曲面絡み目の bridge trisection の対称性について

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結び目の数理

曲面絡み目の bridge trisection 表示における tri-plane diagram の対称性について

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Introduction

Theorem [Meier-Zupan (2017)]

$$\{\text{surface links}\}_{/\sim} \quad \overset{1:1}{\longleftrightarrow} \quad \{\text{tri-plane diagrams}\}_{/\text{tri-plane move}}$$

$$\nwarrow\!\searrow 1:1 \qquad \qquad \updownarrow 1:1$$

$$\left(\{\text{bridge trisections}\}_{/\sim}\right)$$

Tri-plane diagram

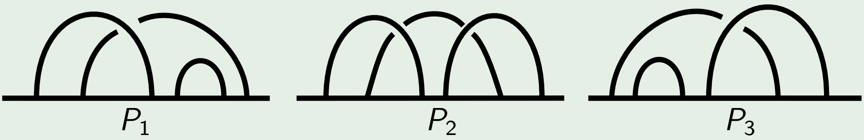
Definition (Tri-plane diagram)

(P_1, P_2, P_3): a tri-plane diagram

$\overset{\text{def}}{\Longleftrightarrow}$

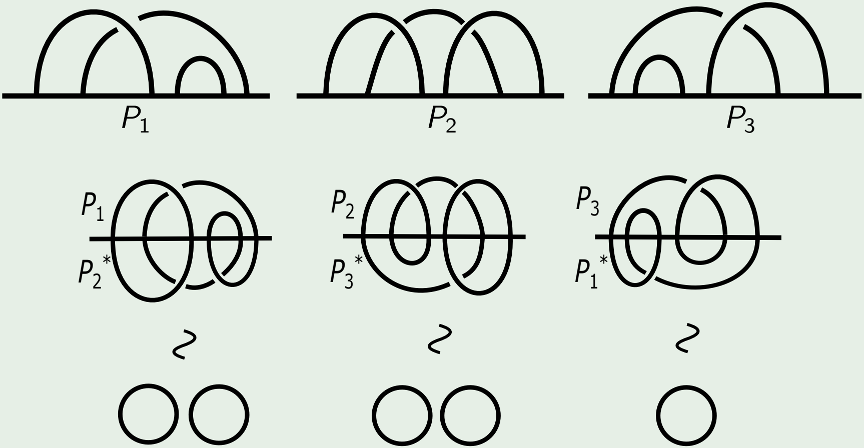
1. P_i: a diagram of a (b-strand) 1-dim trivial tangle (i=1,2,3)
2. $P_i \cup P_j^*$: a diagram of a trivial link ($i, j \in \{1, 2, 3\}$, $i \neq j$)

Example (a trivial S^2-knot)



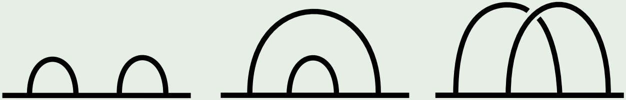
Example

Example

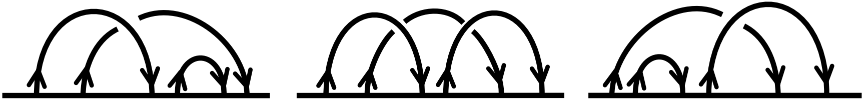


Example

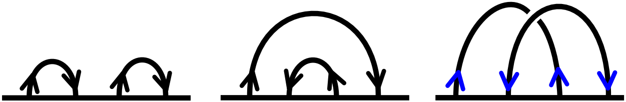
Example (a trivial-$\mathbb{R}P^2$ with $e = -2$)



Orientation of tri-plane diagrams



$\rightsquigarrow$ orientable

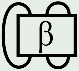


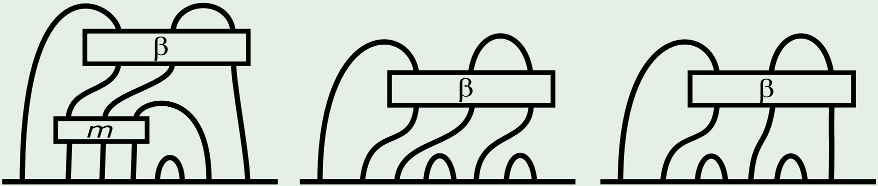
$\rightsquigarrow$ non-orientable

Example

$m \in \mathbb{Z}$, k: a classical knot
$\tau_m(k)$: the m-twist-spining of k

Example ($\tau_m(k)$)

k: a 2-bridge knot $\Big(k=$  $\Big)$



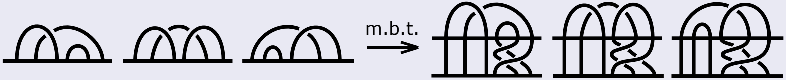
Tri-plane move

Theorem [Meier-Zupan (2017)]

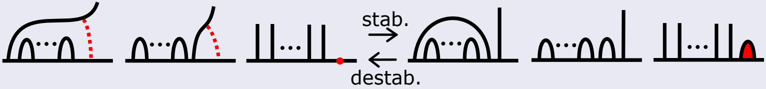
P, P': two tri-plane diagrams of a surface link
$\Longleftrightarrow$ P and P' are related by a finite sequence of tri-plane moves.

Definition (Tri-plane move)

1. interior Reidemester move
2. mutual braid transposition



3. stabilization/destabilization



Main theorem

Theorem 1

(P_1, P_2, P_3): a tri-plane diagram of $\tau_m(k)$
$\Longrightarrow (P_2, P_1, P_3)$ is a tri-plane diagram of $\tau_{-m}(k)$

Main theorem

Theorem 1

(P_1, P_2, P_3): a tri-plane diagram of $\tau_m(k)$
$\Longrightarrow (P_2, P_1, P_3)$ is a tri-plane diagram of $\tau_{-m}(k)$

Theorem 2

(P_1, P_2, P_3): a tri-plane diagram of a surface link F
$\Longrightarrow (P_2, P_1, P_3)$ is a tri-plane diagram of $-(F)^*$

Main theorem

Theorem 1

(P_1, P_2, P_3): a tri-plane diagram of $\tau_m(k)$
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Theorem 2

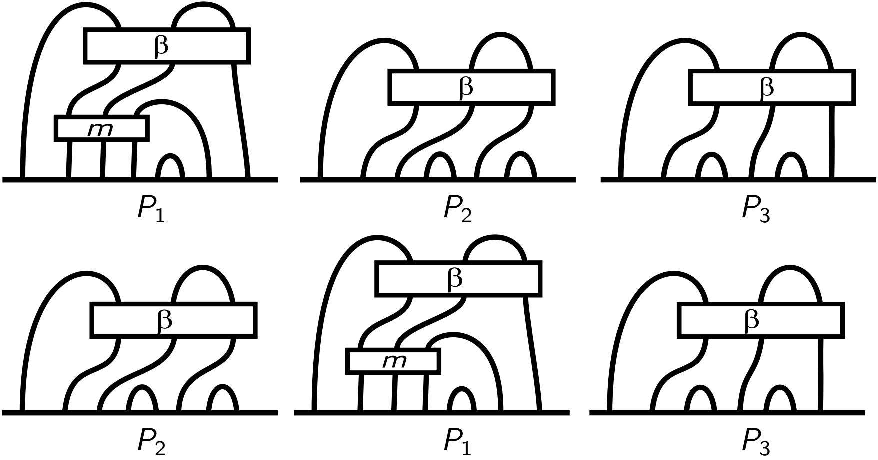
(P_1, P_2, P_3): a tri-plane diagram of a surface link F
$\Longrightarrow$ (P_2, P_1, P_3) is a tri-plane diagram of $-(F)^*$

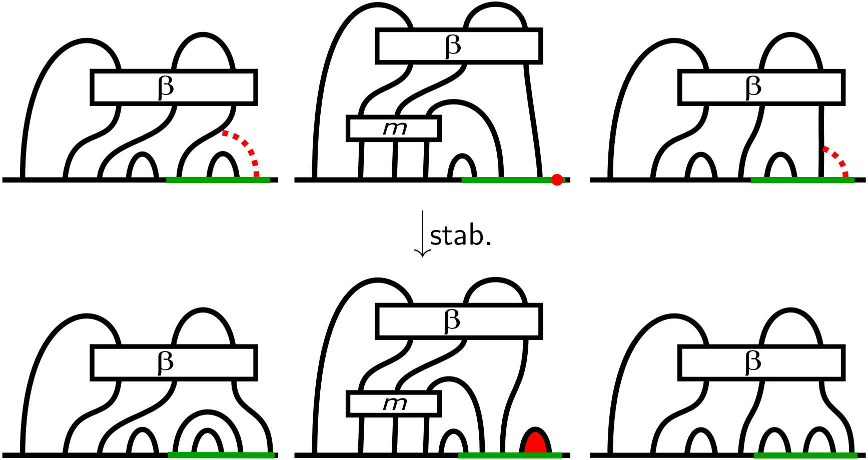
Corollary [Litherland (1985)]

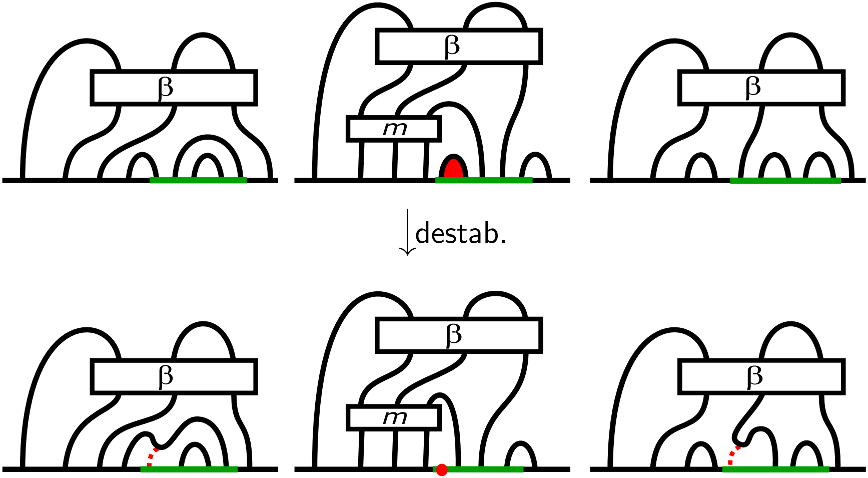
$\tau_{-m}(k) \sim -(\tau_m(k))^*$

Proof of Theorem 1

k: a 2-bridge knot
(P_1, P_2, P_3): a tri-plane diagram of $\tau_m(k)$




β
β
m
β
stab.
β
β
m
β


β
β
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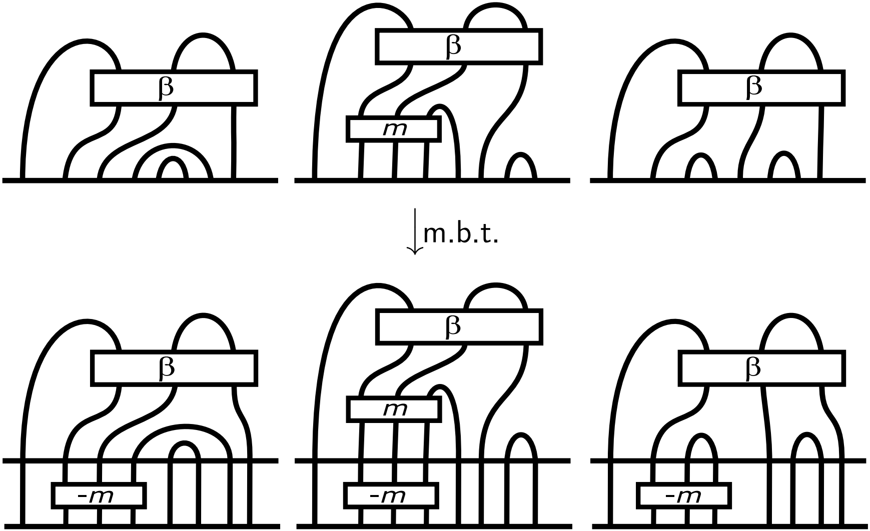
destab.
β
β
m
β


β
β
m
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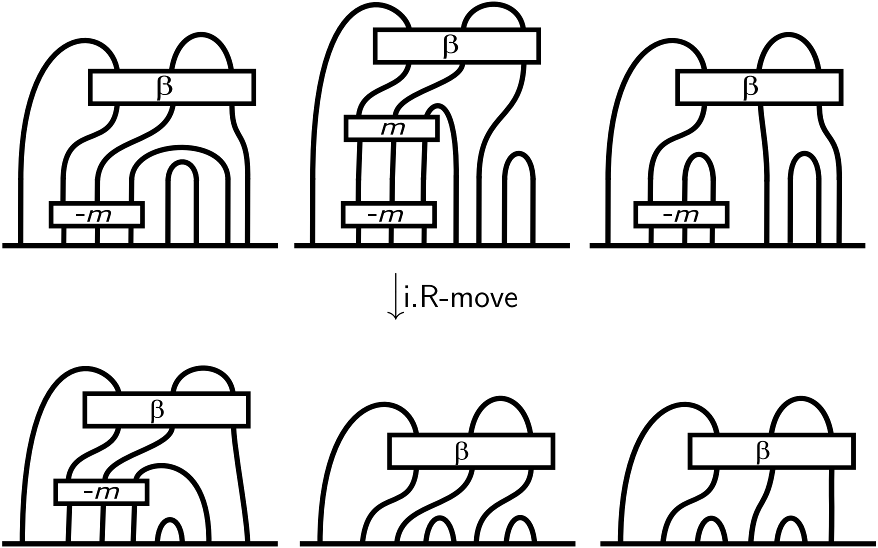
m.b.t.
β
-m
β
m
-m
β
-m


β
-m
β
m
-m
β
-m
i.R-move
β
-m
β
β

□

Proof of Theorem 2

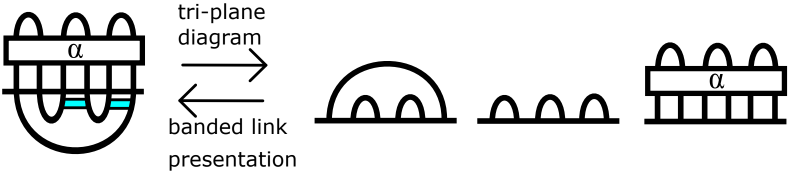
Theorem 2

(P_1, P_2, P_3): a tri-plane diagram of a surface link F
$\Longrightarrow (P_2, P_1, P_3)$ is a tri-plane diagram of $-(F)^*$

Theorem [Meier-Zupan (2017)]

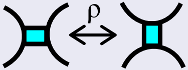
$$\left\{ \begin{array}{c} \text{banded link} \\ \text{presentations} \end{array} \right\}_{/\text{band moves}} \overset{1:1}{\longleftrightarrow} \left\{ \text{tri-plane diagrams} \right\}_{/\text{tri-plane move}}$$

Example.



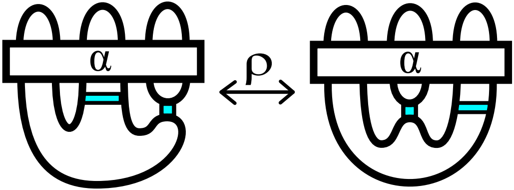
Lemma

$(L, b) \subset S^3$: *a banded link presentation of a surface link* F

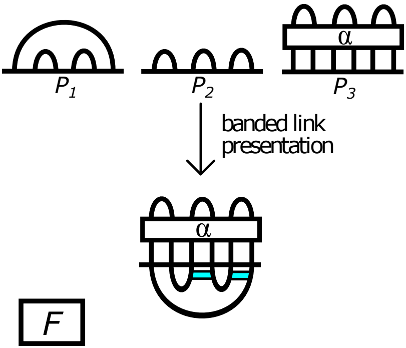
Operation ρ: 

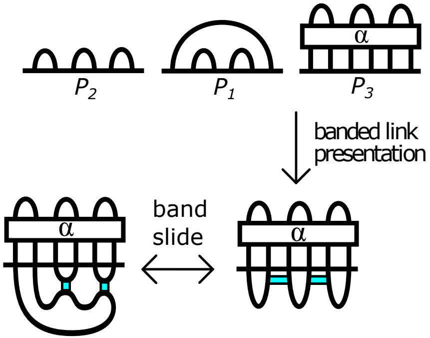
$\Longrightarrow \rho(L, b)$: *a banded link presentation of* $-(F)^*$

Example:

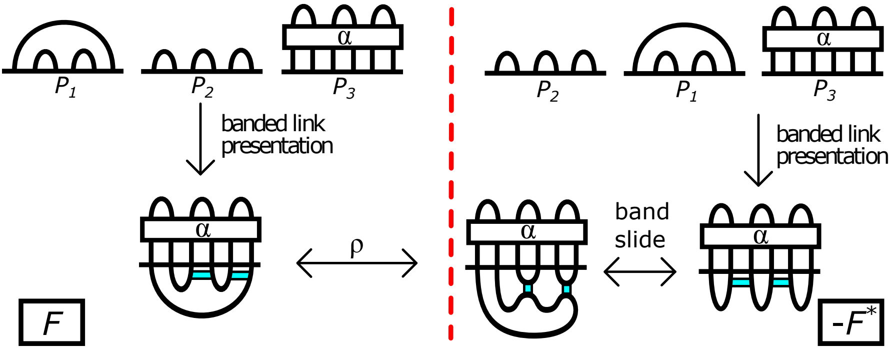


(P_1, P_2, P_3): a tri-plane diagram of F



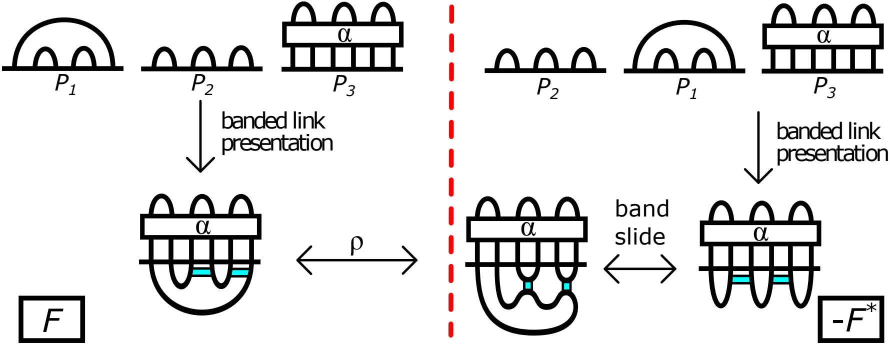


(P_1, P_2, P_3): a tri-plane diagram of F



□

(P_1, P_2, P_3): a tri-plane diagram of F



□

Problem

(P_1, P_2, P_3): a tri-plane diagram of F

$\Longrightarrow (-(P_1)^*, -(P_2)^*, -(P_3)^*)$ *is a tri-plane diagram of* $-(F)^*$

$$\rightsquigarrow (P_2, P_1, P_3) \overset{\text{tri-plane move}}{\sim} (-(P_1)^*, -(P_2)^*, -(P_3)^*)$$

Thank you for your attention!