REGULAR PROJECTIONS OF 6_2 KNOT

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We say that links L_1 and L_2 are *equivalent*, denoted by $L_1 = L_2$, if there exists a homeomorphism of S^3 onto itself which maps L_1 onto L_2, where the homeomorphism is not required to be orientation preserving. Each equivalence class of links is called a *link type*. A *link projection* is the image of an immersion from S^1 to S^2 whose multiple points are only finitely many transversal double points. A *link diagram* is a link projection together with an over/under information at each double point. A link diagram uniquely express a link type. We denote the set of all link types which arise from a link projection $\widehat{L}$ by $\mathrm{LINK}(\widehat{L})$. If a link type L_0 is an element of $\mathrm{LINK}(\widehat{L})$, then we say that $\widehat{L}$ is a projection of L_0. We denote the set of all link projections of a link type L_0 by $\mathrm{PROJ}(L_0)$.

Definition 1. (TANIYAMA [1])
For link types L_1 and L_2, we say that L_1 is a *minor* of L_2, denote by $L_1 \leq L_2$ or $L_2 \geq L_1$, if $\mathrm{PROJ}(L_1) \supset \mathrm{PROJ}(L_2)$. If L_1 is a minor of L_2, then we say that L_2 *majorizes* L_1.

When a knot K is one of the trivial knot, trefoil knot, figure-eight knot, 5_2 knot and (2,5)-torus knot, the knots J with $K \leq J$ are completely determined as follow.

Theorem 0. (TANIYAMA [1])
For each natural number μ, every μ-component link majorizes the μ-component trivial link.

Theorem 1. (TANIYAMA [1])
Every nontrivial knot majorizes the trefoil knot.

Theorem 2. (TANIYAMA [1])
For a knot K, the following (1) and (2) are equivalent:
(1) The knot K majorizes the figure eight knot.
(2) The knot K has a prime factor which is not equivalent to any of the (2,p)-torus knots with $p \geq 3$.

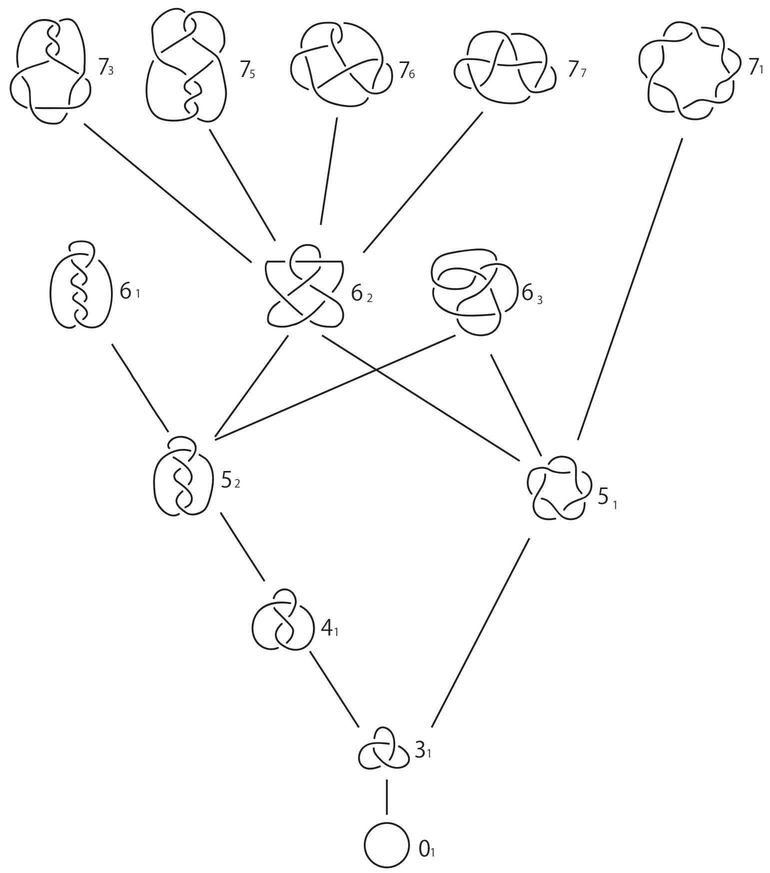
Theorem 3. (TANIYAMA [1])
For a knot K, the following (1) and (2) are equivalent:
(1) The knot K majorizes the 5_2 knot.
(2) The knot K has a prime factor which is not equivalent to any of the (2,p)-torus knots with $p \geq 3$, and the figure eight knot.

Theorem 4. (TANIYAMA [1])
For a knot K, the following (1) and (2) are equivalent:
(1) The knot K majorizes the (2,5)-torus knot.
(2) The knot K has a prime factor which is not equivalent to any of the pretzel knots L(p_1,p_2,p_3) with p_1,p_2 and p_3 odd intergers.

The following is our main theorem.

Theorem 5.
For a knot K, the following (1) and (2) are equivalent:
(1) The knot K majorizes the 6_2 knot.
(2) The knot K has a prime factor which is not equivalent to any of the (2,p)-torus knots with $p \geq 3$, pretzel knots L(p_1,p_2,p_3) with p_1,p_2 and p_3 odd integers, the 6_3 knot, 8_{18} knot, 8_{19} knot or the 8_{20} knot.

We have the following Hasse diagram of knots.



References

[1] K. Taniyama A partial order of knots. Tokyo Journal of Mathematics, 12, (1989), 205-229.
[2] K. Taniyama A partial order of links. Tokyo Journal of Mathematics, 12, (1989), 475-484.