The Third Japan-Taiwan Joint Conference on Differential Geometry

Geometric Quantization on CR Manifolds

Chin-Yu Hsiao (Academia Sinica)

Abstract:
We consider a compact connected orientable CR manifold with the action of a connected compact Lie group. Under natural pseudoconvexity assumptions we show that the CR Guillemin-Sternberg map is Fredholm at the level of Sobolev spaces of CR functions. As an application we study this map for holomorphic line bundles which are positive near the inverse image of zero by the momentum map. We also show that "quantization commutes with reduction" for Sasakian manifolds. This is a joint work with Xiaonan Ma and George Marinescu.

Derived Differential Geometry and Virtual Fundamental Classes

Adeel Ahmad Khan (Academia Sinica)

Abstract:
Virtual counts of pseudoholomorphic curves on a symplectic manifold play an important role in Gromov-Witten theory and Lagrangian Floer theory. These counts are defined using the virtual fundamental class of the moduli space of pseudoholomorphic curves. I will explain a simple new construction of the virtual fundamental class based on a theory of derived differential geometry.

Homotopy Fiber Product of Manifolds

Hsuan-Yi Liao (National Tsing Hua University)

Abstract:
A main motivation of developing derived differential geometry is to deal with singularities arising from zero loci or intersections of submanifolds. Both zero loci and intersections can be considered as fiber products of manifolds. Thus, we extend the category of differentiable manifolds to a larger category in which one has "homotopy fiber products". In this talk, I would like to show a construction, using vector bundles and sections, of homotopy fiber products of manifolds and explain the structures behind the construction. The talk is mainly based on a joint work with Kai Behrend and Ping Xu.

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Algebraicity of Compact Kähler Manifolds via Dual Positive Cones

Hsueh-Yung Lin (National Taiwan University)

Abstract:
Let X be a compact Kähler manifold. The celebrated Kodaira embedding theorem asserts that if the Kähler cone of X contains a rational cohomology class, then X admits a holomorphic embedding into a projective space. Instead of considering Kähler classes, we will study the algebraicity of X when X carries a 1-dimensional positive rational Hodge class.

The Jacobi Spectrum of Null-Torsion Holomorphic Curves in the 6-Sphere

Jesse Madnick (National Center for Theoretical Sciences)

Abstract:
Minimal surfaces are area-minimizing to first order, but not necessarily to second-order. The extent to which a minimal surface is (or isn't) area-minimizing to second-order is encoded by its Jacobi operator. However, for a given minimal surface, computing the spectrum of the Jacobi operator — i.e., the eigenvalues and their multiplicities — is a non-trivial task. In this talk, I will discuss a class of minimal surfaces in the round 6-sphere known as "null-torsion holomorphic curves." These surfaces are of interest to G2 geometry and exist in abundance. Indeed, by a remarkable theorem of Bryant, extended by Rowland, every closed Riemann surface may be conformally embedded into S^6 as a null-torsion holomorphic curve. For null-torsion holomorphic curves of low genus, we will compute the multiplicity of the first Jacobi eigenvalue. Moreover, for all genera, we will give a simple lower bound for the nullity in terms of the area and genus. We expect that these results will have implications for the deformation theory of asymptotically conical associative 3-folds in euclidean R^7.

Quantum Flips and F-embeddings

Chin-Lung Wang (National Taiwan University)

Abstract:
We study analytic continuations of quantum cohomology under simple flips $f : X \to X'$ along the extremal ray variable q^ℓ. Denote by $\Psi : H(X') \to H(X)$ the (inverse) graph correspondence. We show that there is a unique deformation $\widehat{\Psi}$ of Ψ which induces a non-linear imbedding $QH(X') \hookrightarrow QH(X)$ in the category of F (but not Frobenius) manifolds into the regular integrable loci of $QH(X)$ near $q^\ell = \infty$. This is a joint work with Yuan-Pin Lee and Hui-Wen Lin.

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Rigidity and Symmetry of Cylindrical Handlebody-Knots

Yi-Sheng Wang (Academia Sinica)

Abstract:
The theory of handlebody-knots studies handlebodies in three dimensions; in the case of a genus one handlebody embedded in the 3-sphere, the theory is equivalent to the classical knot theory. The talk concerns symmetries of a genus two handlebody-knot measured by its symmetry group, the path components of the space of self-homeomorphisms of the 3-sphere preserving the handlebody-knot setwise. It follows from a recent result of Funayoshi-Koda that a genus two handlebody-knot has a finite symmetry group if and only if it is hyperbolic—the exterior admits a hyperbolic structure with totally geodesic boundary—or irreducible, atoroidal, cylindrical—the exterior contains no essential disks or tori but contains an essential annulus. Little however is known about the structure of these finite groups.

The talk will start with a quick tour through some basics of essential surfaces of non-negative Euler characteristic in a handlebody-knot exterior, and move on from there, I will survey some known results on symmetry groups of cylindrical handlebody-knots.

Lagrangian Mean Curvature Flow with Boundary

Albert Wood (National Taiwan University)

Abstract:
The Lagrangian mean curvature flow is the name given to the remarkable fact that mean curvature flow preserves the class of Lagrangian submanifolds in Kahler-Einstein manifolds. A natural follow-up question that springs to mind is whether there exists a suitable boundary condition for this flow, such that the resulting flow with boundary still preserves the Lagrangian condition. Remarkably, standard Neumann and Dirichlet boundary conditions do not work, but there is a symplectically natural mixed Dirichlet-Neumann boundary condition involving a boundary Lagrangian flow which does. In this talk I will describe the condition and give an overview of the proof, as well as describe some examples of the flow's behaviour.

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Mirror Symmetry and Rigid Structures of Generalized $K3$ Surfaces

Atsushi Kanazawa (Keio University)

Abstract:
Hitchin's invention of generalized Calabi-Yau structures is a key to unify the Calabi-Yau geometry (complex geometry of Calabi-Yau manifolds) and symplectic geometry. Such structures have been extensively studied in 2-dimensions by Huybrechts. Based upon his fundamental work, we introduce a formulation of mirror symmetry for generalized $K3$ surfaces, which generalizes mirror symmetry for lattice polarized $K3$ surfaces. Along the way, we investigate complex and Kahler rigid structures of generalized $K3$ surfaces.

On Exact Triangles Consisting of Projectively Flat Bundles on Higher Dimensional Complex Tori

Kazushi Kobayashi (Chiba University)

Abstract:
In general, for a given complex torus X, a simple projectively flat bundle V on X is constructed from each affine Lagrangian submanifold in a mirror partner of X with a unitary local system along it. In this talk, we focus on a certain class of exact triangles consisting of three simple projectively flat bundles V on a higher dimensional complex torus X, and explain that such an exact triangle on X is obtained as the pullback of an exact triangle on an elliptic curve E by a suitable holomorphic projection $X \to E$.

Curvatures and Austere Property of Orbits of Path Group Actions Induced by Hermann Actions

Masahiro Morimoto (Osaka City University)

Abstract:
It is known that an isometric action of a Lie group on a compact symmetric space gives rise to a proper Fredholm action of a path group on a path space via the gauge transformations. In this talk, supposing that the isometric action is a Hermann action (i.e. an isometric action of a symmetric subgroup of the isometry group) we study the principal curvatures and the austere property of orbits of the path group action. Here an austere submanifold is a minimal submanifold such that the set of principal curvatures in the direction of each normal vector is invariant under the multiplication by minus one. The results show that there exist many infinite dimensional austere submanifolds in Hilbert spaces.

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First-eigenvalue Maximization and Embedding Optimization

Shin Nayatani (Nagoya University)

Abstract:
Maximization problem for the first eigenvalue of the Laplacian began with the seminal work of Hersch (1970), who proved that on the two-sphere the first eigenvalue (multiplied by area for scale invariance) was maximized by the round metrics (and by them only). Since then, this subject has been studied by many geometers and enriched by many interesting results. Among them, I mention a beautiful theorem of Nadirashvili (1996), which states that a metric maximizing the first eigenvalue of the Laplacian admits an isometric minimal immersion into a round sphere of some dimension. Meanwhile, in graph theory, Fiedler (1989) considered a similar maximization problem, and more recently Göring-Helmberg-Wappler (2008, 2011) formulated a problem which is dual (in the framework of mathematical programming) to Fiedler's problem and concerns embeddings of a graph into Euclidean spaces. In this talk, I will introduce an analogue of GHW formalism in differential geometry. In fact, it turns out that the relevant eigenvalue maximization problem concerns the Bakry-Émery Laplacian on a weighed Riemannian manifold rather than the usual Laplacian. I will discuss examples and an analogue of the above mentioned Nadirashvili theorem.

Lagrangian Mean Curvature Flows with Perpendicular Symmetries

Akifumi Ochiai (Tokyo Metropolitan University)

Abstract:
We show a method of constructing an invariant Lagrangian mean curvature flow in a Calabi-Yau manifold with the use of generalized perpendicular symmetries. We use moment maps of the action of Lie groups, which are not necessarily abelian. By our method, we construct non-trivial examples in $\mathbb{C}^n$ including self-similar solutions and translating solitons of mean curvature flows.

Rigid Fibers of Integrable Systems on Cotangent Bundles

Ryuma Orita (Niigata University)

Abstract:
In the talk, we deal with classical integrable systems such as the Lagrangian top and the Kovalevskaya top. Especially, we find a non-displaceable fiber for each of them. To prove these results, we use the notion of superheaviness introduced by Entov and Polterovich. This is a joint work with Morimichi Kawasaki (Aoyama Gakuin University).

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Transversal Properties for Period Maps on Moduli Space of Triply Periodic Minimal Surfaces

Toshihiro Shoda (Kansai University)

Abstract:
Triply periodic minimal surfaces are mathematical objects for surfactant, and they have been studied in many fields. We focus on the genus three case and many one-parameter families have been constructed in physics. In the previous work, we computed Morse indices and nullities for the families, and some bifurcation phenomena, that is, the existence of new one-parameter families issuing from the original one-parameter families were pointed out. The key point is the point where the nullity is greater than three. In this talk, we introduce recent works related to classification of nullities from which a new one-parameter family does not issue, in terms of singularities theory. It is a joint work with Norio Ejiri.

An Example of the Noncompact Yamabe Flow having the Infinite-time Incompleteness

Hikaru Yamamoto (University of Tsukuba)

Abstract:
I explain a recent result on the noncompact Yamabe flow which is joint work with Jin Takahashi at Tokyo Institute of Technology. The noncompact Yamabe flow is complicated compared to the compact case. There are many unexpected phenomena from the viewpoint of the compact Yamabe flow. One of the remaining questions is the following. If each Riemannian metric is complete under the Yamabe flow on a noncompact manifold for all time and the long time limit exists, then is the limit also complete? I give the negative answer to this question by giving a counterexample.