

# 研究集会「結び目のトポロジー VIII」

## -Abstracts-

小沢 誠 (駒澤大学文学部)

Makoto Ozawa

### Closed incompressible surfaces of genus two in 3-bridge knot complements

A. Hatcher と W. Thurston により 2-bridge knot complement 内に closed incompressible surface が無いこと (即ち、2-bridge knot が small であること) が示されています。この講演では、3-bridge knot complement 内の genus two, closed incompressible surface の特徴付けをします。

山本 亮介 (大阪大学大学院理学研究科)

Ryosuke Yamamoto

### On the open book decomposition supporting overtwisted contact structures and Stallings twist

閉 3 次元多様体の (positive) overtwisted contact structure をサポートする open book 分解を overtwisted open book と呼ぶ。この様な open book 分解の特徴付けという問題に関して以下を報告する。

open book 分解の fiber 曲面が Stallings twist と呼ばれる操作を行える simple loop (twisting loop と呼ぶ) を持つなら、それは overtwisted open book である事が分かる。この事実と、“同じ contact structure をサポートする 2 つの open book 分解は、positive Hopf band の plumbing, deplumbing により移り合う” という Giroux による結果に基づき、与えられた open book 分解の fiber 曲面に positive Hopf band をいくつか plumbing することで、twisting loop を持つようにできる場合について考察する。

名倉 真紀 (横浜国立大学工学研究院)

Maki Nagura

### A characterization of 2-component Brunnian links via Seifert matrices

We show some properties of a Seifert matrix of an  $n$ -component Brunnian link. In particular, we give a sufficient condition for a matrix to be a Seifert matrix of a 2-component Brunnian link.

門上 晃久 (大阪市立大学数学研究所)

Teruhisa Kadokami

### Reidemeister torsion and Seifert surgeries on knots in homology 3-spheres

We investigate Seifert surgery along a knot in a homology 3-sphere by using Reidemeister torsion. We suggest that it may be better to classify Seifert surgery into three types such as norm one type, rational type and norm zero type by corresponding the result of surgery to a branched cyclic covering space. In rational type case, a value of the Alexander polynomial at a root of unity has an information of the indices of singular fibers.

斎藤 敏夫 (大阪大学大学院理学研究科)

Toshio Saito

**Lens spaces which are unobtainable by surgery on knots**

(市原 一裕 (大阪産業大学教養部) との共同研究)

The question we consider in this talk is, which lens spaces are obtainable by surgery on knots in the 3-sphere? We first discuss this question for lens spaces of type  $(p, q)$  with small  $p$  or  $q$ . Next, we will particularly show that lens spaces containing Klein bottles are unobtainable by Berge's surgery on doubly primitive knots. We also give an algorithm to determine whether a given lens space is obtainable by Berge's surgery on doubly primitive knots.

山田 裕一 (電気通信大学電気通信学部)

Yuichi Yamada

**Lens space surgeries and plane curves**

J. Berge defined the family "doubly-primitive" knots that yield lens spaces by Dehn surgery, and classifying as several families and listed up them. I will point out that most of such knots are "divide knots" defined by N. A'Campo, and are presented by L-shaped plane curves. By such presentation, we can study the structure of Berge's knot family more. In this talk, I will talk about subfamilies (Berge's type I to VI) of knots in a solid torus yielding solid torus by Dehn surgery.

寺垣内 政一 (広島大学大学院教育学研究科)

Masakazu Teragaito

**Exceptional Dehn fillings at large distance**

This talk reports on some recent results on exceptional Dehn fillings at large distance. The main result completes the determination of best possible upper bounds for the distance between two exceptional Dehn fillings yielding essential small surfaces in all ten cases for large hyperbolic 3-manifolds in the sense of Wu.

新庄 玲子 (早稲田大学教育学部)

Reiko Shinjo

**An infinite sequence of non conjugate braids having the same closures**

By the Classification Theorem of closed 3-braids given by J. Birman and W. Menasco, it is known that there are only finitely many mutually non-conjugate  $n$ -braids ( $n = 1, 2$  or  $3$ ) having the same closure. Moreover they prove that if there is infinitely many mutually non-conjugate  $n$ -braids having the same closure, then all but finitely many of them are related by exchange moves. H. Morton discovered an infinite sequence of pairwise non-conjugate 4-braids whose closures are equivalent to the unknot and E. Fukunaga gave an infinite sequence of pairwise non-conjugate 4-braids whose closures are equivalent to the  $(2, k)$ -torus link for any  $k$ .

For any  $n$ -braid  $b$  ( $n \geq 3$ ) whose closure is a knot, we give an infinite sequence of pairwise non-conjugate  $(n + 1)$ -braids which have the same closures as  $b$  and we show that the closures of the braids in our sequence fall into a single equivalence class by exchange moves.

藤原 健一 (東京工業大学大学院理工学研究科)

Kenichi Fujiwara

### Refined Kirby calculus for rational homology 3-spheres of prime orders

向き付け可能な閉 3 次元多様体は、3 次元球面内の枠付き絡み目に沿った Dehn 手術を使って表示可能である。2 つの表示が同じ多様体を定めるとき、それらは Kirby 移動で移りあう (Kirby の定理)。扱う対象を整ホモロジー球面に絞ると、表示に使う枠付き絡み目を、絡み数が 0 で枠が  $\pm 1$  となるものに制限できる。ただし、ホモロジー球面とは、ホモロジー群が球面のそれに同形な 3 次元多様体のことである。

そのような絡み目の族を念頭に、Habiro は安定化と「band slide」からなる、特殊な Kirby 移動を構成した。「band slide」は代数的に打ち消しあう Kirby 移動の対となっているので、上の絡み目の族の中で、特殊な Kirby 移動は、閉じた絡み目の変形操作となっている。さらに、整係数ホモロジー球面に対する Kirby の定理が Habiro により証明された。

本講演では、素數位数の有理ホモロジー球面を与える枠付き絡み目上で、この特殊な Kirby 移動を行う。とくに、 $4n - 1$  ( $n$  は自然数) の形の素数に対し、Habiro の定理が拡張されることを示す。

濱田 龍義 (福岡大学理学部)

Tatsuyoshi Hamada

### はじめての KNOPPIX/Math

KNOPPIX/Math は数学者のためのコンピュータ環境です。CD を Windows 系コンピュータに入れて再起動するだけで、すぐに日本語 TeX や数学ソフトウェアを使うことができます。収録している数学ソフトウェアは多岐に渡っており、汎用数式処理システム Maxima や Risa/Asir, 結び目理論のための研究ツール Knot や SnapPeaなどを収録しています。KNOPPIX/Math を使えば、ソフトウェアのインストールや設定作業をせずに、すぐに専門的な研究ツールを使い始めることができます。

KNOPPIX/Math は海外出張時に携帯していると便利です。滞在先でネットワークに接続しているコンピュータを借りることができれば、どこにいても日本語でメールを読み書きし、ウェブページを閲覧することができます。既存の環境には何も手を加えませんので、CD を抜いて再起動すれば、元のコンピュータに戻ります。

本講演では KNOPPIX/Math の起動方法やファイルの保存方法、数学ソフトウェアの基本的な使い方について解説致します。

長郷 文和 (東京工業大学大学院理工学研究科)

Fumikazu Nagasato

### Algebraic varieties via a deformation of the Kauffman bracket skein module

In this talk, we will introduce an algebraic variety in an affine space  $\mathbb{C}^N$  constructed via the Kauffman bracket skein module (KBSM) of a knot exterior. One of the main ideas for the construction of the variety is that the polynomial map from  $\mathbb{C}^N$  to itself can be defined by using a representation of the braid group into the endomorphisms of the KBSM of a handlebody. In fact, the algebraic variety turns out to be an invariant of knots in  $S^3$ . In this talk, we will try to get a better understanding of the variety by focusing on the number of its irreducible components. Then we will see a relationship of the variety with so-called the Casson-Lin invariant defined by X.-S. Lin, which in fact inspired the above main idea, and moreover a relationship of the variety with the highest degree of the A-polynomial  $A_K(M, L)$  in terms of  $L$ .

杉村 真之助 (大阪市立大学大学院理学研究科)  
Shinnosuke Sugimura

### Multi-variable Alexander polynomial of certain Brunnian links

Borromean rings を拡張した Brunnian links  $L_{a,b,c}$  の多変数 Alexander polynomial を計算し、この分類について報告します。

川見 将広 (大阪市立大学大学院理学研究科)  
Masahiro Kawami

### $\mathbb{Z}_2$ -symplectic groups and genus 2 surface-knots

Let  $\Sigma_g$  be an oriented connected closed surface of genus  $g$  and  $\mathcal{M}_g$  be the mapping class group of  $\Sigma_g$ ; i.e., the group of all isotopy classes of orientation-preserving self-diffeomorphisms of  $\Sigma_g$ . We define  $SP_g[q]$  as the subgroup of  $\mathcal{M}_g$  consisting of mapping classes which preserve the given spin structure associated to the quadratic form  $q$  on  $H_1(\Sigma_g; \mathbb{Z}_2)$ .

As is well-known, the automorphisms over  $H_1(\Sigma_g; \mathbb{Z}_2)$  form the  $\mathbb{Z}_2$ -symplectic group  $Sp(2g; \mathbb{Z}_2)$ . In this talk, we will observe automorphisms over  $H_1(\Sigma_g; \mathbb{Z}_2)$  induced by the elements of  $SP_g[q]$ , which form the subgroup of  $Sp(2g; \mathbb{Z}_2)$ . We will call this group *the spin-preserving symplectic group*, and determine it explicitly where the case is  $g = 1$  and  $g = 2$  respectively. We call the images of surfaces embedded in the 4-sphere *surface-knots*. Lastly we give an application to the surface-knot of genus two.

岩切 雅英 (広島大学大学院理学研究科)  
Masahide Iwakiri

### Unknotting singular surface braids by crossing changes

C. A. Giller proved that crossing change is an unknotting operation for surfaces in 4-space. In this talk, we present such an unknotting theorem for singular surface braids, which is given when they have no branch points by S. Kamada. As a consequence, we have Giller's unknotting theorem.

田中 心 (東京大学大学院数理科学研究科)  
Kokoro Tanaka

### A canonical form for braid systems of surface braids

The notion of a surface braid was defined by Viro and extensively studied by Kamada. There exists a one-to-one correspondence between the set of (equivalence classes of) surface braids and each of the following two sets, and many results are obtained by using the correspondences. One is the set of (slide equivalence classes of) braid systems, where a braid system is a sequence of elements of the one-dimensional braid group. The other is the set of (C-move equivalence classes of) charts, where a chart is a graph in a two-dimensional disk.

In this talk, we define a canonical form of braid systems, and prove that any braid system can be deformed into a canonical form up to slide equivalence. Though either of braid systems or charts were used in many of previous studies, we obtain the following as an application by interpreting the canonical form of braid systems in terms of charts: Any surface braid can be deformed into an unknotted one by doing some operations, called crossing changes. (Iwakiri has a different proof of the above application.)

佐藤 進 (千葉大学大学院自然科学研究科)  
Shin Satoh

### 3-ブレイド結び目の3-彩色とコサイクル不変量

フォックスの3-彩色に付随する, 3-ブレイド結び目のコサイクル不変量の計算を行う. 具体的にはまず, 3-ブレイド結び目が非自明な ( $3^2$  または  $3^3$  通りの) 3-彩色をもつための必要十分条件を与える. その過程で, あるベクトル空間へのブレイド群の作用によるグラフを導入する. これを利用すると, 3-ブレイド結び目のコサイクル不変量を具体的な形で書き表わすことができる. 時間が許せば, 5-彩色の場合の計算と, 一般の  $p$ -彩色 ( $p$  は素数) に関するいくつかの結果についても述べるつもりである.

斎藤 昌彦 (南フロリダ大学数学科)  
Masahico Saito

### Quandle cocycle invariants and tangle embeddings

After a brief review of quandle cocycle invariants of knots and knotted surfaces, we will present results on the tangle embedding problem we have obtained using cocycle invariants. We use the table of tangles given by T. Kanenobu, H. Saito, and S. Satoh, and examine how much information we obtain from the quandle cocycle invariants, as to which tangles can or can not be embedded in which knots in the knot table. This is a group research project, and the members consist of K. Ameur, M. Elhamdadi, T. Rose, C. Smudde, and the speaker.

石井 敦 (大阪大学大学院理学研究科)  
Atsuchi Ishi

### An algebra for the Alexander-Conway polynomial

We introduce an algebra for the Alexander-Conway polynomial. The Jones polynomial of a closed braid may be obtained from a representation of the Temperley-Lieb algebra. We define a new algebra by a modification of the definition of the Temperley-Lieb algebra. We show that the Alexander-Conway polynomial may be obtained from a representation of this algebra. Furthermore we introduce a bracket polynomial for the Alexander-Conway polynomial and investigate it. We obtain a construction of the Alexander-Conway polynomial through a smoothing decomposition.

Alexander Stoimenow (東京大学大学院数理科学研究科)

### 3-valent graph のウェイト系と種数による交代結び目の双曲体積

I'll try to explain a conjectural relation between the maximal volume of alternating knots of given genus and the  $sl_N$  weight system of Vassiliev invariants of planar 3-valent graphs.

田中利史 (東京大学大学院数理科学研究科)

Toshifumi Tanaka

**Deciding mutation with the colored Jones polynomial**

(Alexander Stoimenow(東京大学大学院数理科学研究科) との共同研究)

We show knots where it is hard to decide whether they are mutants, in particular knots with the same 2-cable (HOMFLY and Kauffman) polynomial invariants, hyperbolic volume, and/or the same colored Jones polynomials.

笠原 泰 (高知工科大学工学部)

Yasushi Kasahara

**Jones 表現の忠実性に関する注意**

V. Jones は絡み目の多項式不変量を定義した有名な論文の中で,  $n$  箱の長方形 Young 図形  $Y$  が与えられるごとに, 対応する組みひも群の岩堀-Hecke 環表現を modify して,  $n$  個の穴あき球面の写像類群の 1 パラメータ付き線形表現を構成している. 本講演では, この表現の忠実性が,  $Y$  から箱一つ除いて得られる Young 図形に対応する組みひも群の岩堀-Hecke 環表現の忠実性と同値であることを示す. また, 特に  $n=6$  の場合, kernel が Birman-Hilden 理論を経由して, 種数 2 の Torelli 群に超楕円的対合を法として含まれることを示す.

栗屋 隆仁 (九州大学大学院数理学府)

Takahito Kuriya

**摂動的不変量のスピンの分解について**

3次元多様体  $M$  の不変量を対  $(M, \quad)$  の不変量の和に分離しようという試みがある ( はスピン構造もしくはコホモロジーの元)。その summand 全体の集合は、特に Betti 数が 1 より大きな多様体に対しては、元の不変量より強力であろうと予想されている。量子不変量に対しては R.Kirby 氏と P.Melvin 氏、C.Blanchet 氏、村上斉氏等により分解は成功している。量子不変量のある種の極限である摂動的な不変量や LMO 不変量に対してはまだ成されていなかったのであるが、ごく最近、ある特別なクラスに対してスピン・コホモロジー分解の摂動的な不変量バージョンが、定義された。この A.Beliakova 氏、C.Blanchet 氏、T.Le 氏による結果の紹介と、これの LMO 不変量版についても考察したいと思っている。

牛島 顕 (金沢大学理学部)

Akira Ushijima

### Rational-fold branched coverings and hyperbolic spatial graphs

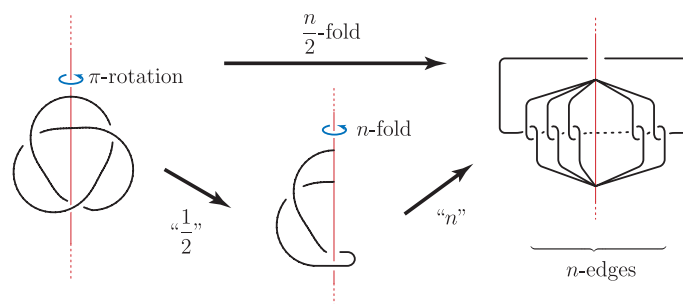
(市原 一裕 (大阪産業大学教養部) との共同研究)

In this talk we will deal with strongly invertible non-trivial knots (and sometimes links) in the three sphere  $S^3$ . A knot  $K$  in  $S^3$  is called *strongly invertible* when there is an involution for  $(S^3, K)$  with axis  $\alpha$  such that  $K \cap \alpha$  consists of exactly two points.

For such a knot, the  $n/2$ -fold cyclic branched covering is defined as follows:

- (i) Take a quotient of  $(S^3, K)$  by the involution  $\iota$ . Then we have  $S^3/\iota$  (which is homeomorphic to  $S^3$ ) with a spatial graph  $G$  as the image of  $K \cup \alpha$ .
- (ii) Take the  $n$ -fold cyclic branched covering of  $(S^3/\iota, G)$  along  $\alpha$ . Then we have the  $n$ -fold cyclic branched covering of  $S^3/\iota$  (which is again homeomorphic to  $S^3$ ) with a spatial graph  $G'$  as the pre-image of  $G - \alpha$  by the covering.

By the definition of the construction, the graph  $G'$  has two vertices with  $n$ -edges. Such a spatial graph is called a  $\theta_n$ -curve.



A spatial graph  $G$  is called *hyperbolic* if its exterior admits suitable complete hyperbolic structure. Since hyperbolic knots are the most common knots, it is natural to study from which knots hyperbolic graphs appear. The main result of this talk is to give a necessary and a sufficient condition about it:

**Theorem 1.** *Let  $K$  be a non-trivial strongly invertible knot in  $S^3$  admitting an involution with axis  $\alpha$ , and  $K_n$  the strongly periodic  $\theta_n$ -curve obtained from  $K$  by the  $n/2$ -fold cyclic branched covering along  $\alpha$ . Then the following three conditions are equivalent:*

1. For any  $n \geq 3$ ,  $K_n$  is hyperbolic.
2. For some  $n \geq 3$ ,  $K_n$  is hyperbolic.
3. There is no essential torus in the exterior of  $K \cup \alpha$ .

We also discuss the relationship between strongly periodic hyperbolic  $\theta_n$ -curves and several particular classes of strongly invertible knots; simple knots and tunnel number one knots.

本橋 友江 (関東学院大学工学部)

Tomoe Motohashi

**A prime decomposition theorem for handcuff graphs in  $S^3$**

We define a prime decomposition for a handcuff graph by 2- spheres each of which intersects the handcuff at exactly three points and prove that every irreducible handcuff graph can be uniquely decomposed into a finite number of prime handcuffs and  $\theta$ -curves.

岡本 美雪 (日本工業大学工学部)

Miyuki Okamoto

**On the intrinsic knottedness of a cat's cradle graph of  $K_5$**

(小林一章 (東京女子大学文理学部) との共同研究)

We exhibit that a cat's cradle graph of the complete graph with five vertices is intrinsically knotted.

新國 亮 (金沢大学教育学部)

Ryo Nikkuni

**Homotopy on spatial graphs and the Sato-Levine invariant**

(Thomas Fleming (University of California San Diego) との共同研究)

Edge-homotopy and vertex-homotopy are equivalence relations on spatial graphs which are generalizations of Milnor's link-homotopy. We introduce some edge (resp. vertex)-homotopy invariants of spatial graphs by applying the Sato-Levine invariant for the 2-component constituent algebraically split links and show examples of non-splittable spatial graphs up to edge (resp. vertex)-homotopy all of whose constituent links are link-homotopically trivial.

The preprint is available from arXiv:math.GT/0509003.

Gengyu Zhang (東京工業大学大学院理工学研究科)

**Crosscap numbers of two-component links**

We define the crosscap number for a 2-component link as the minimum of the first Betti numbers of connected, non-orientable surfaces bounding the link. We discuss some properties of the crosscap numbers of 2-component links.

堤 康嘉 (大阪市立大学数学研究所)

Yasuyoshi Tsutsumi

**キャッソン不変量と2成分絡み目**

2成分絡み目からなる framed link の キャッソン不変量の計算をする。とくにこの講演では、メイザーのホモロジー球面のキャッソン不変量の性質を紹介します。



Rama Mishra (大阪市立大学数学研究所)

**An Energy Function for Polynomial knots**

It is well known that every knot-type has a representative given by a polynomial embedding from  $\mathbb{R}$  to  $\mathbb{R}^3$  and such a representation is not unique. Also two polynomial representations of the same knot-type can be continuously deformed by a one parameter family of polynomial embeddings. In this situation the question of choosing an ideal polynomial representation makes sense. We have made an effort to define an Energy function on the space of polynomial knots and based on this function we call a polynomial representation of a given knot-type with minimum energy to be the ideal one.

市原 一裕 (大阪産業大学教養部)

Kazuhiro Ichihara

**Crosscap numbers of pretzel knots**

(水嶋 滋 (東京工業大学大学院情報理工学研究科) との共同研究)

For a non-trivial knot  $K$  in the 3-sphere, the *crosscap number* is defined as the minimal first betti number of non-orientable spanning surfaces for  $K$ . It was originally defined by B.E.Clark in [1], and, for torus knots and two-bridge knots, formulae of crosscap numbers were obtained in [3, 2], recently. In this talk, we will give a simple formula of the crosscap number for pretzel knots.

参考文献

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堤 幸博 (上智大学理工学部)

Yukihiro Tsutsumi

**On the crosscap numbers of knots and essential tangle decompositions**

(水摩 陽子 (京都大学数理解析研究所, The Fields Institute for Research in Mathematical Sciences) との共同研究)

This is an expository talk. It is well-known that any hyperbolic knot of genus one does not admit 2-string essential tangle decompositions. In this talk we focus on essential tangle decompositions of knots with crosscap number two or three. As an application we give a short proof that the ribbon number of the Kinoshita-Terasaka knot is three. How about Conway's knots?