Unknotting twist knots by forbidden moves

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結び目の数学 VIII

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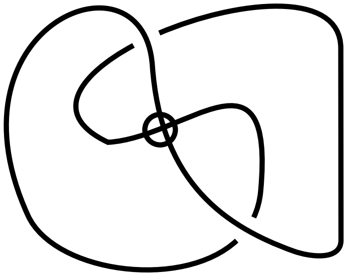
Virtual knot

Definition (virtual knot)

K is a *virtual knot*
$\overset{def}{\Leftrightarrow} K$ is an equivalence class of virtual knot diagrams under Reidemeister moves and virtual Reidemeister moves



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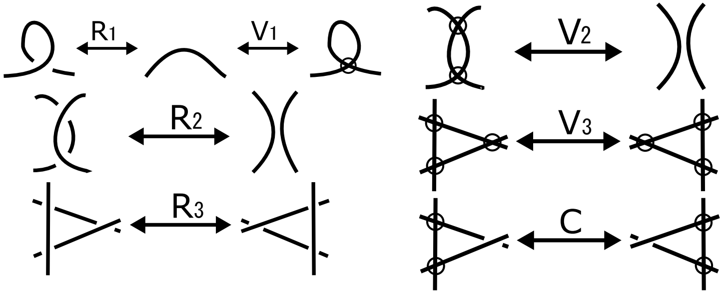
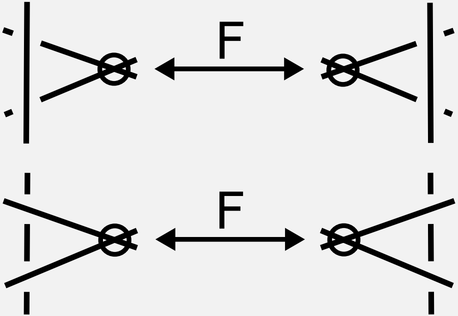


Forbidden move

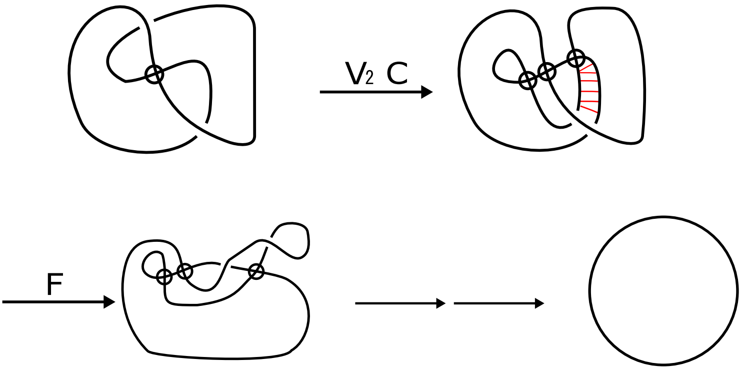
Definition (forbidden move)



Why "forbidden" ?

Theorem [Kanenobu,2001][Nelson,2001]

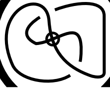
For any virtual knot K,there exists a finite sequence of Reidemeister moves,virtual Reidemeister moves and forbidden moves that takes K to a trivial knot.



Forbidden number F(K)

Definition (forbidden number)

Let K be a virtual knot.The $forbidden\ number$ of K , F(K), is the minimal number of forbidden moves necessary to transform K into a trivial knot.

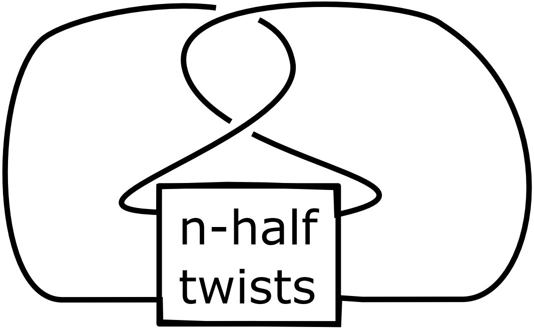
F() = 1

Upper bound

Theorem [Crans-Ganzell-Mellor, 2015]

Let T_n be the twist knot. ($k \in \mathbb{N}$).

$$F(T_{2k-1}) \leq 6k - 2$$
$$F(T_{2k}) \leq 5k - 1$$



Improvement of upper bound

Theorem

Let T_n be the twist knot. ($n \in \mathbb{Z}, k \in \mathbb{N}$).

$$
\begin{aligned}
&F(T_{2k}) \\
&F(T_{2k-1}) \quad \leq 5k-2 \\
&F(T_{-2k-1}) \\
&F(T_{-2k}) \\
&F(T_{-1}) = 0
\end{aligned}
$$

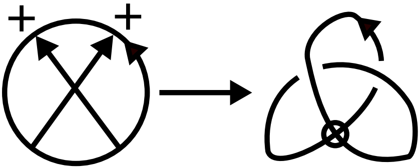
Corollary

$F(\ trefoil\ ) \leq 3$
$F(\ figure\text{-}eight\ ) \leq 3.$

Gauss diagram

Definition (Gauss diagram)

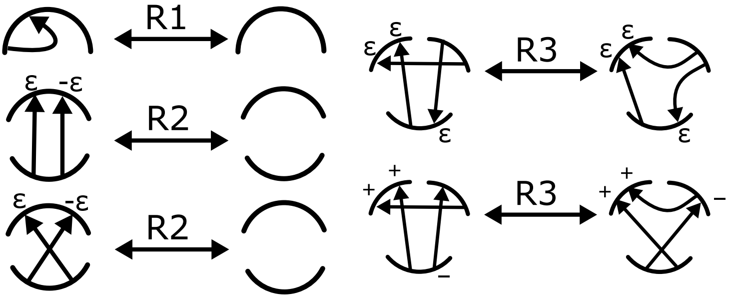
A *Gauss diagram* is the preimage of a virtual knot diagram with information of real crossings .



Theorem [Goussarov-Polyak-Viro, 2000]

There exists a one-to-one correspondence between all virtual knots and all equivalence classes of Gauss diagrams modulo the Reidemeister moves.

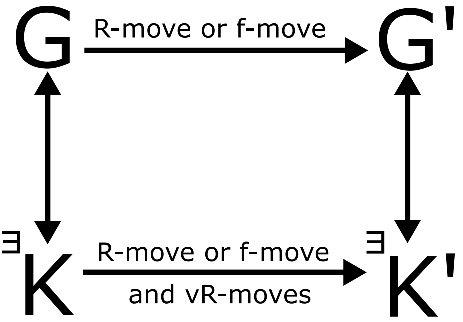
Reidemeister move on Gauss diagram



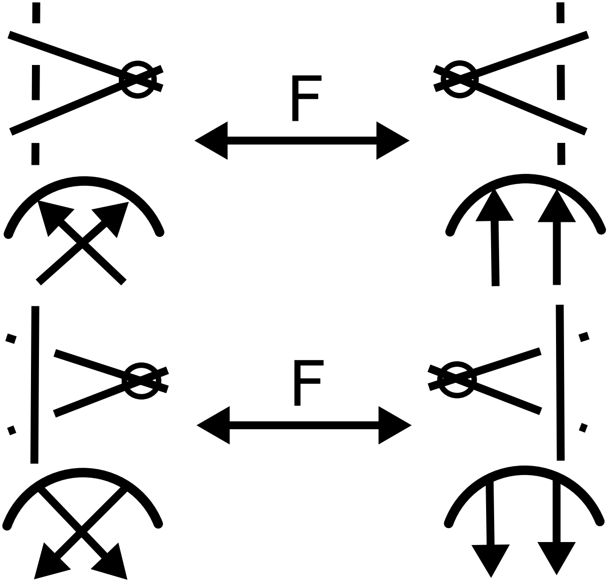
Fact

Fact

Let G,G' be Gauss diagrams
s.t G' is obtained from G by a Reidemeister move or forbidden move.
Then there exist virtual knot diagram K,K' corresponding to G,G' s.t K' is obtained from K by corresponding Reidemeister move or forbidden move with virtual Reidemeister moves.



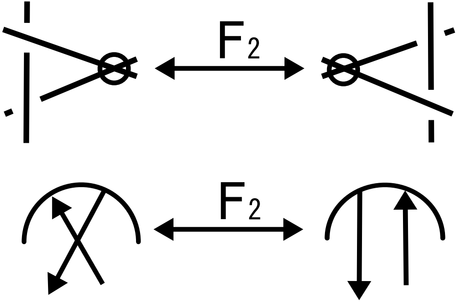
Forbidden move on Gauss diagram



F_2 move

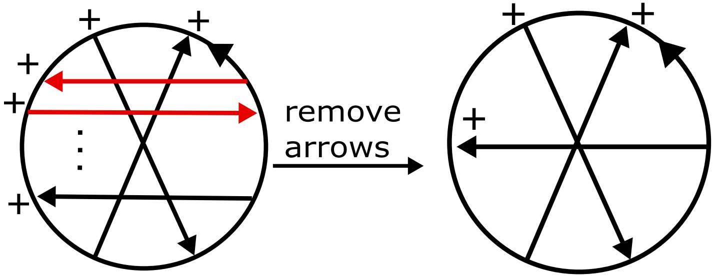
Theorem[Kanenobu,2001]

F_2 move is realized by using forbidden moves twice.

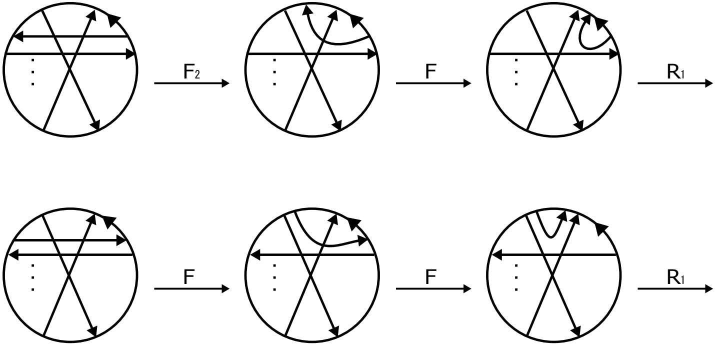


Outline of proof(1)

T_{2k-1}:

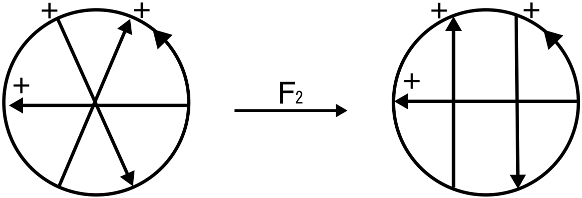


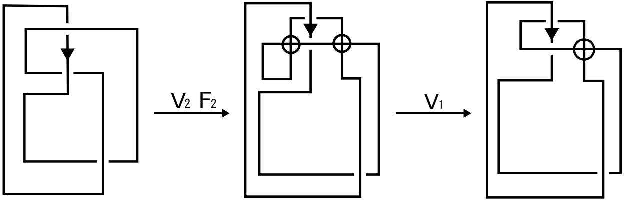
Remove arrows



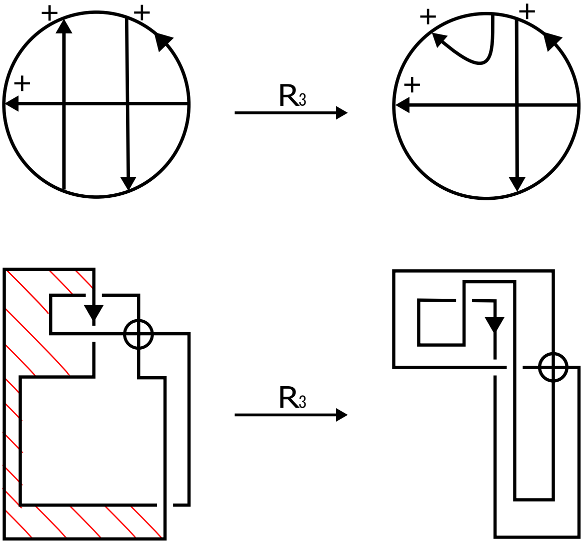
Using forbidden moves : 5 times

Outline of proof(2)

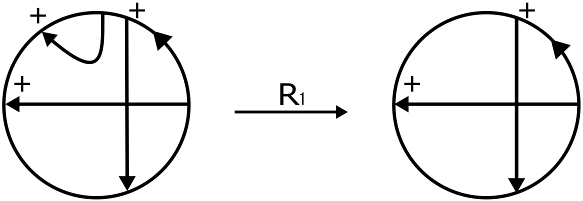


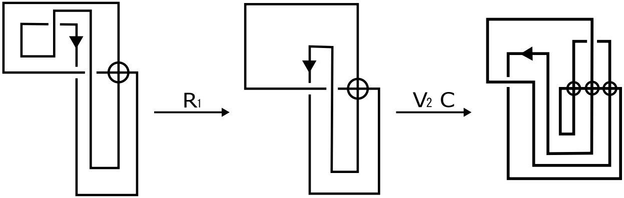


Outline of proof(3)

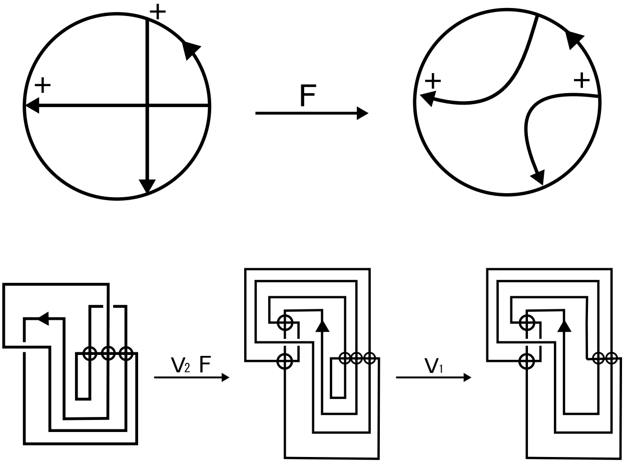


Outline of proof(4)

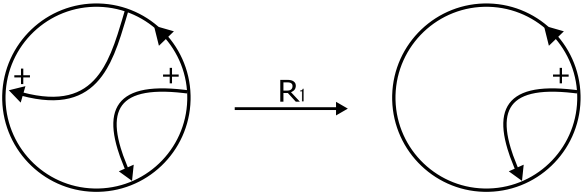


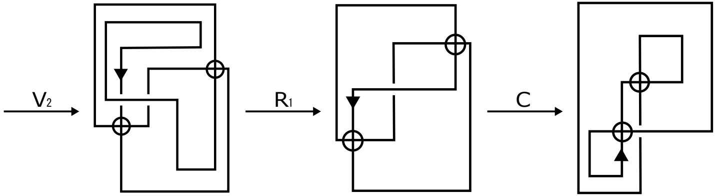


Outline of proof(5)

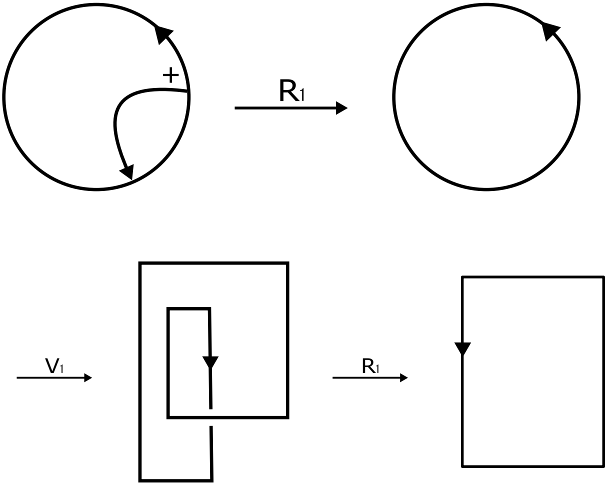


Outline of proof(6)





Outline of proof(7)



Using forbidden moves : 3 times

Question

Is this upper bound best possible ?