The two cultures of mathematics in ancient Greece

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T he notion of 'Greek mathematics' is a key concept among those who teach or learn about the Western tradition and, especially, the history of science.[1] It seems to be the field where that which used to be referred to as 'the Greek miracle' is at its most miraculous. The works of, for example, Euclid or Archimedes appear to be of timeless brilliance, their assumptions, methods, and proofs, even after Hilbert, of almost eternal elegance. Therefore, for a long time, a historical approach that investigated the environment of these astonishing practices was not deemed necessary. Recently, however, a consensus has emerged that Greek mathematics was heterogeneous and that the famous mathematicians are only the tip of an iceberg that must have consisted of several coexisting and partly overlapping fields of mathematical practices (among others, Lloyd 1992, 569). It is my aim here to describe as much of this 'iceberg' as possible, and the relationships between its more prominent parts, mainly during the most crucial time for the formation of the most important Greek mathematical traditions, the fifth to the third centuries BC.

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1. General introductions to Greek mathematics are provided by Cuomo (2001); Heath (1921); Lloyd (1973, chapters 4–5); Netz (1999a).

Reconstructions: Greek practical mathematics

Let us begin with a basic observation. Whoever looks for the first time at a page from one of the giants of Greek mathematics, say, Euclid, cannot but realize an obvious fact: these theorems and proofs are far removed from practical life and its problems. They are *theoretical*.[2] Counting, weighing, measuring, and in general any empirical methods, have no place in this type of mathematics. Somebody, however, must have performed such practices in daily life, for example, in financial or administrative fields such as banking, engineering, or architecture. Some of these fields demand mathematical operations of considerable complexity, for example, the calculation of interest or the comparison of surface areas. Occasionally, ancient authors mention such mathematical practices in passing (for example, at the end of the fifth century BC Aristophanes' play, *Wasps* 656–662). What is known about these practical forms of Greek mathematics?

Not much, obviously. Of the social elite who alone wrote and read for pleasure, most were less interested in practical mathematics, which was apparently not part of common knowledge. Occasionally, one comes across obvious arithmetical blunders, mostly by historians.[3] On the other hand, in most cases the practitioners themselves left no texts. Therefore, of all the manifold forms of practical mathematics that must have existed, only two are known a little, partly through occasional references by authors interested in other topics, partly through preserved artifacts, and, rarely, through the textual traditions of the practitioners themselves.

Pebble arithmetic was used in order to perform calculations of all kinds.[4] 'Pebbles' (*psēphoi*, an appropriate translation would be 'counters') that symbolized different numbers through different forms and sizes, were moved and arranged on a marked surface—what is sometimes called the 'Western abacus' (see Netz 2002b, 326, 342, who remarks that backgammon may well illustrate the principle). Several of these have been found, and the practitioners themselves are mentioned occasionally.[5] These must have been professionals that one could hire whenever some arithmetical problem had to be solved, not so different from professionals renting out their literacy. However, manipulating pebbles on an abacus can lead to the discovery of general arithmetical knowledge, for example the properties of even and odd, or prime numbers, or abstract rules of how to produce certain classes of numbers, for example, square numbers. I call this knowledge 'general' because it no longer has any immediate application. Here 'theoretical'

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2. I avoid here the notions of 'pure' and 'applied' mathematics with their evaluative connotations.
3. For example, Herodotus 7.187.2; Thucydides 1.10.4 f.; Polybius 9.19.6 f. See Netz (2002a, 209–213).
4. Netz (2002b) has recently described this practice and its social setting as a 'counter culture' (for the sake of the obvious pun, he translates *psēphos* as 'counter').
5. Netz (2002b, 325) surveys the archaeological evidence (30 abaci). Pebble arithmetic is mentioned, for example, in Aeschylus, *Aga.* 570; Solon in Diogenes Laertius 1.59.

knowledge emerges from a purely practical-professional background. Some pebble arithmetic probably shows up in later Greek 'Neo-Pythagorean' arithmetic, most notably in Nicomachus of Gerasa (probably second century AD) and, slightly later, in Iamblichus of Chalcis (Knorr 1975, 131–169).

Pebble arithmeticians, as a group or as individuals, never made it into the range of subjects one could write about in antiquity, a fate they shared with most professionals that one could hire to perform specialized tasks (physicians being the most notable of the few exceptions). Therefore, nothing is known about the people who did pebble arithmetic in classical Greece, how their profession was structured, and how they transmitted their knowledge. Their body of knowledge, however, was apparently known to at least some Pythagoreans in fifth-century Greece who used it for their own, semi-religious practices.[6] Also, abstract insight into the properties of numbers, as it is typically gained by arranging pebbles (Becker 1936), must have been already widely known at the beginning of the fifth century in Greece.[7] These two cases show how specialized, practical knowledge could become abstract and move beyond the circle of specialists.

The practitioners of this art in ancient Greece, however, were probably only a tiny part of a long and remarkably stable tradition of such arithmetic professionals that originated somewhere in the ancient Near East (but, admittedly, may have changed along the way). It has recently been demonstrated, by characteristic calculation errors, that Old Babylonian scribes of the early second millennium BC *and* their Seleucid descendants must have used essentially the same accounting board to carry out multiplications of large numbers.[8] In the Middle East, the tradition resurfaces with people that are called 'ahl al-gabr' in Arabic sources of the ninth century AD, the 'algebra people'.[9] At least partly, their knowledge about algebraic problems and solutions goes back to Old Babylonian times (Høyrup 1989). It is not too bold an assumption to understand Greek pebble arithmetic as part of the same tradition (see West 1997, 23–24 for eastern influence on Greek financial arithmetic). Recently, a similar claim has been made concerning the Greek way of dealing with fractions that apparently shows Egyptian influence (Fowler 1999, 359).

The second subgroup of mathematical practitioners was concerned with measuring and calculating areas and volumes. Unlike the pebble arithmeticians, they had textual traditions, of which traces are scarce for ancient Greece, but considerable throughout the ancient Near East. These textual traditions, however, were

6. Aristotle, *Phys*. 203 a 13–15; *Metaph*. 1092 b 10–13; Theophrastus, *Metaph*. 6 a 19–22.

7. Epicharmus (early fifth century BC): fr. 23 B 2.1f. ed. Diels-Kranz; see Knorr (1975, 136); compare Burkert (1972, 434–439).

8. See Proust (2002, esp. 302), who maintains that this device was somehow based on the hand, that is, it would have been an advanced form of finger reckoning.

9. Thābit ibn Qurra in Luckey (1941, 95–96). Al-Nadim, *Fihrist* II, in Dodge (1970, 664–665) (ninth and tenth century, respectively).

sub-literary, that is, they never made it into the traditions of Greek mathematical literature (later we will see why). Therefore, most of these texts have been found inscribed on papyri, mostly written in imperial times, extant only from the Greek population in Egypt because of the favorable conditions of preservation there. There is every reason to assume, however, that in antiquity such texts were widespread in the Greek speaking world, both earlier and later. Here is an example from a first-century AD papyrus, now in Vienna:

Concerning stones and things needed to build a house, you will measure the volume according to the rules of the geometer as follows: the stone has 5 feet everywhere. Make 5 x 5! It is 25. That is the area of the surface. Make this 5 times concerning the height. It is 125. The stone will have so many feet and is called a cube. (Greek text in Gerstinger-Vogel 1932, 17)

The papyrus contained thirty-eight such paragraphs in sixteen columns, obviously meant to codify valid methods or, rather, approved procedures in textbook style (for details, see Fowler 1999, 253). Obviously, these methods are what the text calls 'the rules (*hoi logoi*) of the geometer'. Other papyri contain more difficult procedures, as the following example shows:

If there is given a parallelogram such as the one drawn below: how it is necessary, the 13 of the side squared is 169 and the 15 of the side squared is 225. (Subtract) from these the 169. 56 remains. Subtract the 6 of the base from the 10 of the top. 4 remains. Take a fourth of the 56. It is 14. From these (subtract) the 4. 10 remains, a half of which is 5. So great is the base of the right-angled triangle. Squared it is 25 and the 13 squared is 169. Subtract the 25. 144 remains, the *pleura* (= square root) of which is 12. So great is the perpendicular. And subtract the 5 from the 6 of the base. 1 remains. (Take) the one from the 10 of the top. 9 remains. So great is the remainder of the upper base of the right-angled triangle. And the 12 of the perpendicular by the 5 of the base is 60, a half of which is 30. Of so many *arourōn* (= square units) is the right-angled triangle in it. And the 12 by the 1 is 12. Of so many *arourōn* is the triangle in it. And the 12 by the 9 of the base is 108, a half of which is 54. Of so many *arourōn* is the other right-angled triangle. Altogether it is 96 units. And the figure will be such. (Pap. Ayer, col. III, first to second century AD, transl. after Goodspeed 1898, 31)

The diagram is reproduced according to the papyrus (Goodspeed 1898, 30). The algorithm gives the area of an irregularly shaped figure as the sum of triangles and rectangles, the areas of which have to be found first. In order to ensure that the reader understands the actual procedure and, thereby, the abstract method, the paradigmatic numbers (in Greek, mostly letters) are repeated in the diagram from the text. As with the first text, this is also a part of a collection of such paragraphs. More such collections are known (see Asper 2007, 200): for example, a Berlin papyrus (Schubart 1915/16, 161–170) and the better part of two treatises (*Geometrica, Stereometrica*) ascribed to Hero of Alexandria, an engineer active in Rome and Alexandria in the first century AD.
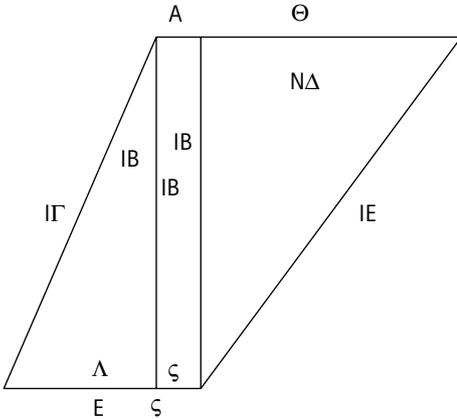
Figure 2.1.1 Diagram in Papyrus Ayer (first to second century AD)

The practical relevance of these procedures is fairly obvious, for example, for practitioners managing construction sites ('how many bricks do I need for a wall with such and such dimensions?') or in surveying ('what is the size of this piece of land?'). Both appeal to commonly shared rules and, thereby, hint at a collective of practitioners whose professional knowledge was codified in such texts.[10]

The rhetorical mood of this codification is clearly one of instruction, of a stylized dialogue between teacher and disciple: strong and frequently iterated imperatives address a second person. More importantly, the method is given as a series of steps, each of which is clearly marked. Often, the end of the procedure is marked as well. That is why these texts remind the modern reader of recipes (Robson 1999, 8). Strangely, the method itself is never explained in general terms, nor is its effectiveness proved. The actual procedure employs paradigmatic numbers that always result in whole numbers (for example, when one has to extract square roots). Obviously, the reader is meant to understand the abstract method by repeatedly dealing with actual, varied cases. The leap, however, from the actual case to the abstract method is never mentioned in these texts. Learning a general method is achieved in these texts by repeatedly performing a procedure, understanding its effectiveness and memorizing the steps by repetition, when one works through the whole text-book. Later, the professional performs his tasks by repeating the method *per analogiam*.

As I have said, these sub-literary Greek texts were written in the first and second centuries AD, mostly in Egypt. The problems they solve are so basic that one can hardly imagine that these methods were not also used much earlier in Greece. They provide, however, a glimpse at a remarkably strong tradition, of which they are probably only a local, rather late branch. Another, older part of

10. I understand the peculiar phrase 'how it is necessary' as shorthand for 'how one has to solve this kind of problem according to the experts'. The Greek is *hōs dei*. Goodspeed translates as 'according to the condition of the problem'.

this tradition is much better known, by thousands of texts preserved on clay tablets found in the Near East. Here is an example, problem 17 of a rather substantial textbook (BM 96954), written in Old Babylonian times (first half of the second millennium BC). The text describes a procedure of how to calculate the volume of a 'grain pile' (probably a cone-shaped body):

A triangular grain-pile. The length is 30, the width 10, the height 48. What is the grain? You: multiply 30, the length, by 10, the width. You will see 5 00. Multiply by 48, the height. You will see 4 00 00. Multiply 1 30 by 4 00 00. You will see 6 00 00 00. The grain capacity is 6 00 00 00 *gur*. This is the method. (transl. Robson 1999, 223)

If one leaves aside the differences, mostly the sexagesimal system, one clearly observes the rhetorical features that were so obvious in the Greek texts: a clearly stated practical problem, the intense appeal to the reader, the recipe-like structure, a procedure that operates with paradigmatic numbers, an abstract method that is not mentioned but illustrated by actual procedures. Admittedly, these texts vary in complexity and in their actual textual conventions. The above listed features, however, apply throughout, and not only to Old Babylonian, but also to Egyptian, Hebrew, Coptic, Arabic, and even Latin texts (compare, for example, Gandz 1929-31, 256–258; Høyrup 1996). The tradition illustrated by these occasional glimpses was alive from at least the second millennium BC well into the Middle Ages. It was so stable that some of the younger texts almost appear to be translations of the oldest ones (see the Coptic examples in Fowler 1999, 259). Moreover, some of the Greek texts show the same methods, and sometimes even the same sets of paradigmatic numbers as much older Egyptian or Babylonian ones (Gerstinger and Vogel 1932, 39, 47–50). In this tradition, the textbooks are complemented by lists of coefficients, certain factors, square roots, etc. (the Greek examples are collected in Fowler 1999, 270–276).

   Although this knowledge, and the textual conventions that were meant to secure its transmission, originated in the ancient Near East, in time it moved westward and spread over the whole Mediterranean. We have some reasons to believe that people who solved practical problems with these methods were active in fifth-century Greece too (and probably much earlier). This argument relies on reconstruction by analogies: expert knowledge of several kinds came to Greece even before the classical age, especially in technical fields as diverse as architecture, writing, and medicine (to name but a few).[11] I do not see why practices that involved calculation should have been the exception. At least in some fields, 'migrant craftsmen', that is, foreigners seem to have been the transmitters. The argument from analogy seems the more compelling as one would expect numeracy to spread along the lines of literacy, especially when both probably

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11. See Burkert (1992, 20–25); West (1997, 23–24, 609–612); for a general introduction to the topic Burkert (2004, 1–15).

took place in the same time and place (and had to be combined in many practices of administration).[12] Early Greek pebble arithmetic, therefore, was quite probably one of the Greek practices that resulted from acculturation with the Middle East in archaic times or even earlier, just like the Greek alphabets (see Netz 2002b, 344) and writing practices more generally.[13]

Back to the texts of these practitioners. There is no notion of definition, proof, or even argument in these texts (and hardly any concept of generality),[14] which has earned them the label 'sub-scientific' (for example, Høyrup 1989). It is important, however, to understand the lack of these features not as a general intellectual 'fault', but to explain them by the social functions of the knowledge concerned: in order to solve important problems, what is needed is not a proof of a general method, let alone of a theorem, but a reliable, accepted procedure that will lead to a reasonable result in every single case. Likewise, it is doubtful whether the notion of an abstract rule was present behind all the actual procedures. It might be a feature of the educational character of these texts that general knowledge is not explicitly stated (*pace* Damerow 2001).

As was the case with the pebble arithmeticians, almost nothing is known about the actual people who were engaged in these practices in Greece. Some assumptions, however, appear to be at least reasonable. First, since this kind of knowledge was of economic importance, it was probably not popular or widespread but rather guarded, perhaps by guild-like social structures. Performing as a practical mathematician in one of these arts was a specialized profession. For some of these people, a Greek name has survived: there was a professional group called *harpēdonaptai* ('rope-stretchers'), obviously surveyors operating with ropes for measuring purposes (Gandz 1929–31). Judging from the stability of the traditions, their group-structures must have been institutionalized somehow, including the education of disciples (maybe in apprenticeship-like relationships). Compared, however, to the complex institutional framework of, for example, the Old Babylonian scribal schools, the migrant craftsmen in Greece must have transmitted their knowledge on a much less institutionalized and, above all, less literate level.

Second, in many realms of professional knowledge, migrant craftsmen had already begun arriving in Greece from the East in the ninth century BC. There existed, for example, Phoenician work shops in seventh-century Athens. The entire vase industry, so prominent especially in Attica, seems to have employed Eastern immigrants with names like Amasis or Lydus (see Burkert 1992, 20–25).

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12. See Netz (2002b, 322–324) on the concept of 'numeracy' and its conceptual interdependence with 'literacy' (and even its antecedence to it) in early Greece and the Middle East.

13. Of course, I do not suggest any *direct* contacts between Near Eastern mathematics and Greek *theoretical* mathematics (including astronomy) at any time before second-century Alexandria (see Robson 2005).

14. There are very few exceptions, most notably in the Egyptian Papyrus Rhind (seventeenth century BC, ed. Chace 1927–9). See Høyrup (2002a, 383); Asper (2007, 201 n727).

In archaic times, mathematical practitioners were probably such migrant crafts-men. Later, these traditions certainly became indigenous, but the knowledge retained its structures, even the textual ones.

Third and most importantly, these practitioners, even if occasionally well-paid, must have been of a rather low social level, viewed from the perspective of the well-off polis citizen. Aristotle's judgment of the craftsmen's social status in past and present probably also applied to these experts.[15] In fifth- and fourth-century Greece, most writers were upper-class citizens writing for their peers, which is why we almost never hear about these practitioners. That does not mean, however, that they were a marginal phenomenon. Rather, as I will argue, they provided the background for the emergence of theoretical mathematics. To think of their knowledge as 'sub-scientific' makes sense, as long as one remembers that *our* understanding of what science is has been heavily influenced by Greek *theoretical* mathematics.[16] The 'sub' here should be taken literally: ancient prac-tical mathematical traditions were certainly all-pervasive in ancient Greece, on top of which theoretical mathematics suddenly emerged, like a float on a river's surface—brightly colored and highly visible, but tiny in size.

Greek theoretical mathematics (and its texts)

Compared with practical traditions such as the ones outlined above, Greek theoret-ical mathematics strikes the reader as very different. Most notably, it is almost exclu-sively geometrical. It consists of a body of general propositions proved by deduction from 'axioms', that is, evident assumptions or definitions—hence the designation of this type of mathematics as 'axiomatic-deductive'. Whereas the practitioners' texts collected problems and provided procedures for solving them, the theoreticians' texts collected general statements with proofs. The language and the structure of these texts are highly peculiar, compared both to the practitioners' texts and to Greek prose of the times in general. From a historian's point of view, this form of mathem-atics is no less remarkable: Greek theoretical mathematics suddenly appears at the end of the fifth century BC. Most of the famous mathematical writers (for example, Euclid, Archimedes, and Apollonius) were active in the third century BC. A rather simple theorem in Euclid (*Elements*, I. 15) may provide a suitable introduction:

If two straight lines cut one another, they make the vertical angles equal to one another. For let the straight lines AB, ΓΔ cut one another at the point E; I say that the angle AEΓ is equal to the angle ΔEB, and the angle ΓEB to the angle AEΔ. For, since the straight line AE stands on the straight line ΓΔ, making the angles ΓEA, AEΔ, the angles ΓEA, AEΔ are equal to two right angles. Again, since the straight line ΔE stands on the straight line

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15. *Politics* III 5, 1278 a 7. 'Craftsmen were either slaves or foreigners.'
16. Maybe one should rather think of them as of a '*science du concret*' (Lévi-Strauss 1966, 1–33).

AB, making the angles AEΔ, ΔEB, the angles AEΔ, ΔEB are equal to two right angles. But the angles ΓEA, AEΔ were also proved equal to two right angles; therefore the angles ΓEA, AEΔ are equal to the angles AEΔ, ΔEB.[17] Let the angle AEΔ be subtracted from each; therefore the remaining angle ΓEA is equal to the remaining angle BEΔ.[18] Similarly it can be proved that the angles ΓEB, ΔEA are also equal. Therefore, if two straight lines cut one another, they make the vertical angles equal to one another. Just what one had to prove. (transl. modified from Heath 1956, I 277–278)

As can be gathered from the text, the reader had to have in front of him a diagram that probably looked like Fig. 2.1.2 (extant in medieval manuscripts of Euclid and probably closely resembling the diagrams illustrating the theorem in the third century BC).[19]

Euclid claims the truth of a general proposition, a theorem (above, given in italics), about what happens when two lines cut each other. First he construes a pseudo-actual case by a diagram, the parts of which are designated by letters. Then he compels his reader to look at the diagram and ask himself which of the already proved or axiomatically accepted truths (both were treated earlier in the first book of the *Elements*) one could use in order to prove the statement. Here, Euclid uses I.13 ('If a straight line set up on a straight line makes angles, it will make either two right angles or angles equal to two right angles.'), and axioms (see notes 10 and 11). From these, already accepted as true, the mathematician can safely deduce the truth of the theorem. The whole proof is implicit, that is, neither does Euclid tell the reader which parts of the axiomatic material or the already proved theorems he uses nor does he ever explain his line of reasoning. At the end of the paragraph, he does facilitate the transition from the pseudo-actual diagram to the general theorem, in exactly the same words that were used in the beginning. The textual unit of theorem, diagram, and proof ends with the explicit and nearly proud assertion that the author has proved what he set out to prove.
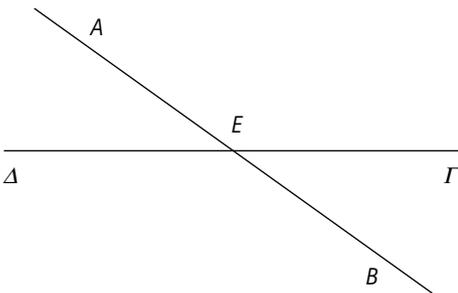

Figure 2.1.2 Diagram illustrating Euclid's *Elements* 1.15 (after Heath 1956, I 277)

17. The argument is based on the so-called postulate 4 ('All right angles are equal to one another.') and so-called 'common notion' 1 ('Things which are equal to the same thing are also equal to one another.'). All definitions, postulates, and common notions relevant to the first book of the *Elements* are gathered at the beginning of the book.

18. Presupposes common notion 3 ('If equals are subtracted from equals, the remainders are equal').

19. On diagrams see Saito, Chapter 9.2 in this volume.

Due to its use in schools well into the twentieth century, Euclid's *Elements* are by far the best known text written in this style, but by no means the only one. Here is a proof from the beginning of the treatise *On the sphere and the cylinder* (I.1), written by the famous Archimedes of Syracuse, probably roughly a contemporary of Euclid (Fig. 2.1.3):

If a polygon be circumscribed about a circle, the perimeter of the circumscribed polygon is greater than the perimeter of the circle. For let the present polygon be circumscribed about a circle. I say that the perimeter of the polygon is greater than the perimeter of the circle. For since BA, AL taken together is greater than the arc BL, because they have the same beginning and end, but contain the arc BL, and because similarly LK, KΘ taken together [is greater] than LΘ, and ZH, HΘ taken together [is greater] than ZΘ, and also ΔE, EZ taken together [is greater] than ΔZ, therefore the whole perimeter of the polygon is greater than the perimeter of the circle.[20] (transl. after Heath 1953, 5)

Archimedes proves a fact that is evident to anyone who takes a look at the diagram. His proof utilizes axiomatic material, too (see note 14). Obviously, the two texts share a number of technical and linguistic or, rather, rhetorical features that further illustrate the theoretical character of these mathematical traditions. A third example shows that the theoretical 'culture' betrayed by these texts is almost obligatory for the authors engaged in the field. This is how the astronomer Aristarchus (probably early third century BC), famous for having claimed heliocentricity, talks about the relation between two spheres (the second proposition of his little treatise *On the sizes and distances of the sun and the moon*):

Figure 2.1.3 Diagram illustrating Archimedes, *On the sphere and cylinder* I.1 (after Heiberg, 1972–5, I 13)
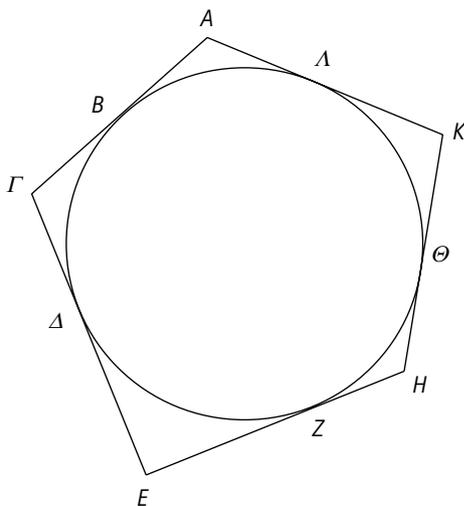
20. The proof is based on the axiomatic 'second assumption' that precedes the first proposition in Archimedes' text.
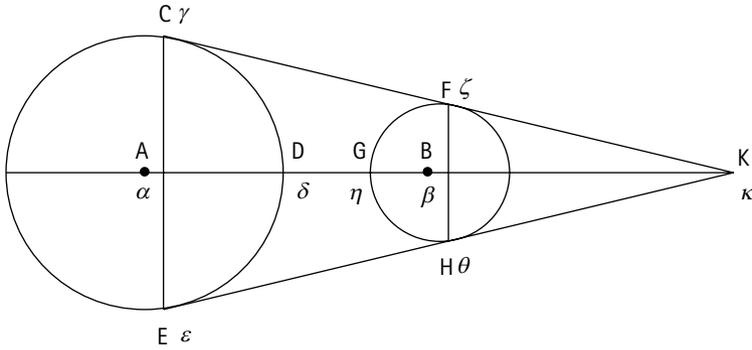
Figure 2.1.4 Diagram illustrating Aristarchus, *On the sizes*, prop. 2 (after Heath 1913, 358)

If a sphere be illuminated by a sphere greater than itself, the illuminated portion of the former sphere will be greater than a hemisphere. For let a sphere the centre of which is B be illuminated by a sphere greater than itself the centre of which is A. I say that the illuminated portion of the sphere the centre of which is B is greater than a hemisphere. For, since two unequal spheres are comprehended by one and the same cone which has its vertex in the direction of a lesser sphere,[21] let the cone comprehending the spheres be (drawn), and let a plane be carried through the axis; this plane will cut the spheres in circles and the cone in a triangle. Let it cut the spheres in the circles CDE, FGH, and the cone in the triangle CEK. It is then manifest that the segment of the sphere towards the circumference FGH, the base of which is the circle about FH as diameter, is the portion illuminated by the segment towards the circumference CDE, the base of which is the circle about CE as diameter and at right angles to the straight line AB; for the circumference FGH is illuminated by the circumference CDE, since CF, EH are the extreme rays. And the centre B of the sphere is within the segment FGH; so that the illuminated portion of the sphere is greater than a hemisphere. (transl. after Heath 1913, 359–361)

Instead of talking about celestial bodies, Aristarchus prefers to 'geometrize' the whole argument and to assume that these are just two given spheres. Illumination is conceptualized as a cone (and illustrated as a triangle). Aristarchus implicitly uses a proposition that has already been proved and accepted by the reader (proposition one, see note 13). Again, both the language and the structure of the argument are completely in line with what Euclid and Archimedes did. This is Greek theoretical mathematics. It seems fair to say that these texts are utterly different from those in which practical mathematicians codified their knowledge. Let us briefly describe the theoretical texts by keeping the characteristic features of the practitioners' textual traditions in mind. They are obviously different in at least three respects:

First, the text's intention is to *prove* a theorem by logical means, which implies that the status of the objects being discussed is general. Therefore, actual numbers

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21. It had been demonstrated in the first proposition that two unequal spheres are contained by one cone.

or measurements have no place in this type of mathematics. Even the diagram introduces only pseudo-individual forms: the 'two straight lines AB, ΓΔ' or the 'two circles CDE, FGH' are in truth any two straight lines or any two circles, respectively.

Second and accordingly, the mathematical writer is interested in the abstract properties of these general geometrical entities, not in calculating any quantitative properties of real objects or classes of real objects.

Third, the rhetoric of the two textual traditions is completely different. Whereas the recipe-like algorithms of the practical tradition employed strong personal appeals to the reader, the theoretical tradition produced highly *impersonal* texts (see Asper 2007, 125–135). This feature is unique, at least to this extent, in the context of all Greek scientific and technical literature and deserves a closer look. With the exception of exactly one formula that serves to introduce the repeated claim and marks the beginning of the proof ('I say that…' in the three examples above), these texts never introduce an authorial voice nor do they ever address the reader. (The introductory letters of Archimedes and Apollonius are not an exception to this rule: these letters employ a style that is indeed personal, but they are not an integral part of the mathematic texts they introduce. Rather, they have the status of 'paratexts'.) And even this 'I' is not personal in the usual sense, since it merely functions as a marker of the internal structure of the proof and will always show up at exactly the same place. Especially remarkable are the impersonal imperatives that regularly feature in the description of the objects concerned: English has to paraphrase theses imperatives with 'let' which makes them less strong: for example, 'let the straight lines AB, ΓΔ cut one another' (Euclid), 'let the polygon be circumscribed about a circle' (Archimedes), or 'let a plane be carried through the axis' (Aristarchus). In the Greek, these are imperatives in the third person, mostly in the passive voice, and often even in the perfect tense.[22] Hundreds of these strange forms exist in the works of theoretical mathematicians. As one would expect, these forms are hardly extant outside of mathematical language, that is, they are part of an exclusive discourse, of a sociolect. The writers present their objects to the reader as independently given, as something that is just there and can be contemplated objectively (Lachterman 1989, 65–67). They write themselves, their perceptions and their operations out of the picture, as it were,[23] which tends to add an air of timeless truth to what they have to say. The admirable rigor of this discourse is, however, achieved at the cost of explanation and, even more, any context of how the proof was found (famously

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22. The Greek forms of the translations quoted are: *temnetōsan* (Euclid: 'they shall cut one another!'), *perigegraphthō* (Archimedes: 'it shall be circumscribed!'), and *ekbeblēsthō* (Aristarchus: 'it shall be extended!').

23. The Aristotelian Aristoxenus (third century BC) explained mathematical imperatives in exactly this way (*Harmonica* II 33, pp 42–43 ed. Da Rios).

criticized by Lakatos 1976). This weird way of writing has no parallel in Greek writing and cannot be anything but a rhetorical stylization. The oral discourse of Euclid as he tried to convince a listener of any given proof would have probably contained many more personal markers (like demonstratives, interjections, or personal pronouns).[24] Thus, the main function of the rhetoric of impersonality is to convey objectivity. Generally, this is still (or again) the case in modern 'hard' science (Storer 1967, 79; Rheinberger 2003, 311–315).

As different as the texts of the two traditions may appear at first glance, they also share at least two features. One is their regular use of diagrams, the second is the thorough standardization of their language.[25]

(a) Greek theoretical mathematics always relies on a lettered diagram in order to add to the logical force of the proof by means of visual evidence. The lettered diagram has proved so powerful that it is still used in modern science, in a nearly unaltered form (occasionally, today it is 'numbered' rather than 'lettered'). Neglected for a long time, the diagram in Greek theoretical mathematics has recently been rediscovered as a very important feature (Netz 1999a), crucial to the communicative success of any proof. However, to anyone examining the traditions and texts of practical mathematics, especially the Near Eastern ones, it becomes quite clear that the specific 'theoretical' Greek lettered diagram is somehow related, like a younger member of the same family, to the 'numbered' diagram that we find in the practitioners' texts (see the example taken from the Papyrus Ayer above, Fig. 2.1.1) and, regularly, in Babylonian problem texts.[26] There, the parts of the diagram are, usually, connected to the relevant portions of the text by repeating the paradigmatic measures given in the text. Greek theoretical mathematics uses generalized indices, that is, letters that, in this case, do not signify numbers. The Greek lettered diagram is thus a generalization of the diagrams employed in the textbooks of practical mathematics and therefore closely related to the former (in my opinion, one of several reasons to think that Greek theoretical mathematics must have emerged from a practitioners' background).

(b) Anyone who works through the contents of either tradition will be amazed by how extremely regulated, even standardized, these texts actually are. The practitioners always use the recipe-structure, within a given text the introduction, the appeal to the second person, and the end of the actual

24. The mathematical passages in Aristotle or Plato (minus the general differences between written prose and oral discourse) might be a model of how mathematicians *talked* about their objects.

25. A third, not dealt with here, is the 'discrete' and, for Greek prose, highly unusual status of the major text-units in both traditions (see Asper 2003, 8–9).

26. See, for example, TMS 1, IM 55357, YBC 8633, YBC 4675 (see Damerow 2001, 230, 234, 244, and 280), all from Old Babylonian times.

problem always use exactly the same language. These texts, being standardized, are somewhat remote from oral discourse.[27] Since these structures are very old and easily cross cultures and languages, one could guess that the most effective way to protect this knowledge was to ensure that it was kept *traditional*, as is evident from the standardization of its textual forms. Although Greek theoretical mathematics is comparatively young, it is no less standardized, albeit in a more complex way and on several levels: the lexicon used is small, nearly free of synonyms, and confined by definitions that precede most extant works. Mathematical syntax is even more regulated: the same elements of any given argument show exactly the same form (Netz 1999a, 133–158 has listed more than a hundred such 'formulas'). Even the whole proof always consists of the same parts, always introduced by the same particles.[28] Of these, the famous phrase 'QED' (*hoper edei deixai*, 'just what one had to prove') is still used today.

Therefore, the language of Greek theoretical mathematics strikes one as being far removed from living oral discourse and its common rhetorical strategies, and being just as far removed from other forms of written argument. Already by the fourth century BC, Aristotle's readers did not appreciate mathematical prose aesthetically, because it was too different.[29] To understand why the theoretical tradition produced such unparalleled texts, why it even emerged this way, one has to dive deeply into historical inquiries, all of which greatly benefit from remembering the strong traditions of Greek practical mathematics that must have been constantly present in the environment of the theoreticians.

Theoretical mathematics in Athens: games of distinction

Theoretical mathematics of the kind outlined in the last paragraph existed only in Greece and was, by comparison with the mighty tradition of practical mathematics, clearly a local phenomenon, the distinctive features of which call for historical explanation or, at least, comment. The traditional, that is, ultimately Aristotelian, narrative of how mathematics and philosophy emerged in archaic Greece tells us that it all began in Asia Minor (modern Turkey), in the sixth century BC, with half-mythical characters such as Thales of Miletus (c 600–550 BC)

27. Eleanor Robson, however, mentions that, at least in Old Babylonian mathematics, the syntax is quite natural and the terminology 'local and ad hoc at best' (personal communication).

28. These parts, slightly different depending on whether the proof is a problem or a theorem, and also consistently standardized only in Euclid, were already isolated, described, and named by the ancient tradition, preserved in Proclus (fifth century AD): see Netz (1999b).

29. Aristotle, *Rhet.* 1404 a 12; *Metaph.* 995 a 8–12.

and his Milesian 'school' or, somewhat later, Pythagoras of Samos (c 550–500 BC) and his followers. It is difficult or even impossible to reach firm ground here (radical skepticism in Dicks 1959 and Burkert 1972). Both Thales and Pythagoras are credited with the discovery of geometrical theorems, for example the theorem above quoted from Euclid is ascribed to Thales by ancient tradition. It is disputed, however, whether these ascriptions are to be trusted (rather not, is my guess). Whatever semi-theoretical practices they and their possible successors might have engaged in, it is quite certain that they did not produce texts showing the characteristics discussed above. It was disputed already in antiquity whether they had even produced texts at all. The earliest history of Greek theoretical mathematics that we can lay our hands on begins in the late fifth century BC at Athens, where the center of theoretical mathematics in the Greek world would be located for some 120 years, until other centers emerged in the beginning of the third century BC, most notably at Alexandria in Egypt.

What is known about the persons involved and the contexts in which such a peculiar body of knowledge emerged? As always, too little. Partly because the impersonality of theoretical mathematics prevented authors from telling us anything about themselves (except in occasional introductory letters), partly because the older tradition was obliterated by the star mathematical writers of the third century BC. The sources are thus mostly indirect, scattered, and do not go back further than the fourth century BC: quotations found in Late Antique commentators from the earliest historical account of mathematics, by Eudemus of Rhodes, an Aristotelian scholar (fourth century BC),[30] and occasional remarks in the works of Plato and Aristotle.

After the shadowy prehistory of theoretical mathematics in eastern Greece,[31] the political and economical power of Athens in the second half of the fifth century attracted Greeks from Asia Minor who probably came as political representatives of their Ionian city-states. Two mathematicians from the island of Chios who were active at Athens are still known: Oenopides, who applied geometrical models to astronomical problems and is credited with a couple of methodological achievements, and, more prominently, Hippocrates, who seems to be the founder of the tradition of *Elements*, that is comprehensive axiomatic-deductive treatises in the later style of Euclid. His is the first theoretical *text* that we have in Greek mathematics: a short passage on the quadrature of 'lunules', that is, certain segments of circles, quoted by Simplicius through Eudemus (see Netz 2004). Although disputed in almost all its details, this text shows all the features that struck the reader as peculiar in the above quoted examples, especially the standardization, the

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30. Proclus in his commentary on the first book of Euclid's *Elements*, Eutocius (sixth century AD) commenting on Archimedes' works, and Simplicius (sixth century AD) on Aristotle's *Physics*. See Zhmud (2002).
31. There are more names, for example Phocus from Samos and Mandrolytus from Priene, but absolutely nothing is known about them.

impersonality, and those strange imperatives. No diagrams have been preserved, but the text obviously relies on several lettered diagrams. Hippocrates is usually dated to 430–420 BC (see Burkert 1972, 314 n 77). Therefore, by this time there must have already existed generic conventions for how to write theoretical mathematics. Furthermore, there must have been a desire to communicate the knowledge to someone, that is, to readers. For Eudemus, writing about a hundred years later, Hippocrates was the founder of the genre that included *Elements* and, apparently, the first 'real' theoretical mathematician in Athens. After him, throughout the fourth century BC, we hear of about twenty names (all in Proclus) and even groups of people in Athens associated with theoretical mathematics, partly in contact with philosophers and astronomers. It is clear, though, that at least some mathematicians were not part of any of these other groups, especially not the Pythagoreans or, later, the followers of Plato. From about 400 BC onwards, at the latest, a mathematical community, however small, must have existed in Athens.

There are reasons to believe that this theoretical knowledge did not suddenly fall from the sky or off the trees, but differentiated itself from the practitioner's knowledge. First, some of the terminology in Euclid betrays a practical origin, for example the term for 'drawing a straight line' (*teinō*, literally 'stretch out'), goes back almost certainly to the aforementioned surveying practices of the 'rope-stretchers'. Similarly, expressions for geometrical entities as angles, certain figures, or the perpendicular go back to craftsmen's traditions.[32] Second, the curious definition of the line in Euclid (Book 1, def. 2) as 'a length without width' makes perfect sense when one realizes that the experts of the practical traditions, when concerned with measuring, always assume a standard width for every line they measure. The theoretician's text is being implicitly, but quite openly polemical here (Høyrup 1996, 61, according to whom the same is true for *Elem.* II.1–10). Third, there is a model for conceptualizing the emergence of theoretical mathematics from practitioners' knowledge. A typical genre of such practitioners' groups and their competitive struggles is the riddle, used by experts to challenge one another. Such riddles are characteristically compound problems, and apply practical methods to improbable problems that already touch upon a theoretical realm (Høyrup 1997, 71–72). According to the traditional histories of Greek mathematics, three problems were at the center of the field's attention from the beginning: how to square the circle, how to trisect an angle, and how to double the cube (Saito 1995). On the one hand, these are *problems to solve*, not theorems to prove, and, therefore, belong to the practical sphere. On the other hand, it is not quite clear in what situation one would be faced with the task of, for example, doubling a cube. Therefore, one model for the transition from practice to

32. Gandz (1929–31, 273–275). More examples in Burkert (1982, 135–136); Høyrup (2002a, 400–405).

theory might indeed be the riddle, which pushed competitive practitioners towards theory.

One can draw another inference from the impersonality and the uniquely coherent terminology of Greek mathematical theory. In other intellectual fields, most notably in medicine and philosophy, one observes the opposite: texts are strongly personal and terminology changes between individuals or, at least, between groups (Netz 1999a, 122 f.).[33] In these cases, both personality and group-related terminology function as instruments in a competitive struggle among the participants in the field, a competition aimed at patients, in the case of the physicians, and at the glory of being right in the philosophers' discussions. As was the case with the authorial 'I', in mathematical texts the reader comes across only one standardized usage of an 'integrative we' (reserved for the formula '…as we will show…'), meant to conjure up a group spirit, so often used in philosophy and medicine. In medicine, personality and polemics in the texts reflect a competitive field, at least partially for economic reasons (Miller 1990, 39). Mathematical practitioners were specialized professionals, paid for their services. The sheer impersonality of the texts, in the case of theoretical mathematics, however, hints instead at a field comparatively free from economic pressures, a field that, for precisely that reason, remained fairly autonomous. The group of theoretical mathematicians in Athens must therefore have been quite homogenous in social terms.

Perhaps it would be adequate to think of theoretical mathematics as some form of *game* rather than something pertaining to a professional occupation, which it has become today, and which practical mathematics has always been. The persons who played this game were certainly at home in the upper circles of Athenian society (evidence collected by Netz 1999a, 279 f.), similarly to Plato and his followers who eagerly absorbed theoretical mathematics. From the majority's perspective, comedians could already make fun of mathematicians in 414 BC.[34] They must have felt like an elitist little group among Athenians. For them, theoretical mathematics was probably a status practice, perhaps enforced by the fact that the most common status practice, that is, politics, became quite dangerous for the old upper class at the end of the fifth century. Mathematics was, as philosophy was to become, a status-conscious way to keep one's head down.[35] There were, however, practitioners around who were, for Athenians from good families, socially unacceptable but who also had some share in mathematical knowledge and its practices. I suggest that many of the odd features of the theorists, such as expressly refusing to mention any practical applications or any useful effect, worked intentionally as distinctive markers, meant to distinguish the precious

33. The last becomes very clear in the writings of the physician Galen (second century AD), especially in his treatise on terminology (*On medical names*, extant only in Arabic).
34. Aristophanes in his *Birds* (v. 1005), targeted at the astronomer Meton.
35. Netz (1999a, 293–294); Fowler (1999, 372). See also Carter (1986, 131–186).

game of distinction[36] from sordid occupations that were carried out by people for hire.[37] Plato himself defines, quite polemically, the difference between practical calculations and theoretical mathematics (*Philebus* 56 D 3–57 A 3, a perspective that is still inherent in the modern opposition of 'pure' and 'applied' sciences). Ancient narratives concerning the emergence of mathematics proper always stress its emancipation from the demands of daily and practical life.[38] Later, this game of distinction effortlessly blended into the Platonic disdain for everything material. (It is difficult to decide whether Platonism adopted theoretical mathematics because it perfectly satisfied the Platonists' desire for immaterial, transcendent truths, or whether Plato and his followers edged theoretical mathematics even further into the ivory tower.) Certainly the game of distinction was already being played before Plato had even dreamed of his forms. A late and, almost certainly, inauthentic anecdote illustrates my point nicely:

Someone who had taken up geometry with Euclid, asked after he had understood the first theorem: 'What is my profit now that I have learned that?' And Euclid called for his servant and said: 'Give him a *triōbolon*, since he must always make a profit out of what he learns'.[39]

The point of theoretical mathematics is precisely that one does *not* gain any material profit from it. The *triōbolon*, here probably synonymous with 'small change', was the day's wage of an unskilled worker in classical Athens, which would bring out the contempt for 'work' on behalf of the mathematicians even better. The anecdote, one of several about Euclid that are all best met with skepticism, probably belongs to a Platonist milieu which began to dominate theoretical mathematics some time after Euclid's lifetime.

For an Athenian gentleman devoted to theoretical mathematics between 420 and 350 BC, however, professional experts of lower social status were not the only group from which it was necessary to demarcate his own pursuits: since the middle of the fifth century, there had been the sophists and, increasingly, the philosophers, both of whom had their own ways for intruding into pure and agoraphobic mathematics. The sophists were professional experts of knowledge and, as such, promised to teach political success, which in a society largely based on public debate depended largely on the use of rhetoric. Some of the sophists, apparently trying to top *all* existing forms of knowledge, tackled the conventions of theoretical mathematics with rather silly objections.[40] Others tried their ingenuity by

36. The expression is taken from Bourdieu (1979, 431 (*jeu distinctif*)). Of course, ancient Athenian upper-classes had, just as their modern equivalents, several 'games of distinction' at their disposal, for example, chariot races.

37. What Netz (2002b) has termed the 'marginalization of the numerical' in theoretical mathematics, I see as one more of these distinguishing moves targeted against the practitioners whose practices were almost exclusively numerical.

38. For example, Aristotle, *Metaph.* 981 b 20–25; Proclus, *In Eucl.* pp. 25.12–26.9.

39. Stobaeus, *Anthologium* 2.31.114 (pp. 228.25–29 ed Wachsmuth/Hense).

40. Protagoras in Aristotle, *Metaph.* 998 a 2 f.; compare *Anal. post.* 76 b 39–77 a 4.

solving the quadrature of the circle, again in pointedly amateurish fashion.[41] By doing this, the sophists aimed probably not at the mathematicians, but at potential customers to whom they could demonstrate that they 'knew everything', which was the typical sophistic claim (Hippias in Plato, *Hippias maior* 285 B 7–286 A 5). Viewed from this perspective, the mathematical excursions of the sophists indicate that they had found a body of theoretical mathematical knowledge at Athens, against which they tried to set themselves up as experts. There is no indication, however, that the mathematicians even bothered to engage with these dilettante newcomers.

A similar argument can be made about the philosophers in Athens.[42] Followers of Socrates took a great interest in theoretical mathematics: First, the Socratic Bryson (c 365 BC) tried to square the circle (Aristotle, *Anal. post.* 75 b 40–76 a 3). Similar to Antipho, he proceeded from premises that were too general and thus failed. Aristotle calls this attempt 'sophistic', thereby indicating an outsider's approach. Second, and far more importantly, Plato and his circle discovered in mathematical knowledge a paradigm of the epistemological quality they were generally after. Plato's criticisms of mathematical methods show that, again, his is an outsider's interest.[43] Most of the people in the Academy were not mathematicians, but were eager to discuss meta-mathematical questions and to apply the deductive logic of mathematical proofs to dialectics and even to science in general (as Aristotle did in his *Second analytics*). There is no reason to assume that the mathematicians were interested in these generalizations. Proclus has preserved a significant statement of the otherwise unknown mathematician Amphinomus (first half of the fourth century BC) who boldly contended that it is *not* the business of the mathematician to discuss the epistemological foundations of his work.[44] The mathematicians' desire to distinguish themselves from other discourses of knowledge obviously worked to distance themselves from the philosophers, too. The clearer the distinction and the more exclusive the group, the more enjoyable were the games of distinction.

The lack of institutions for theoretical mathematics

Since games require peers, but not necessarily readers, the remarkable form of mathematical *texts* deserves our attention, too. They probably also served a

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41. Hippias in Proclus, *In Eucl.* p. 272.7–10; Antipho in Simplicius, *In Arist. Phys.* I 2, p. 54.12–55.11 ed Diels.

42. There is a dubious tradition that Anaxagoras, a friend of Pericles, engaged in theoretical mathematics, in approximately 450 BC (Ps.-Plato, *Amat.* 132 A 5 f.; Proclus, *In Eucl.* p. 65.21–66.1).

43. Plato, *Resp.* 510 C 2–D 2; 533 B 6–C 4.

44. Proclus, *In Eucl.* p. 202.9–12.

certain function within their original contexts of communication. Many of the features of the theoretical tradition have the effect of ensuring the correct understanding of the texts, especially multi-leveled standardization. In a social context where mathematics is regularly explained by a teacher to disciples, texts used in instruction can afford to be less rigorous and less standardized because there is always the option of live dialogue to ensure that the knowledge is transmitted. The texts of the practical tradition were probably always accompanied by personal, oral instruction that filled in the gaps, explained terms, and so on. Unlike these practical texts, the theoretical tradition produced *autonomous* texts, that is, texts that were able to exclude misunderstandings all by themselves, that were able to force readers into a consensus by realizing the mathematical truth. This is the reason for defining crucial terms, standardizing the structure of proofs, and for excluding the context of discovery, every personal trace, and all controversy. Greek mathematical prose in the theoretical tradition is a paradigm of written knowledge transmission, rigorous in a way that still works today for any reader of Euclid or Archimedes. The lettered diagram plays not a small part in this achievement because it transports visual evidence from the author to the reader.[45] For all these reasons, one can happily read these texts alone, without a teacher, and *still* be fairly sure (as much as is possible in written communication, and compared to, say, poetic or historiographic texts) that one understands the argument in the way the authors intended it to be understood. Thus, the theoreticians have created a powerful, very reliable means of purely written communication.

The practitioners imparted their knowledge from generation to generation within a guild-like institutionalized framework. *For theoretical mathematics*, however, *there was no institutional background* in fifth- and fourth-century Greece (of course not, since the point of this socially distinctive game was its being 'useless'). In its infancy, Greek theoretical mathematics lacked institutionalization, which Netz (1999a) has shown convincingly. Plato and Aristotle lament that the city-state has no esteem for and, accordingly, provides no structures for theoretical mathematics.[46] True, Plato makes his guardians learn abstract mathematics—but his point seems to be that in real-life Athens nobody did (*Resp.* 525 B 3–528 E 2). Initially, Greek mathematicians had too few people around to talk to, so they resorted to writing and travel. Mathematicians were forced to write rather than discuss (compare Plato, *Theaetetus* 147 D 3) and, therefore, developed textual forms that could function perfectly in writing alone. In places such as Syracuse or Cyrene, there might not have even been any continuous oral tradition, a scenario quite different from our practical mathematicians whose group-structure ensured that the trade was handed down from generation to generation.

45. See, however, Saito, Chapter 9.2 in this volume on 'over-specification'.
46. Plato, *Resp.* 528 B 6–C 8; Aristotle, fr. 74.1 ed Gigon (= Iamblichus, *Comm. math. sci.*).

Standardization of proof-structure and the theoretical lexicon may have helped to increase the probability of successful knowledge-transmission.

The situation was different in fourth-century Athens and in third- and second-century Alexandria but, by then, the genre of mathematical prose had already emerged with its distinctive features. Besides, even in the third and second centuries, letters and travel were typical for theoretical mathematicians outside of Alexandria, as can be glimpsed from the introductory letters of Archimedes, of Diocles (ed Toomer 1976), and of Apollonius. Instead of walking into a classroom and presenting a new theorem to his graduate students, Archimedes in Syracuse sent letters to the other end of the world challenging his friends in Alexandria to find the proofs of the theorems he has just found.[47] Further, the genre of *Elements* with its peculiar linguistic characteristics emerged as an ideal medium of how to store the pertinent knowledge and is still used in this capacity today.

From a modern perspective it is difficult to imagine that (theoretical) mathematics might not have been institutionalized in some way. In classical Greece, institutionalization proper did not begin with a sudden widespread interest in theoretical knowledge, but with practices of political representation. After the followers of Plato and Aristotle had developed a lively interest in theoretical mathematics throughout the latter half of the fourth century BC, Hellenistic dynasts, above all the Ptolemies in Alexandria, the Seleucids at Antioch in Syria and, on a less grand scale, Hiero at Syracuse in Sicily, sponsored theoretical mathematics just as they funded poets and grammarians: as a contemporary form of pan-Hellenic representation. Intellectuals added to the royal splendor.

Paradoxically, we know next to nothing about Euclid. He may or may not have been the one who migrated from the then thriving mathematical scene of Athens to Alexandria, some time between 320 and 280 BC, and with whom mathematical institutionalization began at Alexandria.[48] There, the Ptolemies had also established some center of engineering, not least because they were keenly interested in siege engines, for which the successful construction and use of practical mathematics was of great importance. There, a tradition of teaching and writing practical mathematics continued into Byzantine or even Arabic times. The most important author of this tradition is the aforementioned Hero of Alexandria, who himself bridged both the practical and the theoretical traditions. Furthermore, the Ptolemies assembled mathematically minded astronomers in Alexandria, for instance Conon of Samos (third century BC) who also wrote on conic sections. Apparently there was a nearby observatory. All these persons[49] must have constantly met and debated with one another. In Archimedes' and Apollonius'

47. Archimedes, *Sph. cyl.* praef. vol 1, pp. 168.3–5 ed Heiberg 1972; *Lin. spir.* praef. vol 2, pp. 2.2–6; *Meth.* praef. vol 2, pp. 426.4–7.

48. At least, this is what Pappus of Alexandria (fourth century AD) tells us (*Coll.* p. 678 ed Hultsch).

49. More names and affiliations in Asper 2003, 27.

introductory letters, we strongly sense the existence of a small 'scientific community', again with notions of elitist distinction.[50] There were several libraries that served scholarly purposes and the famous Mouseion, an institution that gathered and awarded royal stipends to scholars from a number of disciplines, including grammar and, probably, medicine. It was here, with this concentration of various sorts of mathematicians that a stable tradition of theoretical mathematics emerged that betrays signs of teaching and canonization (editions of mathematical 'classics', commentaries, and collections), with the later works more firmly embedded in Platonist philosophy and curriculum. But even then, theoretical mathematics remains a discourse based on writing and confined to very small, socially elevated circles. They were still not professionals in the modern sense, as the mathematical practitioners had always been. Despite the astonishing prominence of theoretical mathematics in modern times, which invites anachronistic re-projections, in ancient Greece theoretical mathematics must always have been an epiphenomenon, or rather, a marginal differentiation, of strong practical traditions.

Conclusion: the two mathematical cultures of ancient Greece compared

Practical mathematics must have been present in all the previously mentioned times and places, albeit socially invisible. Mostly, its practitioners worked with their long-established methods without ever paying attention to the theorists and their games. On the other hand, upper-class theorists must have aimed at staying clear of modest craftsmen. Occasionally, one can suspect a direct, polemical reference by theoretical mathematics directed against the practitioners. Platonic ideology contributed its share to the dichotomy, which was apparently quite strict at times. Rarely did somebody bridge the two traditions, which must have occurred regularly in the very beginnings of the theoretical tradition. One might understand these respective bodies of knowledge as complementary and, almost, as mutually explanative:

Greek practical mathematics:

- was derivative of older traditions that, ultimately, originated in the ancient Near East;

- solved 'real-life' problems;

- communicated actual procedures in order to convey general methods;

- used written texts (if at all) as secondary means of knowledge storage and instruction;

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50. Both ask their addressees to distribute their findings only to those who are deserving: Apollonius, *Con.* II praef. vol 1 pp. 192.5–8 ed Heiberg 1893; Archimedes, *Sph. cyl.* I praef., vol 1 pp. 4.13 f.

- employed 'social' technologies of trust, that is a rhetoric based on institutional authority; for example, the guild's pristine tradition, the specialist status of its practitioners, and the knowledge's commonly accepted usefulness;

- worked within a stable and highly traditional social—that is, institutional—framework.

Greek theoretical mathematics:

- emerged in sixth- to fifth-century Greece, at least partly from a practical background;

- was a theoreticians' game with artistic implications, pointedly removed from 'real life';

- communicated general theorems concerning ideal geometrical entities;

- depended on writing and produced autonomous texts;

- employed epistemological technologies of trust based on evidence and logic;

- was not institutionalized, at least not during its formative stages.

The two fields differ so greatly with respect to their practices, traditions, milieus, functions, methods, and probably also the mindsets of their participants that I could not resist adopting the catchphrase of the 'two cultures'. Thus, when approaching mathematics in ancient Greece, perhaps one should rather think of two mathematical cultures, in many respects neat opposites.[51] To the leading circles of any given ancient Greek community, the practitioners were probably almost socially invisible. As far as sizes of groups and social presence in everyday life are concerned, however, the theorists were never more than an epiphenomenon. Apparently, the unusual characteristics of theoretical mathematics evolved as markers of differentiation, meant to stress a distance from the social and epistemic background that was associated with practical mathematics.

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51. One might also find a dichotomy similar to the one described in this paper in Greek medicine (rationalist Hippocratic medicine versus 'magicians'), and perhaps even in historiography ('serious' historiography versus mere 'storytelling'). Compare also Rihll and Tucker (2002, 297–304).

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