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– August 18 –

An algorithm for detecting intrinsically knotted graphs, yielding many new minor minimal IK graphs.

Ramin Naimi
Occidental College

We describe an algorithm that detects seemingly most (and possibly all) intrinsically knotted (IK) graphs, and can help find knotless embeddings for graphs that are not IK. We have used this algorithm, implemented as a Mathematica program, to find many new minor minimal IK graphs, 200 of which we have verified "manually" to be minor minimal IK. We have also used this algorithm to find knotless embeddings for several graphs, some of which had previously resisted our attempts to do so. (Joint work with Noam Goldberg, Thomas Mattman, and Jonathan Miller.)

Graphs of 20 edges are 2–apex, hence unknotted

Thomas W. Mattman
California State University, Chico

A graph is 2–apex if it is planar after the deletion of at most two vertices. Such graphs are not intrinsically knotted, IK. We investigate the converse, does not IK imply 2–apex? We determine the simplest possible counterexample, a graph on nine vertices and 21 edges that is neither IK nor 2–apex. In the process, we show that every graph of 20 or fewer edges is 2–apex. This provides a new proof that an IK graph must have at least 21 edges. We also classify IK graphs on nine vertices and 21 edges and find no new examples of minor minimal IK graphs in this set.

Unraveling tangles

Catherine Farkas
University of Illinois at Chicago

In 1972, Kinoshita gave an example of a non-planar θ_3 curve which contained no knots. In 1984, Suzuki generalized this example to show that for every $n \geq 3$ there is a non-planar θ_n-curve every subgraph of which is planar. New interest in this area has been sparked by its relationship to entanglements of polymers as well as by the challenge of synthesizing organic molecules with an entangled structure. Motivated by descriptions of entangled molecular structures known as *ravels*, we introduce a method of constructing a θ_4 curve from a tangle T, and show that if the tangle is algebraic then every such θ_4 curve is either planar or contains a knot.

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On strongly almost trivial embeddings of graphs

Ryo Hanaki
Nara University of Education

A continuous map φ from G to $\mathbb{R}^2$ is called a *projection* of G if its multiple points are only finitely many transversal double points away from the vertices. For a spatial embedding f of G, we say that φ is a *projection of* f if there exists a spatial embedding f' of G which is equivalent to f such that $\varphi = \pi \circ f'$ where π is the natural projection. Then we also say that f *is obtained from* φ. A projection φ of G is said to be *trivial* if only trivial spatial embeddings of G are obtained from φ. A spatial embedding f of G is said to be *strongly almost trivial* if f is nontrivial and there exists a projection $\hat{f}$ of f such that $f|_H$ is trivial for any proper subgraph H of G.

In this talk, we present new classes of graphs which have a strongly almost trivial embedding and that of graphs which have no strongly almost trivial embeddings. Then we show that both a property that a graph has a strongly almost trivial embedding and a property that a graph has no strongly almost trivial embeddings are not inherited by minors.

– Lunch time –

Topological symmetry groups and local knotting

Erica Flapan
Pomona College

If a graph Γ is embedded in S^3, then the *symmetries of* Γ can be represented by automorphisms of the abstract graph that are induced by a homeomorphism of S^3. The topological symmetry group, $\mathrm{TSG}_+(\Gamma)$, is the subgroup of the automorphism group of Γ consisting of those automorphisms induced by orientation preserving homeomorphisms of S^3.

In general, we are interested in which groups can occur as $\mathrm{TSG}_+(\Gamma)$ for some graph Γ embedded in S^3. Frucht showed that every finite group is the automorphism group of some abstract graph. By contrast, we showed that while every finite abelian group and every symmetric group can occur as $\mathrm{TSG}_+(\Gamma)$ for some embedded graph Γ, the alternating group A_5 and the cyclic groups of prime order are the only simple groups which can occur. However, it is not known what groups can occur as $\mathrm{TSG}_+(\Gamma)$ for any embedding of a particular graph Γ in S^3.

We are interested in whether all of the subgroups of a given $\mathrm{TSG}_+(\Gamma)$ can occur as $\mathrm{TSG}_+(\Gamma')$, for some re-embedding Γ' of Γ in S^3. We prove the uniqueness up to isotopy of a ball containing all of the local knots on an edge of an embedded graph, and use this result to give a positive answer to this question in many cases. This is joint work with Blake Mellor and Ramin Naimi.

Topological symmetry groups of complete graphs

Blake Mellor
Loyola Marymount University

Given an embedding Γ of a graph in S^3, the *topological symmetry group* $TSG(\Gamma)$ is the subgroup of the automorphism group of the abstract graph which is induced by orientation-preserving homeomorphisms of the pair (S^3, Γ).

Complete graphs are an interesting class of graphs to consider, since they have the largest possible automorphism groups. Flapan, Naimi and Tamvakis have shown that the only possible topological symmetry groups for complete graphs are finite subgroups of $SO(3)$ or $D_m \times D_m$ (with m odd). However, their result does not specify which complete graphs admit embeddings realizing particular topological symmetry groups.

In this talk, we answer this question for the symmetry groups of the regular polyhedra, A_4, S_4 and A_5. For each of these groups, we will determine exactly which complete graphs have an embedding which realizes that group as its topological symmetry group. As part of our investigation, we develop an Edge Embedding Lemma which can be applied to many other groups and graphs as well. This is joint work with Erica Flapan and Ramin Naimi.

Topological symmetry groups of K_1 to K_6 and K_{4r+3}

Dwayne Chambers
Claremont Graduate University

We focus on the complete graphs on n vertices, K_n, embedded in S^3 for n less than 7. In particular we characterize all the groups which can occur as the topological symmetry group of K_n. We also summarize results showing that only certain cyclic, dihedral, and products of two cyclic groups of odd order can occur as the topological symmetry group of an embedding of the complete graph K_{4r+3} in S^3.

Achirality and linking numbers of links

Saori Matsuoka
Tokyo Woman's Christian University

An ordered and oriented n-component link L in the 3-sphere is said to be achiral if it is ambient isotopic to its mirror image ignoring the orientation and ordering of the components. For an ordered and oriented n-component link L, let λ_L be the product of linking numbers of all 2-component sublinks in L. For $n = 4m + 3$, where m is a non-negative integer, we show that if L is achiral then $\lambda_L = 0$. And for $n \neq 4m + 3$, we show that there exists an n-component achiral link L with $\lambda_L \neq 0$ by using achiral embeddings of complete graphs with n vertices K_n.

C_k-classifications on string links and spatial graphs with canonical disk/band surfaces

Akira Yasuhara
Tokyo Gakugei University

This talk is based on joint works with Jean-Baptiste Meilhan and with Ryo Nikkuni.

A family of finite type invariants of string link is given by the HOMFLYPT polynomial of knots that are obtained by various closure operations on (cabled) string links. We call such invariants closure invariants. These closure invariants are useful for C_k-classification of spatial graphs that have 'canonical' disk/band surfaces. Roughly speaking, if the C_k-classification of string links is given by closure invariants, then the C_k-classification of such spatial graphs is also given by the closure invariants. In particular, for $k \leq 5$, the C_k-classification of spatial complete graphs on 4 vertices is given by closure invariants.

– August 19 –

A sufficient condition for intrinsic linking

Ryan Ottman
University of California, Santa Barbara

I will provide a proof that any graph with n vertices and at least $4n-9$ edges is intrinsically linked. I will also mention a similar result for intrinsic knotting. Then I will discuss the key results which were necessary to classify the intrinsic knotting and linking of 1 and 2 deficient complete partite graphs. All of these results are from the paper "Intrinsic Knotting and Linking of Almost Complete Graphs" which grew out of work done in an REU.

Similarities between flat and planar graphs

Joel Foisy
State University of New York at Potsdam

Recall that a graph G is *flat* if G has a spatial embedding $\phi(G)$ for which every cycle of G bounds a disk whose interior is disjoint from $\phi(G)$. In this talk we discuss similarities between flat graphs and planar graphs. In particular, a $3-$connected planar graph has the property that, in a planar embedding, at every vertex the graph looks like a wheel with spokes connected at the center. A similar property can be seen to hold for flat graphs. Studying this property will hopefully lead to a better understanding of flat graphs and maybe even to a new proof of Sachs' linkless embedding conjecture.

A knotted handlebody and a spatial graph

Atsushi Ishii
University of Tsukuba

A knotted handlebody is a handlebody embedded in a 3-dimensional manifold M. We assume that $M = S^3$ in this talk. A regular neighborhood of a spatial graph is a knotted handlebody. Conversely, any knotted handlebody can be obtained from a spatial graph. I will explain a fundamental relationship between them, which shows when two spatial graphs represent the same knotted handlebody. Then I give an invariant for knotted handlebodies. We note that it is also an invariant for spatial graphs.

Bridge position and the representativity of spatial graphs

Makoto Ozawa
Komazawa University

First, we extend Otal's result for the trivial knot to trivial spatial graphs, namely, we show that for any bridge tangle decomposing sphere S^2 for a trivial spatial graph Γ, there exists a 2-sphere F such that F contains Γ and F intersects S^2 in a single loop. This characterizes the bridge position of a trivial spatial graph.

Next, we introduce two invariants for spatial graphs. As a generalization of the bridge number for knots, we define the *bridge string number* $bs(\Gamma)$ of a spatial graph

Γ as the minimal number of $|\Gamma \cap S^2|$ for all bridge tangle decomposing sphere S^2. As a spatial version of the representativity for a graph embedded in a surface, we define the *representativity* of a non-trivial spatial graph Γ as

$$r(\Gamma) = \max_{F \in \mathcal{F}} \min_{D \in \mathcal{D}_F} |\partial D \cap \Gamma|,$$

where $\mathcal{F}$ is the set of all closed surfaces containing Γ and $\mathcal{D}_F$ is the set of all compressing disks for F in S^3. Then we show that the representativity $r(\Gamma)$ does not exceed the half of the bridge string number $bs(\Gamma)$. In particular, if Γ is a knot, then $r(\Gamma) \leq b(\Gamma)$, where $b(\Gamma)$ denotes the bridge number. This generalizes the Schubert's result on torus knots.

Next, we show that for any 2-connected graph G, any positive genus closed surface F embedded in S^3 ($g(F) \geq g(G)$) and any integer n, there exists a spatial graph Γ of G contained in F such that $r(F, \Gamma) \geq n$, where $r(F, \Gamma) = \min_{D \in \mathcal{D}_F} |\partial D \cap \Gamma|$.

Finally, we show that if $r(\Gamma) > \beta_1(G)$, then Γ contains a non-free connected subgraph, where $\beta_1(G)$ denotes the first Betti number of G.

– Lunch time –

Knots and the cyclic polytope

Jorge Ramírez Alfonsín
Université Montpellier 2

The main purpose of this talk is to present a geometric method to explore some knot representation problems. We shall discuss in particular how the cyclic polytope can be used to investigate linear spatial representations and ropes of knots.

Intrinsic linking and knotting in straight-edge embeddings of complete graphs

Lew Ludwig
Denison University

It is well known that the complete graph on six vertices,K_6, is intrinsically knotted and K_7 is intrinsically linked. During a knot theory workshop conducted by Colin Adams, it was asked how many such links or knots would occur in complete graphs if the embeddings consisted of straight edges? In this presentation, we will discuss a three-year project with undergraduate students where we enumerated the number of links in straightedge embeddings of K_6 and K_7 as well as the number of knots in straight-edge embeddings of K_7. We will also discuss several open questions including triple-linking and linear ambient isotopy in straightedge embeddings of complete graphs.

Number of knots and links in linear K_7

Choonbae Jeon
Daeduk University

We present a computational result about the number of knots and links in linear embeddings of the complete graph K_7 into $\mathbb{R}^3$. From this result, we can determine the maximal number of non-trivial knots and links in linear embeddings of K_7.

Counting links in complete graphs

Thomas Fleming
University of California, San Diego

Intrinsic linking has been extensively studied and generalized in many ways, for example to intrinsic n-linking. These generalizations have focused on the minimum complexity of the links contained in graph embeddings. We will consider a different approach: counting the minimum number of nonsplit links (of any type) that must occur in an embedding. We will present results on complete partite graphs on 7 and 8 vertices.

Planar graphs producing knotted projections with three double points

Youngsik Huh
Hanyang University

A generic immersion of a planar graph into $\mathbb{R}^2$ is said to be *knotted*, if it is not a projection of any planar embedding of the graph in $\mathbb{R}^3$. It is known that the number of double points of any knotted projection is at least three, which is a previous work of the speaker. In this talk we determine the planar graphs which have knotted projections with three double points. This is a joint-work with Ryo Nikkuni.

– August 20 –

Multiplicity distance of spatial graphs

Kouki Taniyama
Waseda University

We define multiplicity and multiplicity distance in a category. We define neighbourhood category of spatial graphs and discuss its multiplicity distance. Here the distance is defined on the set of all spatial embeddings of all finite graphs.

Prime knots whose arc index is smaller than the crossing number

Gyo Taek Jin
Korea Advanced Institute of Science and Technology

It is known that the arc index of prime alternating knots is the minimal crossing number plus two and the arc index of prime nonalternating knots is less than or equal to the minimal crossing number. We study some cases when the arc index is strictly less than the minimal crossing number.

Knotting and linking in the Petersen family

Danielle O'Donnol
Rice University

An embedding f of a graph G is *linked* if there is a nontrivial link in $f(G)$. An embedding f of a graph G is *algebraically linked* if there is a link with nonzero linking number in $f(G)$. We will say an embedding f of a graph G is *complexly algebraically linked* (*CA linked*) if there is a link L in $f(G)$ where $|lk(L)| \geq 2$ or if there is more than one link in $f(G)$ with nonzero linking number. An embedding f of a graph G is *knotted* if there is a nontrivial knot in $f(G)$. This talk will examine the relationship between knotting and linking in the Petersen family, the set of minor minimal intrinsically linked graphs. We present a linking condition on the spatial graph that results in a knotted embedding.

On the Conway-Gordon theorems

Ryo Nikkuni
Tokyo Woman's Christian University

Let K_n be the complete graph on n vertices. In 1983, Conway-Gordon showed that for every spatial embedding of K_6, the sum of the linking numbers over all of the constituent 2-component links is congruent to 1 modulo 2, and for every spatial embedding of K_7, the sum of the Arf invariants over all of the Hamiltonian knots is also congruent to 1 modulo 2. In this talk, we introduce the following two topics:
(1) We give an integral version of the Conway-Gordon type theorem for every graph obtained from K_6 or K_7 by a finite sequence of $\triangle Y$-exchanges in terms of the square of the linking number and the second coefficient of the Conway polynomial. This is partially joint work with K. Taniyama.
(2) We say that a graph is intrinsically knotted or completely 3-linked if every spatial embedding of the graph contains a nontrivial knot or a 3-component link any of whose 2-component sublink is nonsplittable. We show that a graph obtained from K_7 by a finite sequence of $\triangle Y$-exchanges and $Y\triangle$-exchanges is a minor-minimal intrinsically knotted or completely 3-linked graph. This is joint work with R. Hanaki, K. Taniyama and A. Yamazaki.