On L–space twisted torus knots

Kazushige Tohki

Nihon University

Graduate School of Integrated Basic Sciences

Joint work with **K. Motegi** (Nihon Univ.)

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L–space

M : a rational homology sphere (i.e. $|H_1(M;\mathbb{Z})| < \infty$)
$\widehat{HF}(M)$: Heegaard Floer homology with coefficients in $\mathbb{Z}_2$

[Ozsváth-Szabó]

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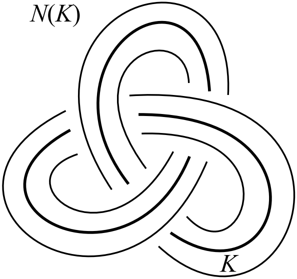
L–space

M is an L–space. $\Leftrightarrow$ $\text{rank}\widehat{HF}(M) = |H_1(M;\mathbb{Z})|$.

Example Lens spaces (except $S^2 \times S^1$) are L–spaces.

Dehn surgery

K : a knot in S^3
$N(K)$: a tubular neighborhood of K



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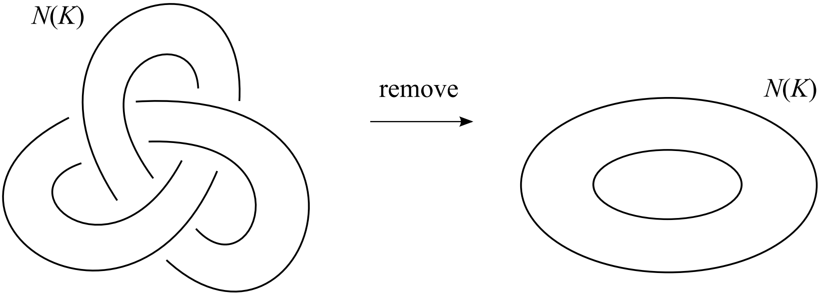

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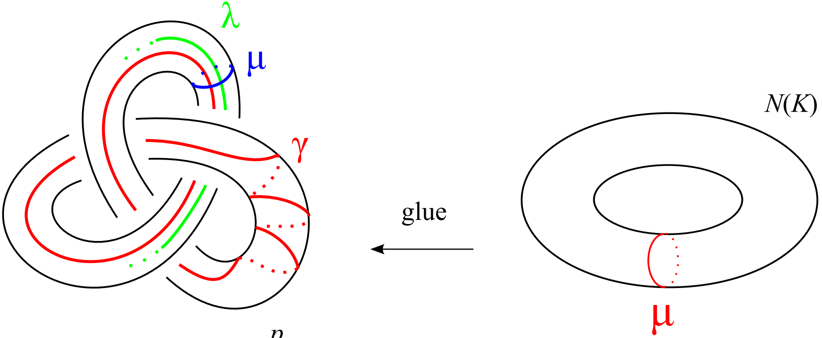

$\gamma = p\mu + q\lambda \quad \longleftrightarrow \quad \frac{p}{q}$

$K(p/q)$: a 3–manifold obtained from S^3 by p/q –Dehn surgery on K

K : a knot in S^3
$K(r)$: a 3–manifold obtained from S^3 by r–Dehn surgery on K

L–space knot

K is an L–space knot.
$\Leftrightarrow$ $K(r)$ is an L–space for some $r \in \mathbb{Q}$.

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Example $T_{p,q}$: Torus knots

Since $T_{p,q}(pq \pm 1)$ is a lens space($\neq S^2 \times S^1$),
$T_{p,q}$ is an L–space knot.

Example K : Berge knots

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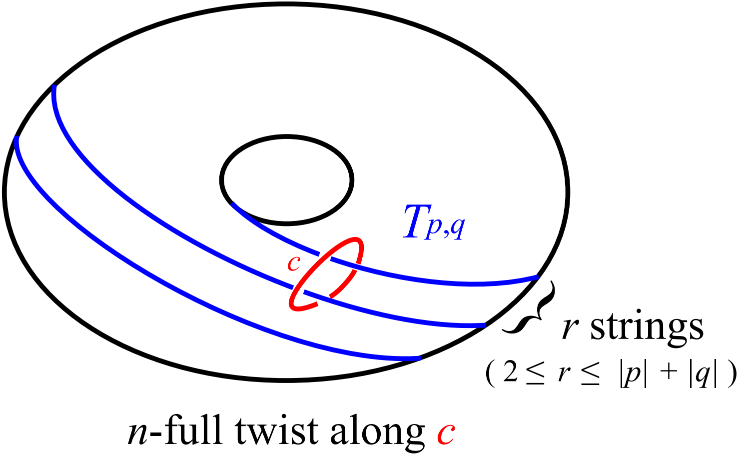
Example K : Berge knots
Since $K(m)$ is a lens space for some integer m,
K is an L–space knot.

Twisted torus knot

[Dean]



Twisted torus knot $K(p,\ q;\ r,\ n)$.

Twisted torus knot

Let $p > q \geq 2$.

Theorem [Motegi]

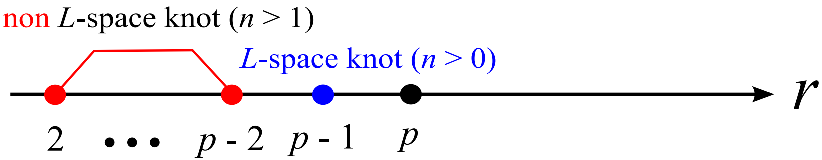
- A twisted torus knot $K(p, q; p+q, n)$ is L–space knot for all integer n.

- A twisted torus knot $K(p, q; p-q, n)$ is L–space knot for any integer $n \geq -1$.

Twisted torus knot

Theorem [Vafaee]

The twisted torus knot $K(p, mp \pm 1; r, n)$ is an L–space knot.
$(p \geq 2, m \geq 1, n > 0, 0 < m < p)$
$\Leftrightarrow$
either $r = p - 1$ or $r \in \{2, p-2\}$ and $n = 1$.



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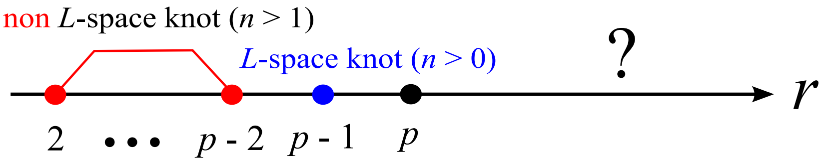

Twisted torus knot

Theorem [Motegi-Tohki]

- $K(p, mp{+}1; mp + 1{-}p, n)$ is an L–space knot for all integer n. $(p \geq 2, m \geq 1)$

- $K(p, mp{-}1; mp - 1{-}p, n)$ is an L–space knot for all integer n. $(p \geq 2, m \geq 3)$

• $K(p, mp-1; mp-1-p, n)$ is an L–space knot for all integer n. $(p \geq 2, m \geq 3)$

If $m = 1$, then $|mp - p - 1| = 1$.

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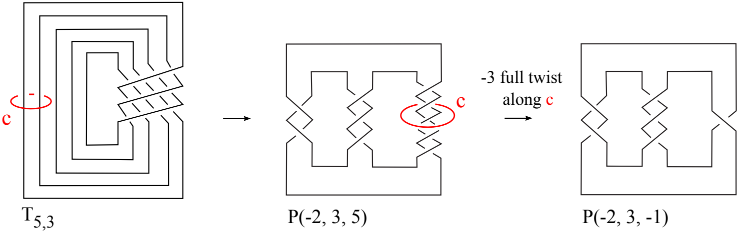
If $m = 2$, let $p = 3$. Then $K(3,\ 5;\ 2,\ n)$.



$P(-2, 3, -1)$ is a non L–space knot. [Lidman–Moore]

Twisted torus knot

Example $K(7,\ 20;\ r,\ n)\ \ (p = 7,\ m = 3)$

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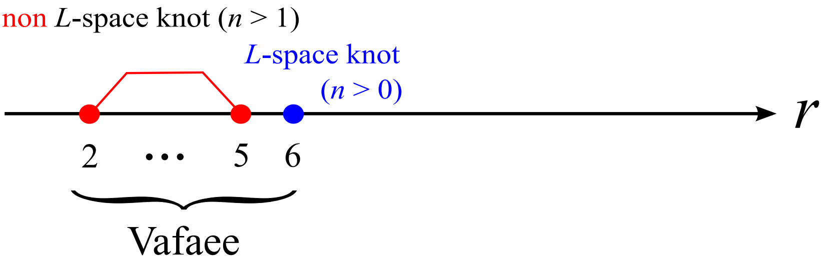
$K(7,\ 20;\ 6,\ n)$ is an L–space knot for all integer $n > 0$.
$K(7,\ 20;\ r,\ n)$ is a non L–space knot for $2 \le r \le 5$.

[Vafaee]



Twisted torus knot

Example $K(7,\ 20;\ r,\ n)$ $(p = 7,\ m = 3)$

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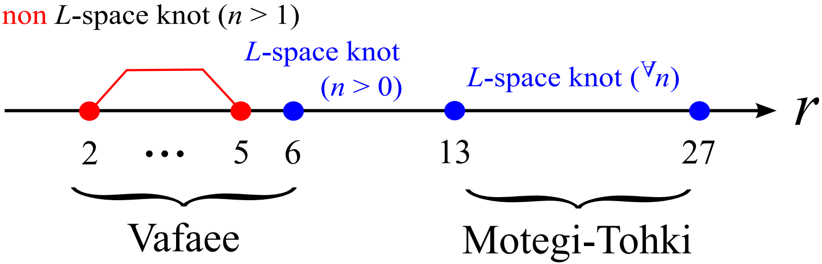
$K(7,\ 20;\ r,\ n)$ is a non L–space knot for $2 \leq r \leq 5$.

[Vafaee]

$K(7,\ 20;\ 13,\ n)$ and $K(7,\ 20;\ 27,\ n)$ are L–space knot for all integer n.

[Motegi-Tohki]

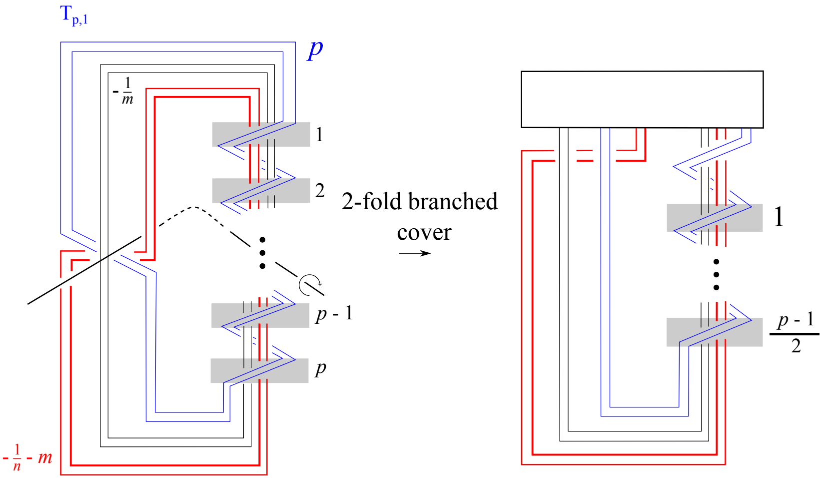


$K(p, mp+1; mp + 1 - p, n)$ is an L–space knot for all integer n.
$(p \geq 2, m \geq 1.)$

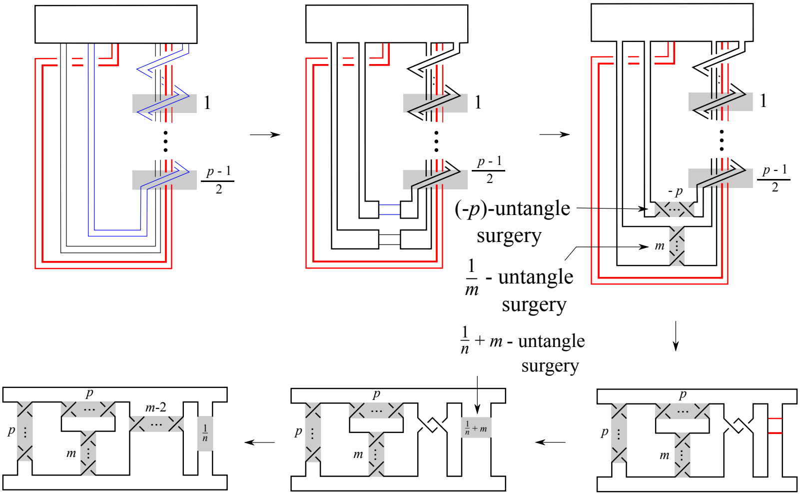
Outline of proof

- $K(p, mp+1; mp + 1 - p, n)$ is an L–space knot for any integer $n \geq -1$.

$p(mp+1) + n(mp-p+1)^2$–surgery on $K(p, mp+1; mp-p+1, n)$,result in a Seifert fiber space.



$K(p, mp+1; mp+1-p, n)$



$M(m-2, \frac{1}{p}, \frac{p}{mp+1}, \frac{1}{n}) \Rightarrow S^2(m-2, \frac{1}{p}, \frac{p}{mp+1}, \frac{1}{n})$ [Montesinos]

$K(p, mp-1; mp-1-p, n)$

$K(p, mp-1; mp-1-p, n)$ is an L–space knot for all integer n.
$(p \geq 2, m \geq 3)$

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