Non-Separability and Sectoral Comovement in a Sticky Price Model*

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Abstract

This paper resolves the sectoral comovment problem between nondurable and durable outputs that arises in response to a monetary shock in a two-sector sticky price model with flexibly priced durable goods. First, we analytically demonstrate that the two-sector sticky price model, if modified to feature the non-separability between aggregate consumption and labor in household preferences, can generate the comovement between nondurable and durable outputs in response to a monetary policy shock. We then estimate the degree of non-separability, together with other parameters, using a Bayesian approach. We find that the non-separable preferences are supported by the data and improve the overall empirical performance of a two-sector sticky price model. Finally, our estimated model with non-separable preferences successfully generates the sectoral comovement in response to a monetary shock.

Keywords: Sticky Price, Durable Good, Comovement, Non-Separable Preferences
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1 Introduction

Durable goods feature prominently in discussions of monetary policy. According to the data, the durable goods sector responds most procyclically to monetary policy.[1] However, as demonstrated by Barsky, House, and Kimball (2003, 2007), it is difficult to match this feature of the data by simply incorporating durable goods into sticky price models with separable preferences. In particular, if durable goods have flexible prices, but nondurable goods prices are sticky, then an expansionary monetary policy leads to an *increase* in nondurable goods production but a *decline* in durable goods production, so that aggregate output may not change at all. The comovement problem arises.

This paper seeks to resolve the comovement problem that arises in response to a monetary shock in a two-sector sticky price model with flexibly priced durable goods. First, we analytically demonstrate that the two-sector sticky price model, if modified to feature the non-separability between aggregate consumption and labor in household preferences, can generate the response of durable spending to a monetary policy shock documented in empirical studies. The key mechanism to generate this result is the complementarity between nondurable consumption and labor implied by the non-separable preferences, absent in the separable preferences. We also show that variable capital utilization with imperfect capital mobility helps expand the threshold level of the non-separability required for the model to generate the sectoral comovement in response to a monetary shock.

Second, we show that the non-separable preferences are indeed supported by the data. To this end, we estimate the intertemporal elasticity of substitution that controls the degree of non-separability, together with other parameters, using a Bayesian approach. Since DiCecio (2009) and Iacoviello and Neri (2010) demonstrate that the introduction of sticky nominal wages helps resolve the comovement problem,[2] we incorporate sticky wages into our estimation to avoid the concern that the degree of non-separability might be over-estimated if they are not included. Our estimates of the intertemporal elasticity of substitution are well below unity suggesting that the data favor non-separable preferences. Furthermore, the Bayesian model comparison favors the model with the non-separable preferences over the one with the separable preferences, unless one has an extremely strong prior belief about the separable preferences. This implies that non-separable preferences should be seriously considered an important ingredient to a two-sector sticky price model.

Finally, we demonstrate that non-separable preferences alone successfully resolve the comovement problem associated with monetary shocks in our estimated model without relying on sticky wages. The estimated degree of non-separability is strong enough to produce an expansion in the durable sector as well as the nondurable sector in response to an expansionary monetary shock.

[1] Erceg and Levin (2006) document that an exogenous increase in the interest rate, estimated through a structural VAR, reduces consumer durables and residential investment spending nearly three times more than nondurable consumption. Barsky, House, and Kimball (2003) also report similar results using the Romer dates as indicators of exogenous changes in monetary policy. Following the Romer date, the production of durables falls far more than that of nondurables.

[2] Barsky, House, and Kimball (2003, 2007) and Carlstrom and Fuerst (2006) also suggest the introduction of a sticky nominal wage as one possible solution to the comovement problem.

Our empirical results complement the previous studies by Basu and Kimball (2002) and Guerron-Quintana (2008), who also report evidence against the separable preferences. However, our empirical specification differs from theirs in two ways. While their empirical specifications abstract from durable spending in estimating the intertemporal elasticity of substitution, we explicitly incorporate durable spending in the estimation. The earlier studies use the limited information approach. Basu and Kimball (2002) employ a method of moments on the Euler equation and Guerron-Quintana (2008) uses a minimum distance estimator. Instead, we employ the full information approach that explores the full range of empirical implications conveyed by the model. The full information approach enables us to investigate whether non-separable preferences not only generates a realistic response of the model in response to a monetary shock, but also improves the overall fit of the model.

Why is the complementarity between nondurable consumption and labor, implied by non-separable preferences, a key for the model generating the comovement in response to a monetary shock?[3] This stems from the fact that the complementarity significantly affects the reaction of a nominal wage to a monetary expansion. Following such an expansion, the nominal wage tends to rise because nondurable-goods producing firms raise the demand for labor inputs to meet their increased demands due to sticky prices. For producers in the flexibly priced durable sector, the increase in factor price is merely an adverse cost shock. Unless there are forces offsetting the rise in the cost of production, the flexible-price sector contracts. However, the complementarity between nondurable consumption and labor supply mitigates the rising pressure on the nominal wage since the increase in nondurable consumption shifts the labor supply curve out. Hence, if the degree of complementarity is large enough, production in the durable sectors could rise.[4] Variable capital utilization in turn strengthens the effects of the complementarity on offsetting a rise in the nominal wage, so that it expands the range of the non-separability consistent with the sectoral comovement.

The remainder of the paper is organized as follows. Section 2 describes a two-sector sticky price model that includes nondurable and durable goods. Section 3 presents an analytical treatment that provides insight into why the non-separable preferences can solve the comovement problem. Section 4 estimates a two-sector sticky price model using a Bayesian approach to show that non-separable preferences are supported by the data and significantly improves the overall empirical performance of the model. It also demonstrates that our estimated model with non-separable preferences generates the sectoral comovement in response to a monetary shock. Section 5 concludes.

[3]The non-separability between aggregate consumption and labor also implies that the service flow from durable goods and labor are complementary. However, the complementarity between the consumption of durable services and labor has little impact on the behavior of the model. It is because the stock of durable goods changes so slightly following the monetary shock as Barsky, House, and Kimball (2003, 2007) show.

[4]Barsky, House, and Kimball (2003, 2007) briefly discuss the possibility that the complementarity between nondurables and labor might temper the negative comovement problem, but have never explored it formally.

2 The Model

In this section, we extend the two-sector sticky price model of Barsky, House, and Kimball (2003, 2007) by incorporating non-separability between aggregate consumption and labor, and by introducing variable capital utilization.

The economy is populated by a constant number of identical, infinitely-lived households, continua of firms in two sectors that respectively produce differentiated durable and nondurable goods, perfectly competitive final goods firms in two sectors, and a monetary authority.

2.1 Households

The representative household receives utility from consumption of the nondurable goods, enjoys the service flow from durable goods, and incurs disutility from hours worked. Let C_t and S_t respectively denote period t consumption of the nondurable goods and the service flow from the durable consumption, and let L_t denote labor supply. The household maximizes the expected lifetime utility, given by

$$U_0 = E_0 \left[\sum_{t=0}^{\infty} \beta^t b_t U(C_t, \upsilon_t S_t, L_t) \right], \tag{1}$$

where $\beta \in (0,1)$ is the subjective discount factor, b_t is a discount factor shock (intertemporal preference shock) and υ_t is a durable preference shock. The intertemporal preference shock is a common ingredient in estimated DSGE models and the durable preference shock is recently introduced by Iacoviello and Neri (2010). These shocks follow the stochastic processes:

$$\log b_t = \rho_b \log b_{t-1} + \xi_{b,t}, \tag{2}$$

$$\log \upsilon_t = \rho_\upsilon \log \upsilon_{t-1} + \xi_{\upsilon,t}. \tag{3}$$

We modify the conventional King-Plosser-Rebelo monetary utility function used by Basu and Kimball (2002) and Shimer (2009) to include the consumption of the service flow from durable goods. The specific form of $U(\cdot)$ adopted in this paper is

$$U(C_t, \upsilon_t S_t, L_t) = \frac{Z_t^{1-\frac{1}{\sigma}} \left(1 + \left(\frac{1}{\sigma} - 1\right) v(L_t)\right)^{\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}}, \tag{4}$$

where $Z_t = \left(\psi_c C_t^{1-\frac{1}{\rho}} + \psi_d \upsilon_t S_t^{1-\frac{1}{\rho}} \right)^{\frac{\rho}{\rho-1}}$ and $v(L_t) = \phi \frac{\eta}{1+\eta} L_t^{\frac{\eta+1}{\eta}}$. Z_t is a quantity index that aggregates the consumption of nondurable goods and durable services, and $v(L_t)$ measures the disutility incurred from hours worked with $v' > 0$, $v'' > 0$.

Our monetary utility formulation departs from Barsky, House, and Kimball (2003, 2007) in that we relax the assumption of additive separability between aggregate consumption and labor. In (4), the degree of non-separability is controlled by a parameter for the intertemporal elasticity of substitution, σ. The lower this parameter is, the larger the non-separability displayed by the utility function. The separable case, for instance, corresponds to $\sigma \to 1$:

$$\lim_{\sigma \to 1} U(C_t, S_t, L_t) = \log(Z_t) - v(L_t).$$

This separable preference is used in most sticky price models, including Barsky, House, and Kimball (2003, 2007).

The stock of durable goods evolves according to

$$D_t = X_t + (1 - \delta)D_{t-1}, \tag{5}$$

where $\delta \in (0, 1)$ is the depreciation rate and X_t denotes newly purchased durables. Following the literature, the service flow from durable goods, S_t, is assumed to be proportional to the stock of the durable goods, D_t and, without loss of generality, the coefficient of proportionality is normalized to 1:

$$S_t = D_t = X_t + (1 - \delta)X_{t-1} + (1 - \delta)^2 X_{t-2} + \cdots .$$

The household enters period t with a stock of private one-period nominal bonds (B_{t-1}), and a fixed stock of capital ($\overline{K}$). During the period, the household receives wages, rentals on capital services, dividends paid by firms, and interest payments on bond holdings. These resources net of the cost of varying capital utilization rate are used to purchase durable and nondurable goods and to acquire assets to be carried over to the next period. Then, the household's budget constraint (in nominal term) is

$$P_{c,t}C_t + P_{x,t}X_t + B_t \leq W_t L_t + \sum_{j=c,x} R_{j,t} u_{j,t} \overline{K}_{j,t} + \Pi_t + (1 + i_{t-1})B_{t-1} - \sum_{j=c,x} P_{j,t} a(u_{j,t}) \overline{K}_{j,t}, \tag{6}$$

where the subscripts c and x denote variables that are specific to the nondurable and durable sector, respectively. $P_{c,t}$ and $P_{x,t}$ are the nominal prices of the nondurable and durable, W_t is the nominal wage rate,[5] Π_t are profits returned to the consumer through dividends, and i_t is the nominal interest rate. In addition, $\overline{K}_{j,t}$ is the productive capital stock in sector $j = c, x$ and $u_{j,t}$ denotes the capital utilization rate in sector $j = c, x$. Hence, $K_{j,t} \equiv u_{j,t}\overline{K}_{j,t}$ represents the capital services used in each sector and $R_{j,t}$ is the rental rate of capital services.[6] The increasing and convex function $a(u_{j,t})\overline{K}_{j,t}$ denotes the cost, in units of the goods in each sector, of setting the capital utilization rate to $u_{j,t}$. Following Christiano, Eichenbaum, and Evans (2005), we impose two restrictions on the capital utilization function, $a(u_{j,t})$. First, we require that $u_{j,t} = 1$ in a steady state. Second, we assume $a(1) = 0$. Under these assumptions, the steady state of the model is independent of the curvature of the function $a(\cdot)$ in steady state, $\chi \equiv \frac{a''(1)}{a'(1)}$. The parameter χ governs the elasticity of capital utilization. A high value of χ corresponds to a small elasticity, implying that varying utilization is highly costly.

[5] Note that we assume that labor can flow freely between sectors. Hence, wage rates are identical between sectors.

[6] Rental rates in different sectors might not be the same because we consider the case where capital stock is imperfectly mobile between sectors.

The first order conditions associated with the optimal choice of C_t, L_t and X_t are

$$\frac{\gamma_{c,t}}{P_{c,t}} = \frac{\gamma_{x,t}}{P_{x,t}}, \tag{7}$$

$$-U_L(C_t, D_t, L_t) = \gamma_{x,t}\frac{W_t}{P_{x,t}} = \gamma_{c,t}\frac{W_t}{P_{c,t}}, \tag{8}$$

where $\gamma_{c,t} \equiv U_C(C_t, D_t, L_t)$ denotes the marginal utility of nondurable consumption and $\gamma_{x,t}$ denotes the shadow value of durable consumption. As in Barsky, House, and Kimball (2007), $\gamma_{x,t}$ can be written as

$$\gamma_{x,t} = MU_t^D + \beta(1-\delta)E_t\left[\gamma_{x,t+1}\right], \tag{9}$$

where $MU_t^D \equiv U_D(C_t, D_t, L_t)$ denotes the marginal utility of the service flows from an additional unit of the durable at time t.

2.2 Firms

We assume the existence of a continuum of monopolistically competitive firms, indexed by $s \in [0, 1]$, producing differentiated intermediate goods in each sector. A final good in each sector is produced by a perfectly competitive, representative firm. The firm produces the final good by combining a continuum of intermediate goods.

2.2.1 Final goods firms

The final good in each sector is aggregated by the constant elasticity of substitution (CES) technology:

$$C_t = \left[\int_0^1 c_t(s)^{\frac{\varepsilon_c - 1}{\varepsilon_c}} ds\right]^{\frac{\varepsilon_c}{\varepsilon_c - 1}} \quad \text{and} \quad X_t = \left[\int_0^1 x_t(s)^{\frac{\varepsilon_x - 1}{\varepsilon_x}} ds\right]^{\frac{\varepsilon_x}{\varepsilon_x - 1}}, \tag{10}$$

where $c_t(s)$ and $x_t(s)$ are the quantity of intermediate goods s used as an input in each sector. Solving a cost minimization problem for the final good producer in each sector delivers the conditional demand for the intermediate goods

$$c_t(s) = \left(\frac{p_{c,t}(s)}{P_{c,t}}\right)^{-\varepsilon_c} C_t \quad \text{and} \quad x_t(s) = \left(\frac{p_{x,t}(s)}{P_{x,t}}\right)^{-\varepsilon_x} X_t, \tag{11}$$

where $p_{j,t}(s)$ is the price of intermediate good s in sector $j = c, x$ and $P_{j,t}$ is the aggregate price level in sector $j = c, x$. Finally, the zero-profit condition implies that

$$P_{j,t} = \left[\int_0^1 p_{j,t}(s)^{1-\varepsilon_j} ds\right]^{\frac{1}{1-\varepsilon_j}}, \quad \text{for} \quad j = c, x. \tag{12}$$

2.2.2 Intermediate goods firms

Intermediate good producers in each sector are monopolistically competitive. Each intermediate goods firm produces its differentiated goods using the following production function:

$$c_t(s) = A_t A_{c,t} F(k_{c,t}(s), l_{c,t}(s)) = A_t A_{c,t} k_{c,t}^{\alpha}(s) l_{c,t}^{1-\alpha}(s), \tag{13}$$

$$x_t(s) = A_t A_{c,t} F(k_{x,t}(s), l_{x,t}(s)) = A_t A_{x,t} k_{x,t}^{\alpha}(s) l_{x,t}^{1-\alpha}(s), \tag{14}$$

where A_t is an aggregate total factor productivity (TFP) shock and $A_{j,t}$ is a sectoral TFP shock in sector $j = c, x$, $l_{j,t}(s)$ and $k_{j,t}(s)$ are labor and capital in firms s in sector $j = c, x$ at time t. We assume that these aggregate and sectoral technology shocks follow the stochastic processes:

$$\log(A_t) = \rho_A \log(A_{t-1}) + \xi_t, \tag{15}$$

$$\log(A_{j,t}) = \rho_{A_j} \log(A_{j,t-1}) + \xi_{j,t} \quad \text{for} \quad j = c, x. \tag{16}$$

Intermediate goods firms are assumed to set nominal prices in a staggered fashion, according to the stochastic time dependent rule proposed by Calvo (1983). Each firm in sector $j = c, x$ resets its price with the probability of $1 - \theta_j$ each period, independently of the time elapsed since the last adjustment. Thus, for each period a measure $1 - \theta_j$ of firms reset their prices, while a fraction θ_j firms keep their prices from the previous period. An intermediate goods firm resetting its price in period t in sector $j = c, x$ will seek to maximize the present value of expected future real profits generated while that price remains effective,

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \theta_j^t \gamma_{j,t} \frac{\Pi_{j,t}}{P_{j,t}} \right], \tag{17}$$

subject to the sequence of demand constraints (11). Here $\gamma_{j,t}$ is the shadow value of the good produced in sector j, and $\Pi_{j,t}/P_{j,t}$ measures the real value of an intermediate goods firm's profit in sector j in period t. It is easy to show that the optimal reset prices in sector $j = c, x$, denoted as $p^*_{j,t}$, are

$$p^*_{c,t} = \frac{\varepsilon_c}{\varepsilon_c - 1} \frac{E_t \sum_{k=0}^{\infty} \beta^k \theta_c^k \left(\frac{\zeta_{c,t+k}}{P_{c,t+k}} \right) (P_{c,t+k})^{\varepsilon_{c,t}} \gamma_{c,t+k} C_{t+k}}{E_t \sum_{k=0}^{\infty} \beta^k \theta_c^k (P_{c,t+k})^{\varepsilon_{c,t}-1} \gamma_{c,t+k} C_{t+k}}, \tag{18}$$

and

$$p^*_{x,t} = \frac{\varepsilon_x}{\varepsilon_x - 1} \frac{E_t \sum_{k=0}^{\infty} \beta^k \theta_x^k \left(\frac{\zeta_{x,t+k}}{P_{x,t+k}} \right) (P_{x,t+k})^{\varepsilon_{x,t}} \gamma_{x,t+k} X_{t+k}}{E_t \sum_{k=0}^{\infty} \beta^k \theta_x^k (P_{x,t+k})^{\varepsilon_{x,t}-1} \gamma_{x,t+k} X_{t+k}}, \tag{19}$$

where $\zeta_{j,t}$ is the nominal marginal cost in sector j. Finally, the equation describing the dynamics for the aggregate price level in sector $j = c, x$, is given by $P_{j,t} = \left[(1 - \theta_j)(p^*_{j,t})^{1-\varepsilon_j} + \theta_j P_{j,t-1}^{1-\varepsilon_j} \right]^{1/(1-\varepsilon_j)}$.

2.3 Monetary Authority and Market Clearing

The monetary authority conducts monetary policy using the short-term nominal interest rate as the policy instrument. The gross nominal interest rate $R_t^n \equiv 1 + i_t$ follows a Taylor rule of the following type:

$$R_t^n = (R^n)^{(1-\rho_R)}(R_{t-1}^n)^{\rho_R}(\pi_t)^{\rho_\pi(1-\rho_R)}(Y_t^z)^{\rho_Y(1-\rho_R)}\exp(\xi_{R,t}), \tag{20}$$

where $\xi_{R,t}$ is a monetary policy shock, π_t is an economy-wide inflation rate, Y_t^z is the deviation of real GDP from its steady state. While a monetary policy shock is an unanticipated change in money supply in Barsky, House, and Kimball (2003, 2007), we replace it with a Taylor rule. It is widely accepted that the Taylor rule best describes the U.S. monetary policy over 1984-2007 period. To estimate the model, therefore, it seems more appropriate to model the monetary policy shock as an exogenous change in the policy interest rate rather than in the money supply. However, the key results of the paper go through if the monetary policy shock is introduced as an unanticipated change in the money supply.

The economy-wide total output (real GDP) Y_t is given by

$$Y_t = \frac{P_{c,t}}{P_t}C_t + \frac{P_{x,t}}{P_t}X_t, \tag{21}$$

where P_t is the aggregate price index (GDP deflator). Following Erceg and Levin (2006), we define the aggregate price index P_t as

$$P_t = P_{c,t}^{\omega_c}P_{x,t}^{1-\omega_c} \tag{22}$$

where ω_c is the steady state output share of the nondurable goods sector. This aggregate price index P_t is used for calculating an economy-wide inflation.

Finally, in equilibrium net private debt $B_t = 0$ and the labor and capital markets equilibriums require

$$L_t = L_{c,t} + L_{x,t} \quad \text{and} \quad \overline{K} = \overline{K}_{c,t} + \overline{K}_{x,t}, \tag{23}$$

where $L_{j,t} = \int l_{j,t}(s)ds$ and $\overline{K}_{j,t} = \int \overline{k}_{j,t}(s)ds$ is labor and the stock of capital used in sector $j = c, x$.

3 Analytical Discussion

In this section, we analytically show that the response of a two-sector sticky price model with flexibly priced durable goods to monetary policy shocks depends crucially on how we assume separability between aggregate consumption and labor in preferences. In particular, we find that the larger the non-separability displayed by the utility function, the more likely the model is to generate sectoral comovement in response to monetary policy shocks. We then show that variable capital utilization with imperfect capital mobility expands the threshold level of non-separability needed to generate the sectoral comovement. Since we focus on the reaction of the model to a monetary policy shock, we assume that there are no technology shocks, discount factor shock, and durable preference shock in this analytical discussion. We will allow these shocks to operate in the estimation.

3.1 Why does non-separability affect the behavior of the model?

We analytically present the underlying mechanism through which non-separability affects the behavior of a two-sector sticky price model with flexibly priced durable goods. In so doing, it is useful to rewrite the labor supply condition (8) in the following manner:

$$-U_L(C_t, D, \underbrace{L_{x,t} + L_{c,t}}_{L_t}) = \gamma_x \frac{W_t}{P_{x,t}} = \frac{\gamma_x}{\mu \zeta_{x,t}} W_t, \tag{24}$$

where the last equality is implied by the fact that the assumption of sticky prices in the durable sector is dropped and thus the flexible price of durables is a constant markup (μ) over its marginal cost: $P_{x,t} = \mu \zeta_{x,t}$. Note that we drop the time script of D_t and $\gamma_{x,t}$ in the equation. Barsky, House, and Kimball (2003, 2007) show that the stock-flow ratio is high so that even large changes in purchases following a monetary shock have only minor effects on the total quantity of the durable good. Small deviations from the steady state of the economy virtually do not alter the stock of durables, and thus their shadow value ($\gamma_{x,t}$) is nearly constant at cyclical frequencies.

The nominal marginal cost $\zeta_{x,t}$ is the cost of hiring an additional unit of a productive input multiplied by the number of inputs required to produce an additional unit of durable goods. For now, let us assume that both physical capital and labor is perfectly mobile across sectors but the capital utilization rate in each sector remains constant. This corresponds to the case considered by Barsky, House, and Kimball (2007). Because the production functions in both sectors have constant returns to scale, and because physical capital and labor can flow freely across sectors, all firms have the same marginal cost and choose the same capital-to-labor ratios. Thus,

$$\zeta_{x,t} = \frac{W_t}{F_2(\overline{K}_{x,t}, L_{x,t})} = \frac{W_t}{F_2(\overline{K}, L_t)} = \frac{W_t}{f(L_t)}, \tag{25}$$

where $f(L_t) = F_2(\overline{K}, L_t) = (K/L_t)^\alpha$. Combining (24) and (25) yields

$$-U_L(C_t, D, \underbrace{L_{x,t} + L_{c,t}}_{L_t}) = \frac{\gamma_x}{\mu} f(\underbrace{L_{x,t} + L_{c,t}}_{L_t}). \tag{26}$$

This equation shows that the nature of the comovement problem is closely related to the separability between nondurable consumption and labor. First, suppose that the preference is separable ($\sigma = 1$). In this case, the marginal disutility from labor is only a function of labor, so that (26) becomes

$$v'(\underbrace{L_{x,t} + L_{c,t}}_{L_t}) = \frac{\gamma_x}{\mu} f(\underbrace{L_{x,t} + L_{c,t}}_{L_t}). \tag{27}$$

This equation says that if the production of nondurables rises, then employment in the durable sector must fall in a model with separable preferences. The intuition behind this result is straightforward. Following a monetary expansion, nondurable goods firms with sticky prices increase production to meet demand instead of raising their prices. The incipient increase in output in the nondurable goods sector increases the demand for labor, which in turn raises marginal costs (i.e., an increase in disutility of work $v^{'}(\cdot)$ and a decrease in marginal product of labor $f(\cdot)$). For producers in the flexibly priced durable goods sector, the increase in marginal costs is merely an adverse supply shock. Because there are no forces that can offset a rise in the cost of production, this definitely lowers the labor employed in the durable goods sector and thus the production of durable goods falls.

However, things will be different in a model with non-separable preferences. When $\sigma < 1$, the cross-partial derivative, $-U_{LC}$ is negative in our monetary utility function, (4), which means that the increased level of nondurable consumption would reduce the marginal disutility of work. According to (26), the fact that the cross-partial derivative $-U_{LC} < 0$ implies that increased nondurable consumption shifts the labor supply curve out. This mitigates the rise in the nominal wage and marginal cost of producing durables and therefore it moderates the contraction in durable production. Since the complementarity between the consumption of nondurables and labor is decreasing with the parameter σ (i.e.,$\partial \left|-U_{LC}\right| / \partial\sigma < 0$), the extent to which production in the durable sector contracts gets smaller as σ takes lower values than unity.

3.2 What determines the range of the σ consistent with sectoral comovement?

It is instructive to log-linearize the equation (26) around a deterministic steady state to understand what factors determine the range of the parameter σ in which the model generates sectoral comovement. Define $\eta_{LL} \equiv \left(\frac{-U_{LL}L}{-U_L}\right)\Big|_{ss} > 0$ as the own elasticity of marginal disutility from labor and $\eta_{LC} \equiv \left(\frac{-U_{LC}C}{-U_L}\right)\Big|_{ss}$ as the cross-elasticity of marginal disutility from labor with respect to nondurable consumption, evaluated at the steady state. These elasticities are expressed as

$$\eta_{LL} = \frac{(1-\sigma)^2}{\sigma}\frac{WL}{P_C C}\frac{\psi_c C^{1-1/\rho}}{\psi_c C^{1-1/\rho} + \psi_d D^{1-1/\rho}} + \frac{1}{\eta} \tag{28}$$

and

$$\eta_{LC} = \left(1 - \frac{1}{\sigma}\right)\frac{\psi_c C^{1-1/\rho}}{\psi_c C^{1-1/\rho} + \psi_d D^{1-1/\rho}}, \tag{29}$$

respectively. Log-linearizing (26) around a non-stochastic steady state yields

$$\eta_{LC}\widehat{C}_t + \eta_{LL}(\omega_c \widehat{L}_{c,t} + \omega_x \widehat{L}_{x,t}) = -\alpha(\omega_c \widehat{L}_{c,t} + \omega_x \widehat{L}_{x,t}), \tag{30}$$

where a circumflex ("hat") over a variable represents proportionate deviations of that variable from its steady state and $\omega_j = L_j/L$ in sector $j = c, x$. Using the fact that all firms choose the same capital-to-labor ratio, we can write $C_t = \overline{K}_{c,t}^{\alpha} L_{c,t}^{1-\alpha} = \left(\frac{\overline{K}}{L_t}\right)^{\alpha} L_{c,t}$ and log-linearize it as follows:

$$\widehat{C}_t = \kappa_c \widehat{L}_{c,t} - \omega_x \widehat{L}_{x,t}. \tag{31}$$

Here $\kappa_c \equiv \frac{\partial \widehat{C}_t}{\partial \widehat{L}_{c,t}}$ is the elasticity of nondurable production with respect to labor in the nondurable sector and is equal to $(1 - \alpha\omega_c)$ when both physical capital and labor are perfectly mobile across sectors but capital utilization rate remains constant. Combining equation (31) with equation (30) yields

$$(-\eta_{LC}\alpha + \eta_{LL} + \alpha)\,\omega_x \widehat{L}_{x,t} = (-\eta_{LC}\kappa_c - (\eta_{LL} + \alpha)\omega_c)\,\widehat{L}_{c,t}. \tag{32}$$

This equation confirms a previous discussion that unless labor supply and the consumption of nondurables are complementary (i.e., $\eta_{LC} < 0$), it is impossible to obtain sectoral comovement. Given that $\eta_{LC} < 0$, the condition that generates the sectoral comovement is

$$-\,\eta_{LC}\kappa_c > \nu, \tag{33}$$

where $\nu = (\eta_{LL} + \alpha)\omega_c$. This condition has an intuitive interpretation. As discussed, when $L_{c,t}$ rises to meet higher demand in the nondurable goods following a monetary expansion, it has two offsetting effects on the costs of durable good production. The first term, $-\eta_{LC}\kappa_c$, quantifies the extent to which an increase in $L_{c,t}$ lowers costs of durable good production through the complementarity between labor supply and nondurable consumption. Higher values for κ_c strengthen the effects of the complementarity on lowering costs of durable good production. The second term, ν, measures the extent to which an increase in $L_{c,t}$ raises costs of durable good production by inducing a higher disutility of work ($\eta_{LL}\omega_c$) and a lower marginal product of labor ($\alpha\omega_c$). When the former dominates the latter, a positive response of durable goods production is obtained.

More importantly, the condition above clearly identifies what factors determine the range of the complementarity consistent with sectoral comovement. Higher values of κ_c and lower values of ν enable the model to generate sectoral comovement with a smaller degree of the complementarity, $-\eta_{LC}$. Below, we show that variable capital utilization with imperfect capital mobility increases the value of κ_c and lowers the value of ν.

To understand why imperfect capital mobility and variable capital utilization lead to a wider range of σ that generates sectoral comovement, it is useful to inspect something analogous to (26) in this case, which is given by

$$-\,U_L(C_t, D, L_{x,t} + L_{c,t}) = \frac{\gamma_x}{\mu}\left(\frac{u_{x,t}\overline{K}_x}{L_{x,t}}\right)^{\alpha}. \tag{34}$$

With imperfect capital mobility, labor in the nondurable sector has no impact on the marginal product of labor in the durable sector. Increased labor demand in the nondurable sector raises the cost of production in the durable sector only through a higher disutility of work. Hence, ν, which measures the extent to which an increase in $L_{c,t}$ raises the costs of durable production, decreases from $(\eta_{LL}+\alpha)\omega_c$ to $\eta_{LL}\omega_c$. Thus, a smaller complementarity is required to offset the rise in costs of production.

In addition, a variable rate of capital utilization also helps expand the range of the parameter σ that is consistent with sectoral comovement since it increases the value for $\kappa_c \equiv \frac{\partial \widehat{C}_t}{\partial \widehat{L}_{c,t}}$. Variable capital utilization makes the supply of capital services strongly responsive to changes in the labor so that it is sometimes described as leading to short-run production that is nearly linear in labor.[7] As Shapiro (1993) shows, for example, increases in labor are accompanied by increases in the workweek of capital, one measure of capital utilization. Loosely speaking, this observation allows us to write the production function as

$$C_t = (u_{c,t}\overline{K}_c)^{\alpha} L_{c,t}^{1-\alpha} = \left(\frac{u_{c,t}\overline{K}_c}{L_{c,t}}\right)^{\alpha} L_{c,t} \simeq BL_{c,t},$$

where $B = \left(\frac{u_c\overline{K}_c}{L_c}\right)^{\alpha}$. Thus, variable capital utilization might increase κ_c, the elasticity of non-durable goods production with respect to $L_{c,t}$, from $(1-\alpha\omega_c)$ to 1.

Obviously, the extent to which variable capital utilization increases κ_c depends on the costliness of varying capital utilization, which is controlled by the parameter χ. When varying utilization becomes more costly (i.e., higher χ), an increase in κ_c tends to be smaller, so that a stronger complementarity is required for the model to obtain sectoral comovement.

4 Bayesian Estimation

In this section, we estimate our two-sector sticky price model, using a Bayesian approach. First, we show that non-separable preferences are supported by the data and significantly improves the overall empirical performance of the model. We then show that our estimated model with non-separable preferences generates the sectoral comovement in response to a monetary shock.

4.1 Empirical Specification and Data

While our previous analytical results demonstrate that non-separable preferences are important for resolving the comovement problem in a two-sector sticky price model with flexibly priced durable goods, they are not the only way to generate the sectoral comovement. As DiCecio (2009) and Iacoviello and Neri (2010) show, sticky nominal wages can be an important ingredient to explain the comovement across sectors. In order to avoid the possibility of exaggerating the role of non-separable preferences in a two-sector sticky price model, we introduce sticky wages into our baseline estimation. We assume that households are monopolistic suppliers of labor and firms employs a CES aggregator of household labor with an elasticity of substitution equal to ε_w. In particular, the

[7] Previous papers that have used this type production function include Bils and Cho (1994) and Ramey and Shapiro (1998).

labor aggregator, L_t, takes the form of

$$L_t = \left[\int_0^1 L_t(s)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} ds\right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}}, \tag{35}$$

where $L_t(s)$ is the labor supply of a representative household s. We follow Kim (2000) by modeling nominal wage rigidity through the cost of adjusting nominal wages. The wage adjustment cost function is assumed to be quadratic and zero at the steady state.[8] The nominal total adjustment cost for a representative household s is given by

$$\mathrm{AC}_t^w(s) = \frac{\phi_w}{2}\left(\frac{W_t(s)}{W_{t-1}(s)} - 1\right)^2 W_t(s), \tag{36}$$

where ϕ_w is the adjustment cost scale parameter for the wage. This nominal adjustment cost, AC_t^w, enters the budget constraint of the household, (6).

In our estimation, we consider including the following observables:

$$\begin{aligned}
dc_t &\equiv \log\left(\frac{C_t}{C_{t-1}}\right) = \text{Per capita nondurable consumption growth} \\
dx_t &\equiv \log\left(\frac{X_t}{X_{t-1}}\right) = \text{Per capita durable consumption growth} \\
\pi_{c,t} &\equiv \log\left(\frac{P_{c,t}}{P_{c,t-1}}\right) = \text{Nondurable price inflation} \\
\pi_{x,t} &\equiv \log\left(\frac{P_{x,t}}{P_{x,t-1}}\right) = \text{Durable price inflation} \\
dw_t &\equiv \log\left(\frac{w_t}{w_{t-1}}\right) = \text{Real wage growth} \\
h_t &\equiv \text{Log per capita hours worked} \\
f\!f_t &\equiv \text{Federal funds rate (quarterly rate)}
\end{aligned}$$

We define the nondurable consumption expenditure as the sum of nondurables and services, excluding expenditure on housing and utility. Durables consumption expenditure consists of consumer durable spending, housing and utility expenditure, and residential investment. Based on these definitions, we construct dc_t, dx_t, $\pi_{c,t}$, and $\pi_{x,t}$ using data from the NIPA tables. Details are explained in the Appendix.

We retrieve the effective Federal funds rate (FEDFUNDS), Hours of All Persons (Nonfarm Business Sector, HOANBS), Compensation Per Hour (Nonfarm Business Sector, COMPNFB), GDP deflator (GDPDEF), and Total Population (POP) from FRED of the St. Louis Fed. The

[8] Alternatively, we could follow Erceg, Henderson, and Levin (2000) by modeling nominal wage rigidity in the Calvo fashion. However, doing so substantially complicates the solution of the model whose preferences are non-separable between aggregate consumption and labor. In other words, the degree of heterogeneity across households resulting from different labor supply, which is in turn due to non-separable preferences, is significantly increased. Each household would choose a different level of nondurable consumption, hours worked, and durable spending. To get around this issue, we adopt a quadratic adjustment cost of nominal wage, which allows for a symmetric equilibrium.

monthly Federal Funds rate is converted to quarterly observations by taking the quarterly averages. The annualized percentage Fed funds rate is converted to quarterly rate (decimal number) by dividing it by 400. We deflate the nonfarm business sector compensation per hour by the GDP deflator to get the real wage index. The monthly population data is converted to quarterly by taking the end-of-quarter observations. All of these data are demeaned. The sample starts from 1959:Q2 and ends at 2010:Q4.

Ideally, we wish to include the quality-adjusted data on the stock of consumer durable goods as observables in the estimation. However, the quality-adjusted series on consumer durable goods are difficult to measure and the ones constructed by Gordon (1990) are only available up to 1983:Q4. Recently, Cummins and Violante (2002) update the Gordon's quality-adjusted series all the way to 2000 for producer durable goods such as equipment and software, not for consumer durable goods. To get around the problem with the data on the consumer durable stock, we use the expenditure data on consumer durable goods spending (X_t). As pointed out by Guerron-Quintana (2010), a choice of observables could affect the estimation outcomes. However, we think that the use of the flow variable conveys sufficient information on estimating our parameters of interest.

While we have seven observable variables, there are six structural shocks in the model (aggregate and two sectoral technology shocks, discount factor shock, durable-good preference shock, and monetary policy shock). In order to avoid the stochastic singularity problem, we include measurement errors, as in Sargent (1989). The model variables are related to the observable variables through the following measurement equations:

$$dc_t = \hat{C}_t - \hat{C}_{t-1}, \tag{37}$$

$$dx_t = \hat{X}_t - \hat{X}_{t-1} + e_{x,t}, \tag{38}$$

$$\pi_{c,t} = \pi_t^c, \tag{39}$$

$$\pi_{x,t} = \pi_t^x + e_{\pi_x,t}, \tag{40}$$

$$dw_t = \hat{W}_t - \hat{W}_{t-1} + e_{w,t}, \tag{41}$$

$$h_t = \hat{L}_t, \tag{42}$$

$$ff_t = \hat{R}_t^n, \tag{43}$$

where $\pi_t^c = P_{c,t}/P_{c,t-1}$, $\pi_t^x = P_{x,t}/P_{x,t-1}$, and $e_{x,t}$, $e_{\pi_x,t}$, and $e_{w,t}$ are measurement errors associated with durable consumption growth, durable price inflation, the real wage growth, respectively. Those measurement errors are assumed to be uncorrelated with each other at any lags and leads. Even though we have carefully constructed durable expenditure data, the underlying price indices are subject to unaccounted changes in the quality. This motivates us to include measurement errors for dx_t and $\pi_{x,t}$. Another measurement error is included in the real wage growth. It is because the compensation-based wage growth is more volatile and noisier than the earnings-based wage growth and thus it is likely to be subject to measurement errors.

4.2 Prior

We impose the following parameter values that are used in Barsky, House, and Kimball (2007), so that our results are comparable to their specification. We set $\beta = 1.02^{-0.25}$ and $\delta = 0.0125$, such that the annual discount rate is 2% and the annual depreciation rate becomes 5% per year. The capital share in the economy α is set to 0.33. We set η to 1, which corresponds to the Frisch labor supply elasticity when preferences are separable.[9]

The values of ε_c and ε_x equal to 11, so that the steady-state markup becomes 10% in both the nondurable and durable sectors. Finally, setting $\omega_c = 0.75$ results in the nondurable sector accounting for 75% of GDP in a steady state. Furthermore, we set $\varepsilon_w = 7.67$, implying a steady state wage markup of 15%. In addition, since the discount factor shock (b_t) and the durable preference shock (v_t) enter the first-order conditions linearly, we impose $\rho_v = 0$ to avoid a potential identification issue.

We impose the Calvo parameters such that there are the sticky-price nondurable sector and flexible-price durable sector. Specifically, we set $\theta_c = 2/3$ and $\theta_x = 0$. This corresponds to the case in which the comovement problem arises in Barsky, House, and Kimball (2007). Iacoviello and Neri (2010) also assume sticky nondurable prices and flexible durable prices. In this sense, the estimated parameter values are conditional on these Calvo parameters.

Parameters to be estimated and the associated prior distributions are summarized in Table 1. The prior distribution for $1/\sigma$, which is an inverse of intertemporal elasticity of substitution, is adopted from Smets and Wouters (2007). The prior mean for χ, which is the curvature parameter for utilization costs, is set to 0.01, which is the value used in Christiano, Eichenbaum, and Evans (2005). The prior mean for ρ, which is the elasticity of substitution between nondurable and durable consumption, is set based on the GMM estimate and the corresponding standard error in Ogaki and Reinhart (1998). We use the Gamma distribution with mean of 50 and the standard deviation of 20 for the prior distribution for ϕ_w. This prior is the same as the one employed in Gerali, Neri, Sessa, and Signoretti (2010). With the separable preferences, the prior mean can be roughly translated to the Calvo wage parameter being 0.358. Other priors are standard. Priors for the persistence parameter, the Taylor rule coefficients, and standard deviations of the structural shocks are all adopted from Smets and Wouters (2007). We set the priors for the measurement errors to be the same as those for the structural shocks.

[9] As Guerron-Quintana (2008) shows, the non-separability affects the Frisch elasticity. When non-separable preferences include nondurable and durable goods, the Frisch labor supply elasticity (FE) is given by

$$\mathrm{FE} = \left[\left(\eta_{LL} - \frac{1}{\eta} \right) \frac{Z_c - 1}{(1 - Z_c)\frac{\rho}{\sigma} - 1} + \frac{1}{\eta} \right]^{-1}, \tag{44}$$

where $0 < Z_c \equiv \frac{\psi_c C^{1-1/\rho}}{\psi_c C^{1-1/\rho} + \psi_d D^{1-1/\rho}} < 1$. As the preferences become separable ($\sigma \to 1$), the Frisch elasticity converges to η because $\eta_{LL} \to \frac{1}{\eta}$ as in (28). As the utility function becomes more non-separable (i.e., as σ gets lower), the Frisch elasticity declines. This is consistent with Guerron-Quintana (2008). Also, the degree of the complementarity between nondurable and durable goods in the utility function (ρ) affects the Frisch elasticity. As nondurable and durable goods become more complements (i.e., $\rho \to 0$), the Frisch elasticity decreases.

Table 1: Parameters and Prior Distributions

Parameter	Description	Distribution	Mean	Std. Dev.
$1/\sigma$	Inverse of intertemporal elasticity of substitution	Normal	1.50	0.37
χ	Curvature of $a(u)$, $\chi \equiv a''(1)/a'(1)$	Inverse Gamma	0.01	2
ρ	Elasticity of substitution between C_t and D_t	Normal	1.17	0.1
θ_c	Calvo parameter for nondurable firms	Beta	0.5	0.1
θ_x	Calvo parameter for durable firms	Beta	0.5	0.1
ϕ_w	Wage adjustment cost parameter	Gamma	50	20
ρ_A	Persistence of aggregate productivity shock	Beta	0.5	0.2
ρ_{A_c}	Persistence of nondurable-specific technology shock	Beta	0.5	0.2
ρ_{A_x}	Persistence of durable-specific technology shock	Beta	0.5	0.2
ρ_b	Persistence of discount factor shock	Beta	0.8	0.1
ρ_R	Interest smoothing coefficient in the Taylor rule	Beta	0.75	0.1
ρ_π	Coefficient on the inflation rate in the Taylor rule	Normal	1.5	0.25
ρ_Y	Coefficient on the output gap in the Taylor rule	Normal	0.12	0.05
σ_ξ	Std. Dev. of aggregate technology shock	Inverse Gamma	0.05	2
σ_{ξ_c}	Std. Dev. of sectoral technology shock in nondurable	Inverse Gamma	0.05	2
σ_{ξ_x}	Std. Dev. of sectoral technology shock in durable	Inverse Gamma	0.05	2
σ_{ξ_b}	Std. Dev. of discount factor shock	Inverse Gamma	0.05	2
σ_{ξ_b}	Std. Dev. of durable-good preference	Inverse Gamma	0.05	2
σ_{ξ_R}	Std. Dev. of monetary policy shock	Inverse Gamma	0.05	2
σ_{e_π}	Std. Dev. of measurement error for $\pi_{x,t}$	Inverse Gamma	0.05	2
σ_{e_X}	Std. Dev. of measurement error for dx_t	Inverse Gamma	0.05	2
σ_{e_W}	Std. Dev. of measurement error for dw_t	Inverse Gamma	0.05	2

4.3 Estimation Results for the Baseline Specification

The solution to the log-linearized system of equations characterizing the equilibrium conditions in the model are expressed in the form of the first-order difference equations. The first-order difference equations and the observation equations (37)–(43) together form the state-space model that we will estimate with a standard Bayesian approach, which is explained in detail by An and Schorfheide (2007) and Fernández-Villaverde (2009), for example. Some details are outlined in the Appendix. We first numerically find the posterior modes, and the posterior distributions of parameters of our interest are obtained from the random-walk Metropolis-Hastings algorithm with 300,000 draws (the first 10% of draws are discarded as a burn-in period). We set the scaling parameter such that the average acceptance rate is about 30%.

4.3.1 Posterior Distributions of the Parameter

Table 2 reports the posterior distributions of parameters for the baseline specification that incorporates the wage adjustment cost. The columns under the label of *Non-Separable Sticky Wage* show the posterior means and the 90% credible set of parameters when σ is allowed to be estimated. The columns under the label of *Separable Sticky Wage* present the posterior distributions when $\sigma = 1$.

The posterior mean of $1/\sigma$ is 2.2085. The associated 90% credible set is from 1.9028 to 2.5063. The implied elasticity of intertemporal substitution (σ) is 0.4527. The separable preferences (i.e.,

Table 2: Posterior Distributions of the Parameters for the Baseline Specification

		Posterior Distributions					
		Non-Separable Sticky Wage			*Separable Sticky Wage*		
Parameter	Prior Mean	Mean	5%	95%	Mean	5%	95%
$1/\sigma$	1.500	2.2085	1.9028	2.5063	1	n.a.	
χ	0.010	24.2978	4.6963	47.7969	2.4914	1.3512	3.6176
ρ	1.170	1.6244	1.5171	1.7288	1.4122	1.2680	1.5518
ϕ_w	50	287.8196	281.6112	294.2184	293.9376	292.8607	295.1809
ρ_A	0.500	0.9586	0.9487	0.9683	0.9426	0.9313	0.9545
ρ_{A_c}	0.500	0.6926	0.6361	0.7496	0.7251	0.6431	0.8051
ρ_{A_x}	0.500	0.9814	0.9728	0.9901	0.9724	0.9633	0.9811
ρ_b	0.800	0.9982	0.9971	0.9995	0.9969	0.9953	0.9984
ρ_R	0.750	0.1739	0.1229	0.2229	0.1991	0.1334	0.2614
ρ_π	1.500	1.0433	1.0371	1.0488	1.0403	1.0309	1.0495
ρ_Y	0.120	-0.0159	-0.0214	-0.0103	-0.0238	-0.0301	-0.0173
σ_ξ	0.050	0.0075	0.0065	0.0083	0.0095	0.0085	0.0105
σ_{ξ_c}	0.050	0.0092	0.0079	0.0105	0.0086	0.0070	0.0100
σ_{ξ_x}	0.050	0.0077	0.0066	0.0088	0.0090	0.0076	0.0105
σ_{ξ_R}	0.050	0.0063	0.0059	0.0068	0.0066	0.0060	0.0072
σ_{ξ_b}	0.050	0.1729	0.0648	0.2888	0.0554	0.0271	0.0841
σ_{ξ_v}	0.050	0.0062	0.0059	0.0066	0.0080	0.0066	0.0094
σ_{e_π}	0.050	0.0060	0.0059	0.0061	0.0060	0.0059	0.0061
σ_{e_X}	0.050	0.0213	0.0173	0.0253	0.0397	0.0343	0.0451
σ_{e_W}	0.050	0.0064	0.0059	0.0068	0.0071	0.0065	0.0077
Log Marginal Densities			4672.49			4634.50	

Note: The posterior distributions are obtained using the random walk Metropolis-Hastings algorithm with 300,000 draws (the first 10% of draws are discarded as a burn-in period). We use the modified Harmonic mean estimator of Geweke (1999) to obtain the log marginal density.

$1/\sigma = 1$) are clearly outside the credible set and the data strongly support the non-separability between consumption and leisure. This finding is in line with earlier studies, such as Basu and Kimball (2002) and Guerron-Quintana (2008). Our Bayesian estimate of the elasticity of intertemporal substitution is somewhere between 0.5 from the single-equation estimate of Basu and Kimball (2002) and 0.12 of Guerron-Quintana (2008) with the minimum distance estimator.

The posterior mean of the wage adjustment cost parameter (ϕ_w) is fairly large in contrast to the prior mean or the posterior mean reported in Gerali, Neri, Sessa, and Signoretti (2010). It is possible to translate the wage adjustment cost parameter into the Calvo parameter for wage rigidity when preferences are separable. Given the values for $\varepsilon_w = 7.67$ and $\phi_w = 287.8196$, the implied Calvo parameter for wage rigidity is about 0.643 when preferences are separable. This is slightly smaller than typical estimates.

Hence, our estimated baseline model indicates that both the degrees of non-separability and wage rigidity are substantial in the data even when they are jointly estimated. This suggests that two possible alternatives for resolving the comovement problem, the non-separable and the sticky wages, are all important elements in a two-sector sticky price with flexibly priced durables. While sticky wages have been considered important in the two-sector sticky price model, the novel feature of our paper is to unveil the importance of non-separable preferences in the model.

The posterior mean of the elasticity of substitution between nondurable and durable consumption is 1.6244 with the non-separable preferences and is 1.4122 with the separable preferences. The 90% credible set for the elasticity of substitution between nondurable and durable goods lies above unity irrespective of whether preferences are separable or not. This suggests that nondurable and durable are substitutes, which is line with earlier studies by Ogaki and Reinhart (1998) and Piazzesi, Schneider, and Tuzel (2007).

Using the posterior distributions of σ and ρ, we can calculate the implied posterior mean of the Frisch labor elasticity, given in (44). The resulting posterior mean of the Frisch elasticity is 0.9650 with the 90% probability interval ranging from 0.9571 to 0.9733.

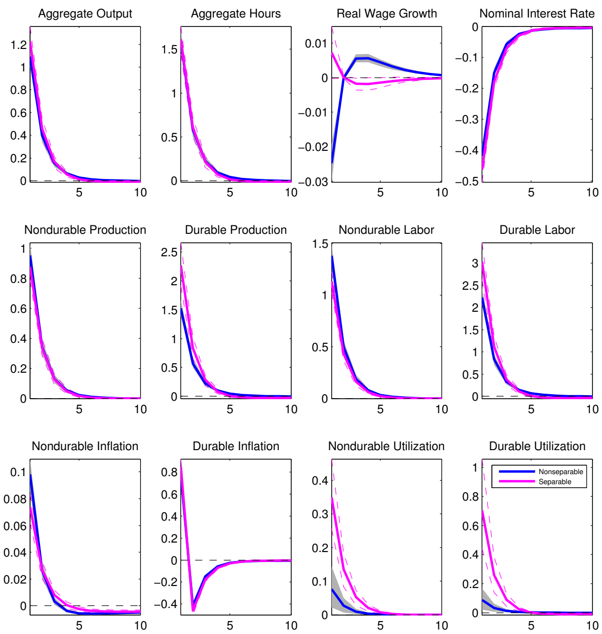
The curvature parameter for $a(u)$ is estimated to be 24.2978 with the non-separable preferences. The posterior mean is 2.4914 with the separable preferences, suggesting that the cost of variable capital utilization is much less costly under the separable preferences. In general, the difference in preferences does not alter posterior means of parameters. However, the utilization cost parameter is heavily affected by the preference specification.

The posterior means suggest that the aggregate technology shock and discount factor shock appear to be very persistent. The persistence parameter for the nondurable-sector technology shock is estimated to be much less persistent.

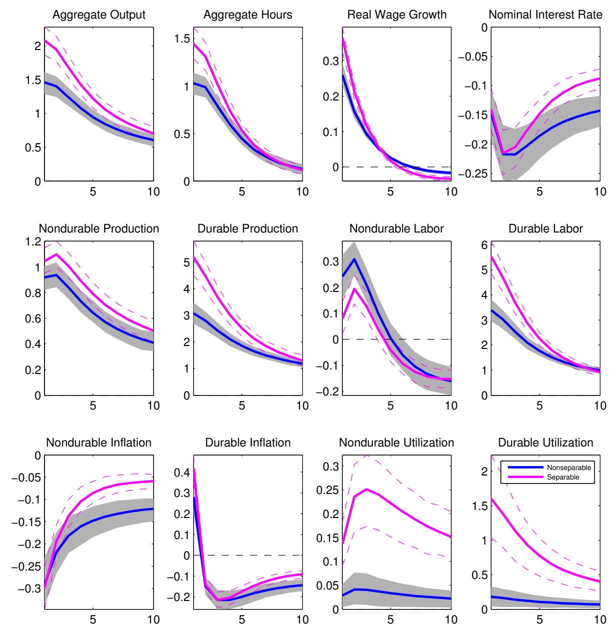
The posterior means of the interest smoothing coefficient (ρ_R) and the inflation rate (ρ_π) show less interest rate smoothing and weaker stance to the inflation in the monetary policy rule than typically reported in the literature. The estimated coefficient on the output gap is somewhat troublesome for models with both non-separable and separable preferences. Unlike what is typically reported in the literature, the posterior means of output gap coefficients take negative values. Even though the magnitude is very close to zero, the credible set does stay negative.

The posterior means of the standard deviations of the structural shocks and measurement errors are relatively unaffected by the restriction on preferences, except for the discount factor shock. The standard deviation of the discount factor shocks is substantially much larger with non-separable preferences than the one with separable preferences.

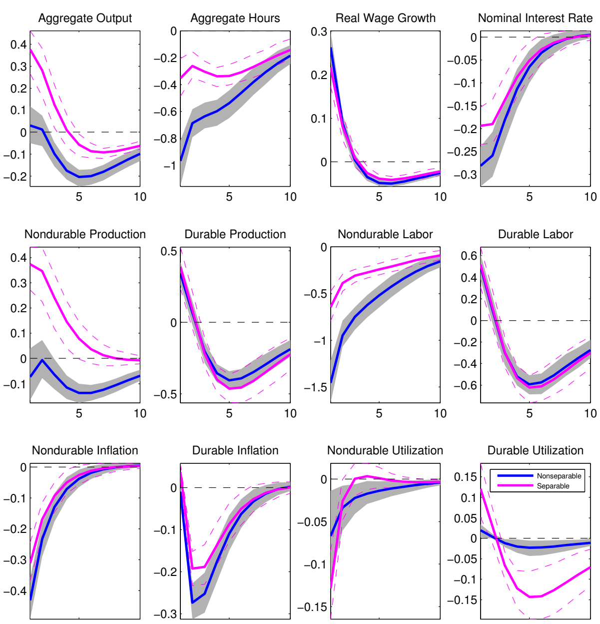
Figure 1 presents the estimated responses of the economy to different types of shocks. Each plot compares dynamic behavior of variables with and without the non-separability. Solid lines represent the posterior mean responses and the shaded areas (dashed lines for the separable preferences) correspond to the 5% and 95% posterior intervals. We are particularly interested in responses of variables to the expansionary monetary policy shock (i.e., one standard deviation decrease in the interest rate), which are shown in Figure 1(a). Responses to the monetary shock are qualitatively and quantitatively quite similar regardless of preference, especially sectoral outputs and labor, which show positive comovement. The only noticeable difference is the response of real wage growth. With the non-separable preferences, the real wage growth responds negatively at the



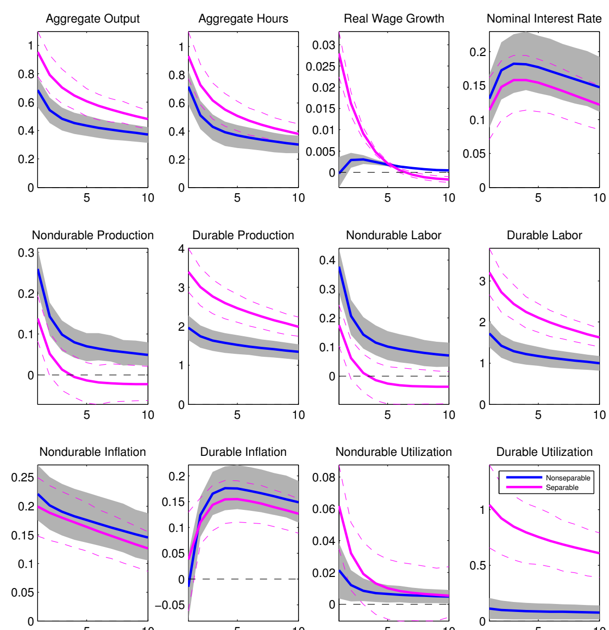
(a) Monetary Shock



(b) Aggregate TFP Shock



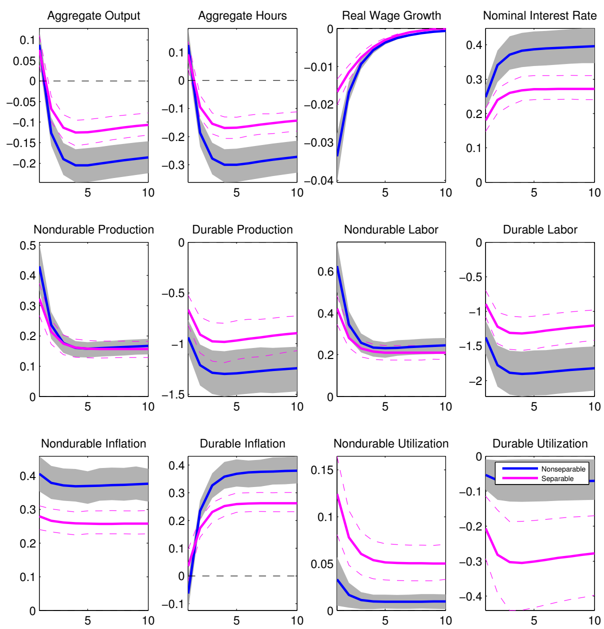
(c) Consumption Sector TFP Shock



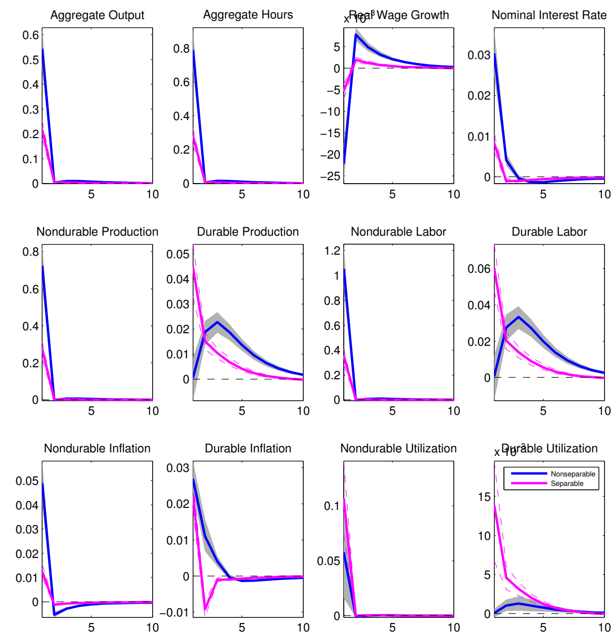
(d) Investment Sector TFP Shock

Figure 1: Impulse Response Functions in the Baseline Specification

Note: The horizontal axes take the quarters after the shock and the vertical axes measure percentage deviations from the steady state. The above figures depict the responses of the economy to the one standard-deviation expansionary monetary policy shock. The solid lines represent the posterior mean responses, and the shaded areas (or dashed lines for the separable case) correspond to the 5% and 95% posterior intervals.



(e) Discount Factor Shock



(f) Durable Preference Shock

Figure 1: Impulse Response Functions in the Baseline Specification (Cont.)

impact period and starts to increase from the second period on, whereas the real wage growth shows the opposite dynamics, initially responding positively and decreasing afterward. This is because the increased nondurable consumption reduces the marginal disutility of labor supply in the non-separable preferences, which makes households set lower nominal wages on impact. In the estimated model, the effect of this complementarity turns out to dominate the effect of future increase in nominal wages, which makes households increase their nominal wages on impact, so that real wages initially decline in the non-separable preferences. The differences in responses of the variable capital utilization in both sectors are driven by the significantly different estimates of the cost of utilization.

Responses to other types of shocks presented in Figures 1(b)–1(f) are also qualitatively similar regardless of the specification of preferences. There are some differences across preference specifications. In response to the consumption-sector TFP shock, nondurable production moves negatively with the non-separable preferences, whereas it does positively with the separable preferences. As a result, aggregate output can respond positively to the consumption sector TFP shock for a couple of periods. Another thing to note is that once the durable preference shock hits the economy, durable labor shows hump-shaped responses with the non-separable preferences. Consequently, the durable production also shows hump-shaped responses. However, we observe very monotonic responses of durable labor and production with separable preferences. Except for these two, different specifications of preferences yield qualitatively very similar results.

4.3.2 Bayesian Model Comparison

It is in our interest to examine whether the data support the non-separability even when nominal wage rigidity is explicitly incorporated in the estimation. This is particularly important because the dynamic behavior of the models is qualitatively very similar. By using the Bayes factor, we can formally compare the model with and without the non-separability. The Bayes factor can tell us the strength of evidence provided by the data. As discussed above, the majority of parameters are qualitatively unaffected by the separable preferences.

With the Bayes factor, we can compare the model with the non-separable preferences against the one with the separable preferences. We use the modified Harmonic mean estimator of Geweke (1999) to obtain the log marginal density. The log marginal density of the model with the non-separable preferences is 4672.49. On the other hand, the one associated with the separable preferences is 4634.50. In order to choose the separable preferences over the non-separable preferences, we need a prior probability of the separable preferences $\exp(37.99) \approx 3.15 \times 10^{16}$ times larger than that over the non-separable preferences. This difference seems to be overwhelmingly large and suggests that the data decisively support the non-separable preferences.

4.3.3 Posterior Second Moments

To better understand why the model with non-separable preferences outperforms the one with separable preferences, we compare posterior standard deviations, posterior autocorrelations, correlation between nondurable and durable spending, and correlation between nondurable and durable price inflation predicted with the data.

The top panel of Table 3 reports posterior standard deviations of the observable variables. The most noticeable difference between the models with non-separable and separable preferences (reported in the second and third columns of Table 3) is volatility of durable output. The model with non-separable preferences does a better job than separable preferences. This might be one of the contributing factors to the relatively good data fit of the model with non-separable preferences. Besides this, both models produce a fairly similar pattern. Both models with non-separable preferences and separable preferences tend to generate too much volatility of the observable variables relative to the data, except for hours worked and wage growth. Both models under-predict the volatility of hours worked relative to the data, but match the volatility of wage growth very well.

In terms of matching the autocorrelations of the observable variables in the data, there is not much of a noticeable difference between the model with non-separable preferences and the one with separable preferences. The bottom panel of Table 3 reports posterior autocorrelations of the observable variables. Both models with non-separable preferences and separable preferences explain the autocorrelations of nondurable and durable price inflation, and hours worked fairly well. However, they generate too strong a serial correlation of wage growth and fail to match the persistence in nondurable and durable spending growth.

We now turn our attention to the correlation of quantities and prices in the nondurable and durable sector. The top panel of Table 4 reports unconditional and conditional correlations between

Table 3: Posterior Second Moments

(a) Standard Deviations

	Data	Non-Separable Sticky Wage	Separable Sticky Wage
Nondurable Output	0.0057	0.0152	0.0129
Durable Output	0.0170	0.0360	0.0608
Nondurable Inflation	0.0081	0.0406	0.0287
Durable Inflation	0.0065	0.0416	0.0302
Hours	0.0778	0.0297	0.0302
Wage Growth	0.0060	0.0060	0.0065
Interest Rate	0.0085	0.0425	0.0300

(b) Autocorrelations

	Data	Non-Separable Sticky Wage	Separable Sticky Wage
Nondurable Output	0.4999	-0.2780	-0.1179
Durable Output	0.3037	-0.0813	-0.0751
Nondurable Inflation	0.7873	0.9479	0.9406
Durable Inflation	0.8834	0.8887	0.8080
Hours	0.9896	0.6792	0.7194
Wage Growth	0.0615	0.1510	0.2115
Interest Rate	0.9528	0.9479	0.9388

Note: The top panel presents standard deviations and the bottom panel shows autocorrelations. The first column is standard deviations and autocorrelations of the observable variables. The second and third columns report the posterior standard deviations and autocorrelations of variables with the models with non-separable and separable preferences, respectively.

nondurable and durable good spending growth under different specifications. While the model with separable preferences generates a too strong correlation relative to the data, the one with non-separable preferences makes it very close to the data. This may be another contributing factor to the better performance of the model with non-separable preferences. The bottom panel of Table 4 reports unconditional and conditional correlations between nondurable and durable inflation rates under different specifications. Both models with non-separable and separable preferences overshoot the unconditional correlation observed in the data.

Table 5 reports the fraction of the variance of each variable explained by the shocks in the model and measurement errors. The importance of shocks in explaining variations in the observable variables to some extent varies, depending on the preferences specifications. The demand shocks (the discount factor shock and the durable preference shock) play a somewhat more important role in explaining the real variables in the model with non-separable preferences than the one with separable preferences. For example, the durable preference shock and the discount factor shock account for a larger fraction of nondurable output and hours worked, respectively, in the model with non-separable preferences. In accounting for the nominal variables such as nondurable and

Table 4: Unconditional and Conditional Correlations

(a) Nondurable and Durable Output

		Data	Non-Separable Sticky Wage	Separable Sticky Wage
Unconditional Correlation		0.5444	0.5989	0.7723
Conditional on	TFP		0.9935	0.9809
	Nondurable TFP		-0.2182	0.7877
	Durable TFP		0.9453	0.8800
	Monetary		0.9999	1.0000
	Discount Factor		-0.7378	-0.7578
	Durable Preference		-0.6265	0.9712

(b) Nondurable and Durable Inflation

		Data	Non-Separable Sticky Wage	Separable Sticky Wage
Unconditional Correlation		0.8070	0.9924	0.9609
Conditional on	TFP		0.7934	0.3437
	Nondurable TFP		0.5352	0.4070
	Durable TFP		0.9469	0.9595
	Monetary		0.6993	0.6168
	Discount Factor		0.9983	0.9977
	Durable Preference		0.8483	0.9580

Note: The top panel reports unconditional and conditional correlations between nondurable and durable output under different specifications. The bottom panel shows those between nondurable and durable inflation. The second and third columns show correlations in the models with non-separable and separable preferences, respectively.

durable inflation, the discount factor shock is the most important regardless of different preference specifications. The aggregate TFP shock plays a less important role once the non-separable preferences are introduced. This is especially true in nondurable output and hours.

There is a non-negligible portion of variations in durable output growth and wage growth that cannot be explained by the shocks regardless of preferences. It is not so surprising since models considered here do not have many frictions and different types of shocks, compared to larger scale models, which intend to explain the majority of variations in the data. The fact that about 20% of variations in durable output growth and 60% of variations in wage growth cannot be explained by the model does not imply that our models are completely misspecified. Since variations due to the measurement errors remain almost unchanged regardless of the specification of preferences, it suggests that those measurement errors do not have a significant impact on the importance of the non-separability.

4.3.4 Marginal Densities under Different Assumptions about the Calvo Parameters

Thus far, our empirical analysis is based upon our maintained assumption from Barsky, House, and Kimball (2007) that the nondurable goods sector has sizable price rigidity and that firms in the durable sector are flexible to adjust their prices (i.e., $\theta_c = 0.66$ and $\theta_x = 0$). Here we investigate the

Table 5: Variance Decomposition under Different Specifications (%)

(a) Non-Separable Sticky Wage									
	ξ_t	$\xi_{c,t}$	$\xi_{x,t}$	$\xi_{R,t}$	$\xi_{b,t}$	$\xi_{v,t}$	$e_{\pi_x,t}$	$e_{x,t}$	$e_{w,t}$
Nondurable Output	21.9	0.5	1.9	42.0	3.3	30.4	0	0	0
Durable Output	42.7	1.2	15.9	18.5	2.4	0	0	19.2	0
Nondurable Inflation	2.7	0.9	2.1	0.1	94.3	0	0	0	0
Durable Inflation	3.0	0.6	1.8	3.5	89.9	0	1.2	0	0
Hours	23.5	17.2	17.7	24.5	12.3	4.9	0	0	0
Wage Growth	17.6	12.5	0	0.2	0.2	0.1	0	0	69.5
Interest Rate	2.8	0.6	1.9	0.8	93.9	0	0	0	0
(b) Separable Sticky Wage									
	ξ_t	$\xi_{c,t}$	$\xi_{x,t}$	$\xi_{R,t}$	$\xi_{b,t}$	$\xi_{v,t}$	$e_{\pi_x,t}$	$e_{x,t}$	$e_{w,t}$
Nondurable Output	42.2	7.2	1.0	41.2	2.1	6.2	0	0	0
Durable Output	47.5	0.8	18.1	12.5	0.4	0	0	20.7	0
Nondurable Inflation	3.3	1.8	4.4	0.1	90.5	0	0	0	0
Durable Inflation	4.9	1.2	3.0	9.1	78.6	0	3.1	0	0
Hours	41.2	6.6	28.5	20.5	2.6	0.6	0	0	0
Wage Growth	29.6	10.3	0.2	0	0	0	0	0	60.0
Interest Rate	3.7	1.3	3.3	2.5	89.2	0	0	0	0

Note: Numbers reported here are percentages of variance of each variable explained by the shocks in the model and those due to measurement errors. The first to sixth columns correspond to the structural shocks (aggregate technology, nondurable technology, durable technology, monetary, discount factor, and durable preference, respectively) and the last three columns are measurement errors (for durable inflation, durable output, and wage growth, respectively).

following two questions: i) Does our empirical evidence supporting the non-separable preferences depend on the maintained assumption on the Calvo parameters? ii) Is the assumption of fully flexible durable prices compatible with the data?

To address these questions, we consider estimating two additional versions of our model that feature a different assumption about the Calvo parameters. One is to estimate the Calvo parameters for both nondurable and durable firms, together with other parameters. Prior distributions for the Calvo parameters are the Beta distribution with mean 0.5 and standard deviation of 0.1. This is adopted from Smets and Wouters (2007). The other is to estimate the Calvo parameters for nondurable firms, while we impose price flexibility for durable firms (i.e., $\theta_x = 0$). Table 6 summarizes the posterior distributions of the selected parameters and the resulting log marginal densities, together with the one from our baseline specifications.

Regarding the robustness of our empirical evidence supporting the non-separability, Table 6 clearly shows that the Bayes factor continues to favor the model with non-separable preferences, even when we consider different assumptions about the Calvo parameters. Furthermore, we also find that the posterior mean of $1/\sigma$ remains barely changed and the separable preferences remains clearly outside the credible set. The estimated posterior mean value of $1/\sigma$ is 2.33 with the 90% probability interval ranging from 2.05 to 2.63 when both the Calvo parameters are allowed to

Table 6: Comparison of Different Specifications

	Baseline		Estimated Calvo		Flexible Durable	
	Non-Separable	Separable	Non-Separable	Separable	Non-Separable	Separable
$1/\sigma$	2.21	1	2.33	1	2.33	1
	[1.90, 2.51]	n.a.	[2.05, 2.63]	n.a.	[2.05, 2.60]	n.a.
θ_c	0.66	0.66	0.80	0.51	0.80	0.52
	n.a.	n.a.	[0.73, 0.88]	[0.46, 0.58]	[0.71, 0.88]	[0.46, 0.58]
θ_x	0	0	0.11	0.11	0	0
	n.a.	n.a.	[0.06, 0.15]	[0.07, 0.16]	n.a.	n.a.
$\log(p(\mathcal{Y}_T\|\mathcal{M}))$	4672.49	4634.50	4652.65	4619.85	4672.05	4637.32

Note: Values reported here are the posterior means (the corresponding 90% probability intervals in brackets) of selected parameters and the log marginal densities $\log(p(\mathcal{Y}_T|\mathcal{M}))$ associated with different specifications. In the baseline specifications (the first and second columns), both Calvo parameters are imposed ($\theta_c = 0.66$, $\theta_x = 0$). The third and fourth columns represent specifications in which both Calvo parameters are estimated. The fifth and sixth columns correspond to specifications that impose flexible durable price ($\theta_x = 0$), while the Calvo parameter for nondurable firms is estimated.

be estimated. We obtain almost the same results when the Calvo parameter for the nondurable goods is estimated, but fully flexible durable prices are imposed. Therefore, the empirical evidence favoring the non-separability in the utility function does not depend on a particular choice of the Calvo parameters and the data generally support the non-separability.

We now check the feasibility of flexible durable prices. When we estimate the Calvo parameters for both nondurable and durable firms, we find that there is some degree of price stickiness in the durable prices, even though durable goods prices are estimated to be more flexible than nondurable goods. In the model with non-separable preferences, the posterior means of the nondurable and durable Calvo parameters are 0.80 and 0.11, respectively. They are 0.51 and 0.11, respectively, in the model with separable preferences. However, imposing full flexibility durable prices (i.e., $\theta_x = 0$) always yields higher log marginal densities regardless of whether preferences are separable or not. This suggests that a model with fully flexible durable prices fits the U.S. data better than a model with some degree of price stickiness in the durable sector. Our result here is somewhat in contrast with Rabanal and Aspachs-Bracons (2010) who show that using Spanish data, it is difficult to accept full flexibility in the durable prices, even though it is true that durable goods prices are more flexible than nondurable goods. While flexible durable prices are supported by the data, they are not decisive about the exact degree of nondurable price stickiness. When we compare baseline specifications and flexible durable specifications, the log marginal densities are very close to each other in Table 6.

4.4 Can Non-Separable Preferences Alone Resolve the Comovement Problem?

The previous section has shown that non-separable preferences are strongly supported by the data, even when we incorporate sticky wages, considered in the literature as an alternative to explaining the sectoral comovement, into our estimation. In this section, we investigate whether, as our

Table 7: Posterior Distributions of the Parameters Without Sticky Wages

		Posterior Distributions					
		Non-Separable			*Separable*		
Parameter	Prior Mean	Mean	5%	95%	Mean	5%	95%
$1/\sigma$	1.500	2.929	2.5349	3.3276	1	n.a.	
χ	0.010	0.2061	0.1258	0.2835	0.1458	0.0518	0.2406
ρ	1.170	1.7226	1.6521	1.8061	1.5566	1.4157	1.6964
ρ_A	0.500	0.9489	0.9375	0.9608	0.9478	0.9359	0.9595
ρ_{A_c}	0.500	0.7441	0.6899	0.8023	0.7088	0.5943	0.8212
ρ_{A_x}	0.500	0.8523	0.7857	0.9224	0.9794	0.9702	0.9891
ρ_b	0.800	0.9917	0.9877	0.9952	0.9927	0.9881	0.9963
ρ_R	0.750	0.3789	0.3274	0.4301	0.3900	0.3391	0.443
ρ_π	1.500	1.2316	1.1724	1.2893	1.0554	1.0328	1.0806
ρ_Y	0.120	-0.0095	-0.0132	-0.0057	-0.0134	-0.017	-0.0098
σ_ξ	0.050	0.0085	0.0075	0.0095	0.0107	0.0095	0.0120
σ_{ξ_c}	0.050	0.0115	0.0099	0.0131	0.0091	0.0072	0.0109
σ_{ξ_x}	0.050	0.0072	0.0061	0.0083	0.0080	0.0066	0.0094
σ_{ξ_R}	0.050	0.0060	0.0059	0.0062	0.0060	0.0059	0.0063
σ_{ξ_b}	0.050	0.0537	0.0333	0.0741	0.0200	0.0115	0.0285
σ_{ξ_v}	0.050	0.0061	0.0059	0.0063	0.0077	0.0065	0.0090
σ_{e_π}	0.050	0.0062	0.0059	0.0066	0.0063	0.0059	0.0068
σ_{e_X}	0.050	0.0600	0.0542	0.0653	0.0689	0.0629	0.0748
σ_{e_W}	0.050	0.0108	0.0099	0.0117	0.0132	0.0121	0.0143
Log Marginal Densities			4618.42			4562.47	

Note: The posterior distributions are obtained using the random walk Metropolis-Hastings algorithm with 300,000 draws (the first 10% of draws are discarded as a burn-in period). We use the modified Harmonic mean estimator of Geweke (1999) to obtain the log marginal density. This table reports posterior distributions with non-separable and separable preferences with the Calvo parameters $\theta_c = 2/3$ and $\theta_x = 0$ when there are no sticky wages ($\phi_w = 0$).

previous analytical results suggest, non-separable preferences alone can resolve the comovement problem associated with a monetary shock in our estimated model. Toward this end, we re-estimate our baseline two-sector sticky price model assuming flexible wages (i.e., $\phi_w = 0$). While we estimate the degree of the complementarity between nondurable consumption and durable service consumption (ρ) and the degree of non-separability ($1/\sigma$), we also explore how the variations in ρ affect the degree of non-separability required to induce the comovement in response to a monetary shock.

Posterior Distributions of the Parameters and Marginal Densities Table 7 reports the posterior distributions of parameters when sticky wages are not included in the estimation, and most importantly, whether incorporating sticky wages in the estimation has a limited impact on the estimates of non-separability ($1/\sigma$). The posterior mean of $1/\sigma$ increases from 2.2085 to 2.929 once flexible wages are imposed. The posterior means of the rest of the parameters remain almost

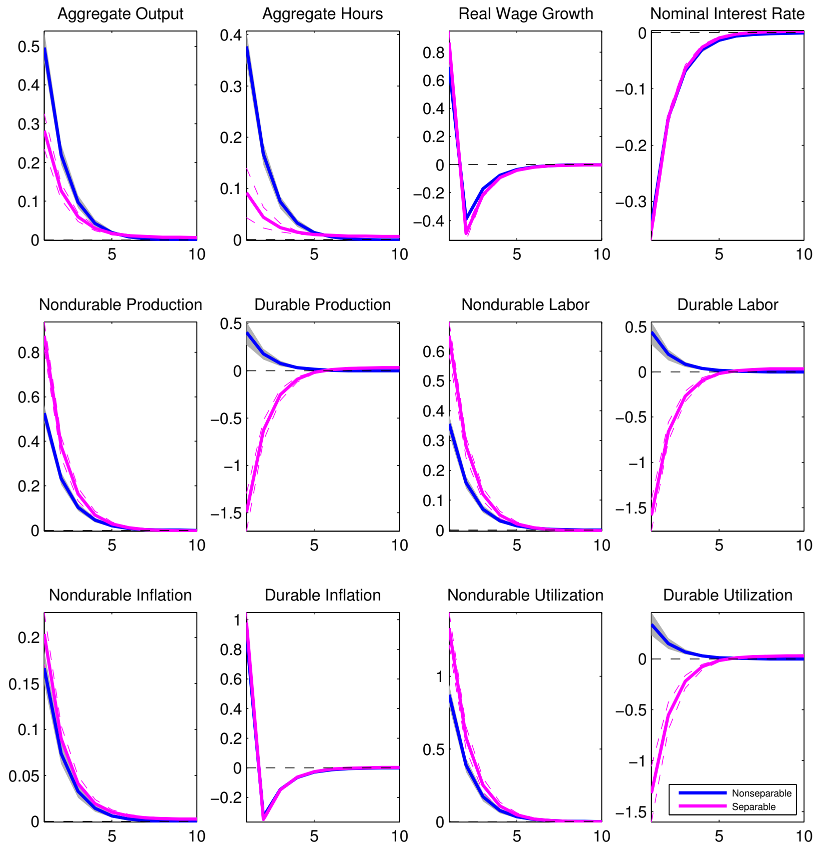
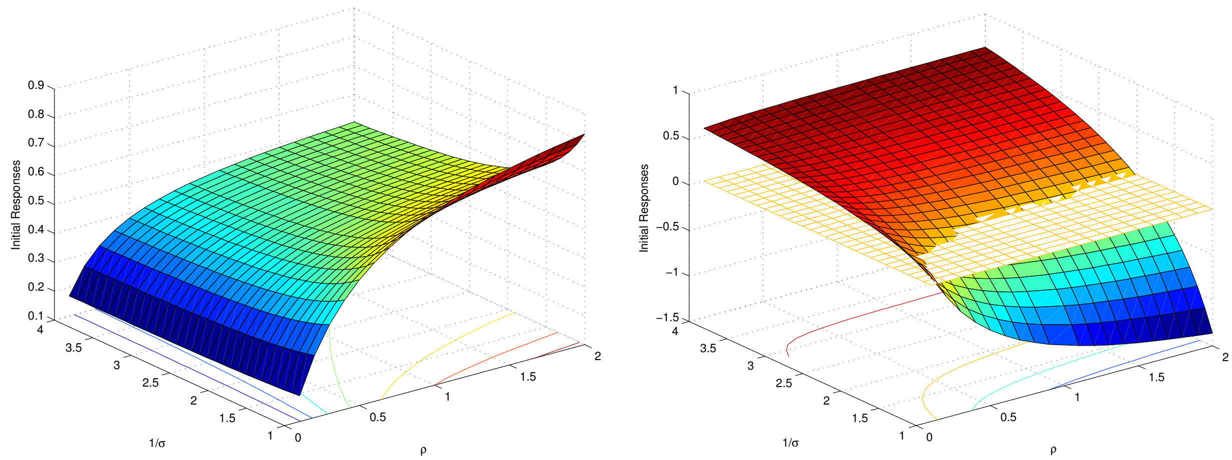


Figure 2: Responses to the Monetary Policy Shock with Flexible Wages

Note: The horizontal axes take the quarters after the shock and the vertical axes measure percentage deviations from the steady state. The above figures depict the responses of the economy to the one standard-deviation expansionary monetary policy shock. The solid lines represent the posterior mean responses, and the shaded areas (or dashed lines for the separable case) correspond to the 5% and 95% posterior intervals.

unchanged, except for χ. The posterior mean of χ becomes much smaller with flexible wages. As in the baseline specification, the Bayes factor strongly supports the non-separable preferences. Comparing the marginal data densities of models with and without nominal wage rigidity (those reported in Table 2 and Table 7) suggests that wage stickiness improves the overall fit of the model in addition to helping resolve the comovement puzzle.

Impulse Response Function In order to summarize the dynamic behavior of the model, Figure 2 plots the estimated responses to the expansionary monetary policy shock (i.e., one standard deviation decrease in the interest rate) with and without the non-separability. Solid lines represent the posterior mean responses and the shaded areas (dashed lines for the separable preferences) correspond to the 5% and 95% posterior intervals. The impulse responses are tightly estimated. With the separable preferences, the model exhibits the sectoral comovement problem. Following the expansionary shock, there is a large contraction of the production of durable goods, whereas the production of nondurable goods expands. Labor in each sector moves in the opposite direction.



(a) Nondurable Production

(b) Durable Production

Figure 3: Initial Responses of Nondurable and Durable Spending to the Monetary Policy Shock

Note: Surfaces summarize the initial response of nondurable production and durable production to the expansionary monetary policy with different combinations of ρ and $1/\sigma$, holding other parameter values at the posterior mean.

As our previous analytical results show, however, the model with the non-separable preferences drastically changes the reaction of the macroeconomic variables to a monetary shock. Even without sticky wages, our estimated degree of the non-separability is indeed strong enough to enable the model to produce a sectoral comovement in response to the monetary shock. With the non-separable preferences, output, hours worked, capital utilization in each sector increase all together following a monetary expansion.

The Role of ρ While we have estimated the degree of the complementarity between nondurable consumption and durable service consumption (ρ) and the degree of non-separability ($1/\sigma$), it is theoretically interesting to examine the role that ρ plays in determining the degree of nonseparability required to induce the comovement in response to a monetary shock. To see this, Figure 3 plots the initial responses of nondurable and durable spending to the expansionary monetary shock as functions of ρ and $1/\sigma$, holding other parameters at the posterior means.

The figure shows that the threshold level of the non-separability needed to generate the comovement is inversely related to the degree of the complementarity between nondurable consumption and durable service consumption (ρ). Needless to say, nondurable production responds positively to the monetary shock, regardless of the values of ρ and $1/\sigma$. As ρ gets lower (i.e., the degree of the complementarity becomes higher), smaller values of $1/\sigma$ (i.e., the lower degree of the non-separability) is sufficient to generate a positive initial response of durable production. The intuition behind this result is straightforward. The increased complementarity between nondurable consumption and durable service consumption suppresses the incentive to consume more of the nondurable because the durable service consumption (the durable goods stock) changes little in response to the expansionary monetary shock. This in turn puts less pressure on the cost of the flexibly priced durable sector, so that a smaller degree of the non-separability is required to offset the rise in the cost of production in the durable sector. As ρ becomes smaller, therefore, the smaller degree of non-separability can generate a positive response of the durable goods spending.

5 Conclusion

In the data, strong procyclical fluctuations in the production of durable goods are the most prominent feature of the response to monetary shocks. This paper investigates the role of preferences in matching this feature of the data in a two-sector sticky price model with flexibly priced durables. We analytically demonstrate that the non-separability between aggregate consumption and labor supply plays an important role in shaping the reaction of durable goods production to a monetary shock. When preferences are separable in aggregate consumption and labor, the model exhibits counterfactual behavior. Flexibly priced durable goods production contracts substantially following a monetary expansion if preferences are separable. In contrast, the sticky price model with non-separable preferences can replicate the empirically plausible response of durable goods spending to a monetary expansion.

We also examine whether the non-separable preferences are empirically compatible with the data. To this end, we estimate a two-sector sticky price model, using a Bayesian approach. We incorporate sticky wages, which are considered in the literature as an alternative to explain the sectoral comovement, into our estimation. Interestingly, the inclusion of sticky wages does not undermine the importance of non-separable preferences. Our estimates of the intertemporal elasticity of substitution are well below unity, suggesting that the data favor non-separable preferences. Furthermore, non-separable preferences substantially improve the overall empirical performance of the model. This stresses the importance of the non-separable preferences in a two-sector sticky price model.

Finally, we show that non-separable preferences alone can successfully resolve the comovement problem associated with monetary shocks in our estimated model without relying on sticky wages. Our estimate for the intertemporal elasticity of substitution is well below the threshold level needed for the sticky price model to produce a strong procyclical response of durable goods spending to an expansionary monetary policy shock. As a result, the estimated degree of non-separability resolves the comovement problem in the sticky price model with flexibly priced durable goods. The key to the model's success is due to the fact that non-separable preferences imply the complementarity between nondurable consumption and labor supply, which is absent in separable preferences.

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Appendix

Data Construction

We need to construct the following variables:

$$
\begin{aligned}
C_t &= \text{real expenditure on nondurables and services,} \\
X_t &= \text{real expenditure on durable consumption and residential investment,} \\
\pi_{c,t} &= \text{nondurable consumption price inflation,} \\
\pi_{x,t} &= \text{durable consumption price inflation.}
\end{aligned}
$$

In order to construct C_t and X_t, we need to have nominal consumption expenditures on various consumption goods from the NIPA Table 2.3.5 and corresponding price indices from the NIPA Table 2.3.4. Also, nominal residential investment and the associate price index are obtained from the NIPA Table 5.3.5 and Table 5.3.4, respectively.

One problem is that we cannot add or subtract the levels of chain aggregated price or quantity index.

We want to exclude expenditure on housing services from C_t and include it in X_t. The nominal nondurable consumption expenditure is given by

$$
\begin{aligned}
TC_t &= P_{c,t}C_t \\
&= \text{Nondurable expenditure} + \text{Expenditure on services} - \text{Expenditure on housing and utilities} \\
&= (\text{line 8 in Table 2.3.5}) + (\text{line 13 in Table 2.3.5}) - (\text{line 15 in Table 2.3.5})
\end{aligned}
$$

We need to construct $P_{c,t}$ by using the Trönqvist index. However, one thing we need to be careful about is that we do not want to include the price index for housing services. In general, the formula for the Törnqvist index aggregating $y_1, \cdots, y_n$ is given by

$$\Delta \ln(Y_t) = \sum_{i=1}^{n} \bar{\omega}_{i,t} \Delta \ln(y_{i,t}), \tag{45}$$

where $\bar{\omega}_{i,t}$ is the aggregating weight on each component and given by

$$\bar{\omega}_{i,t} = \frac{1}{2}(\omega_{i,t} + \omega_{i,t-1}), \tag{46}$$

and $\omega_{i,t}$ is the expenditure share of the category i, such that

$$\omega_{i,t} = \frac{p_{i,t}q_i}{\sum_{i=1}^{n} p_{i,t}q_i}. \tag{47}$$

To be concrete, let us suppose that we need to construct an aggregate price index of nondurable consumption (ND) and services (S), excluding housing services (s_1), which is a subcategory of services. Basic strategy is as follows:

(i) Construct an index Z that aggregates ND and S.

(ii) Then, construct the index of our interest C by using Z and s_1.

The first step is just applying the formula (45) to create new aggregate Z. We calculate

$$\Delta \ln(Z_t) = \frac{1}{2}(\omega_{ND,t} + \omega_{ND,t-1})\Delta \ln(ND_t) + \frac{1}{2}(\omega_{S,t} + \omega_{S,t-1})\Delta \ln(S_t), \tag{48}$$

where $\omega_{ND,t} = \dfrac{P_{ND,t}ND_t}{(P_{ND,t}ND_t + P_{S,t}S_t)}$ and $\omega_{S,t} = 1 - \omega_{ND,t}$.

The second step is to recognize that the index Z consists of s_1 and C, so that we have

$$\Delta \ln(Z_t) = \frac{1}{2}(\omega_{s_1,t} + \omega_{s_1,t-1})\Delta \ln(s_{1,t}) + \frac{1}{2}(\omega_{C,t} + \omega_{C,t-1})\Delta \ln(C_t), \tag{49}$$

where $\omega_{C,t} = 1 - \omega_{s_1,t}$. Here $\omega_{s_1,t} = \frac{P_{s_1,t}s_{1,t}}{P_{z,t}Z_t}$.

Thus, we can calculate the index of our interest C_t as follows:

$$\Delta \ln(C_t) = \frac{2}{(2 - \omega_{s_1,t} - \omega_{s_1,t-1})}\left\{\Delta \ln(Z_t) - \frac{1}{2}(\omega_{s_1,t} + \omega_{s_1,t-1})\Delta \ln(s_{1,t})\right\} \tag{50}$$

and use these growth rates to uncover the level of the variable.

We can also construct X_t and $\pi_{x,t}$ in a similar fashion. The total expenditure on X_t is defined as follows:

$$\begin{aligned} TX_t &= P_{x,t}X_t \\ &= \text{Durable expenditure} + \text{Expenditure on housing and utilities} + \text{Residential investment} \\ &= (\text{line 3 in Table 2.3.5}) + (\text{line 15 in Table 2.3.5}) + (\text{line 17 in Table 5.3.5}) \end{aligned}$$

Estimation Details

Let $\boldsymbol{\theta}$ denote a vector of parameters to be estimated and use $p(\boldsymbol{\theta})$ to denote the prior distribution of the parameter vector. $p(\mathcal{Y}_T;\boldsymbol{\theta})$ represents the likelihood of the data given the parameter $\boldsymbol{\theta}$, where $\mathcal{Y}_t$ stacks all observations as $\mathcal{Y}_T = [\boldsymbol{y}_1, \cdots, \boldsymbol{y}_T]$ and $\boldsymbol{y}_t$ is a vector of observables at time t.

By the Bayes rule, the posterior distribution of $\boldsymbol{\theta}$ is given by

$$p(\boldsymbol{\theta}|\mathcal{Y}_T) = \frac{p(\mathcal{Y}_T|\boldsymbol{\theta})p(\boldsymbol{\theta})}{p(\mathcal{Y}_T)}. \tag{51}$$

All results reported in the paper are based on 300,000 draws from the posterior distribution $p(\boldsymbol{\theta}|\mathcal{Y}_T)$, which are generated from the random walk Metropolis-Hastings algorithm. Note that the first 30,000 draws are discarded as a burn-in period. The posterior mode and the associated variance-covariance matrix are used to configure a starting point for the random walk Metropolis-Hastings algorithm.

Using the state-space representation of the model and the Kalman filter, we can easily evaluate the log-likelihood, $\log p(\mathcal{Y}_T|\boldsymbol{\theta})$. We numerically maximize the log posterior kernel, $\log p(\mathcal{Y}_T|\boldsymbol{\theta}) + \log p(\boldsymbol{\theta})$, with respect to $\boldsymbol{\theta}$ in order to find the posterior mode. The initial mean of jumping distribution is set at the posterior mode, and the variance-covariance matrix is set to the inverse of the Hessian evaluated at the posterior mode.

At the i^{th} Metropolis-Hastings draw, a candidate parameter vector $\boldsymbol{\theta}^*$ is drawn from $N(\boldsymbol{\theta}^{(i-1)}, c\boldsymbol{\Sigma})$, where c is a scaling factor and $\boldsymbol{\Sigma}$ is the inverse of the Hessian at the posterior mode. Then we compute the acceptance ratio

$$r = \frac{p(\boldsymbol{\theta}^*|\mathcal{Y}_T)}{p(\boldsymbol{\theta}^{(i-1)}|\mathcal{Y}_T)} = \frac{p(\mathcal{Y}_T|\boldsymbol{\theta}^*)p(\boldsymbol{\theta})}{p(\mathcal{Y}_T)} p(\mathcal{Y}_T|\boldsymbol{\theta}^{(i-1)})p(\boldsymbol{\theta})p(\mathcal{Y}_T) \tag{52}$$

and we accept $\boldsymbol{\theta}^*$ and set $\boldsymbol{\theta}^{(i)} = \boldsymbol{\theta}^*$ with probability $\min(r, 1)$ and reject otherwise (i.e., $\boldsymbol{\theta}^{(i)} = \boldsymbol{\theta}^{(i-1)}$). The scaling factor c is set, such that the fraction of candidate draws that are accepted becomes about 30%.

Given two specifications $\mathcal{M}_1$ and $\mathcal{M}_2$, the posterior odds ratio is given by

$$\frac{p(\mathcal{M}_1|\mathcal{Y}_T)}{p(\mathcal{M}_2|\mathcal{Y}_T)} = \underbrace{\frac{p(\mathcal{M}_1)}{p(\mathcal{M}_2)}}_{\text{Prior odds}} \underbrace{\frac{p(\mathcal{Y}_T|\mathcal{M}_1)}{p(\mathcal{Y}_T|\mathcal{M}_2)}}_{\text{Bayes factor}}. \tag{53}$$

For the Bayesian model comparison, we need to compute the marginal data density, $p(\mathcal{Y}_T|\mathcal{M}_j)$ for $j = 1, 2$, which is difficult to evaluate. Geweke (1999) proposes to use the following estimator

$$\hat{p}(\mathcal{Y}_T|\mathcal{M}_j) = \left[\frac{1}{K}\sum_{k=1}^{K} \frac{f(\boldsymbol{\theta}_j^{(k)})}{p(\mathcal{Y}_T|\boldsymbol{\theta}_j^{(k)}, \mathcal{M}_j)p(\boldsymbol{\theta}_j^{(k)}|\mathcal{M}_j)}\right]^{-1}, \tag{54}$$

where $\boldsymbol{\theta}_j^{(k)}$ is a Metropolis-Hastings draw from $p(\boldsymbol{\theta}|\mathcal{Y}_T, \mathcal{M}_j)$ and K is the number of the Metropolis-Hastings draws. $f(\boldsymbol{\theta})$ is specified as the density of a truncated multivariate normal distribution,

$$f(\boldsymbol{\theta}) = \tau^{-1}(2\pi)^{-\frac{d}{2}} |\boldsymbol{V}_{\boldsymbol{\theta}}|^{-\frac{1}{2}} \exp\left[-\frac{1}{2}(\boldsymbol{\theta} - \bar{\boldsymbol{\theta}})'(\boldsymbol{V}_{\boldsymbol{\theta}})^{-1}(\boldsymbol{\theta} - \bar{\boldsymbol{\theta}})\right] \times \mathbb{1}\left\{(\boldsymbol{\theta} - \bar{\boldsymbol{\theta}})'\boldsymbol{V}_{\boldsymbol{\theta}}^{-1}(\boldsymbol{\theta} - \bar{\boldsymbol{\theta}}) \le F_{\chi_d^2}^{-1}(\tau)\right\}, \tag{55}$$

where $\bar{\boldsymbol{\theta}}$ and $\boldsymbol{V}_{\boldsymbol{\theta}}$ are the posterior mean and variance-covariance matrix computed from the Metropolis-Hastings algorithm, d is the size of $\boldsymbol{\theta}$, $F_{\chi_d^2}$ is the cumulative density function of a χ^2 random variable with d degrees of freedom, $\tau \in (0, 1)$, and $\mathbb{1}\{\cdot\}$ is an indicator fuction.