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INTRODUCTION TO MODULI SPACES OF FLAT CONNECTIONS

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There are many references for this topic as it appears in several different fields of mathematics. Here are just a few of them that are related to or will be used in this lecture series.

Motivation

1. M. Atiyah, The Geometry and Physics of Knots

Background on geometry and Lie groups

- 1. L. Duistermaat, Lie Groups (Chapter 1.1-1.4: Lie groups and Lie algebras; Chapter 2.1-2.4: group actions and associated bundles)
- 2. J. Milnor, Characteristic Classes (Chapter 1-3: manifolds and vector bundles)
- 3. M. Audin, The Topology of Torus Actions on Symplectic Manifolds.
- 4. L. Jeffrey, *Hamiltonian Group Actions and Symplectic Reduction*, IAS/Park City Mathematics Series.
- 5. E. Meinrenken, Lecture Notes on Symplectic Geometry.
- 6. A. Cannas da Silva, Lectures on Symplectic Geometry.

Yang-Mills theory

- 1. V. Guillemin and S. Sternberg, Symplectic Techniques in Physics (Chapter 3)
- 2. J. Jost, Riemannian Geometry and Geometric Analysis (Chapter 3, 4.1-4.2)
- 3. S. Donaldson, The Geometry of Four-Manifolds, (Chapter 2, 4, 6)

Principal bundles, holonomy, and surface group representations

- 1. S. Kobayashi, Foundations of Differential Geometry I (Chapter 2)
- 2. C. Taubes, Differential Geometry: Bundles, Connections, Metrics and Curvature
- 3. A. Alekseev, A. Malkin, and E. Meinrenken, Lie group valued moment maps

- 4. W. Goldman, The symplectic nature of fundamental groups of surfaces
- 5. W. Goldman and J. Millson, The deformation theory of representations of fundamental groups of compact Kaehler manifolds, IHES 1988 (Sections 4, 5)

Vector bundles and their moduli spaces

- 1. S. Kobayashi, Differential Geometry of Complex Vector Bundles
- 2. R. Gunning, Lectures on Vector Bundles over Riemann Surfaces
- 3. S. Mukai, An Introduction to Invariants and Moduli
- 4. O. Garcia-Prada, Moduli spaces and geometric structures (appendix in third edition of R. Wells, Differential Analysis on Complex Manifolds)

Complex geometry

- 1. R. Wells, Differential Analysis on Complex manifolds (third edition)
- 2. D. Huybrechts, Complex Geometry

Fundamental papers

- 1. M. Atiyah and R. Bott, The Yang-Mills equations over Riemann surfaces
- 2. N. Hitchin, The self-duality equations on a Riemann surface
- 3. M. Atiyah, Collected Works, Volume 5 Gauge theories