( α \geq -w \), always defects.
- Moderate State A, for which \( α < -w \), defects if \( α \in [0, x] \) exceeds some critical value \( \hat{α} \).
  - Ex ante probability that State A defects is

\[
Pr(α > \min(\hat{α}, -w)) = \begin{cases} 
1 - F_A(-w) & \text{if } \hat{α} < -w \\
1 - F_A(-\hat{α}) & \text{if } \hat{α} > -w 
\end{cases}
\]

- A type-contingent Bayesian strategy for State B is analogously defined.
Similarly, given $\sigma_A(\alpha)$, State B’s expected payoffs are:

$$EU_B(C) = (1 - \sigma_A(\alpha))r + \sigma_A(\alpha)(0) = (1 - \sigma_A(\alpha))r$$
$$EU_B(C) = (1 - \sigma_A(\alpha))r + \sigma_A(\alpha)(w + \beta) = \sigma_A(\alpha)(w - t) + (t + \beta)$$

State B will cooperate in equilibrium if

$$\begin{cases} 
\beta \leq \bar{\alpha} \psi - w \equiv \bar{\beta} & \text{if } \bar{\beta} < -w \\
\beta \leq -w^2 - 2w \equiv \bar{\beta} & \text{if } \bar{\beta} > -w 
\end{cases}$$

We now have two critical values to characterize the equilibrium, which should hold simultaneously.

Since $\bar{\alpha} \neq \bar{\beta}$, the solution to the system is given by

$$\begin{align*}
\bar{\alpha} &= -w^2 - 2w \quad \text{and} \quad \bar{\beta} = -w^2 - 2w & \text{if } \bar{\alpha}, \bar{\beta} > -w \\
\bar{\alpha} &= -w^2 - 2w \quad \text{and} \quad \bar{\beta} = \bar{\alpha} \psi - w & \text{if } \bar{\alpha} > -w > \bar{\beta} \\
\bar{\alpha} &= \bar{\beta} \psi - w \quad \text{and} \quad \bar{\beta} = -w^2 - 2w & \text{if } \bar{\beta} > -w > \bar{\alpha} \\
\bar{\alpha} &= \bar{\beta} \psi - w \quad \text{and} \quad \bar{\beta} = \bar{\alpha} \psi - w & \text{if } \bar{\alpha}, \bar{\beta} < -w
\end{align*}$$

The Bayesian Nash Equilibrium to the Security Dilemma Subgame

- State A’s equilibrium type-contingent (behavioral) strategy is
  $$s_A(\alpha) = \begin{cases} 
  \text{Cooperate} & \text{if } \alpha \leq \bar{\alpha} \\
  \text{Defect} & \text{if } \alpha > \bar{\alpha}
  \end{cases}$$

- State B’s equilibrium type-contingent (behavioral) strategy is
  $$s_B(\beta) = \begin{cases} 
  \text{Cooperate} & \text{if } \beta \leq \bar{\beta} \\
  \text{Defect} & \text{if } \beta > \bar{\beta}
  \end{cases}$$

where $\bar{\alpha}$ and $\bar{\beta}$ are given above.