This paper investigates the resources the student-teachers used to design and implement grade 4 mathematics lessons in the context of a project-based international exchange programme between Switzerland and Japan. The lesson, initially planned together by nine student-teachers of the two countries, was implemented separately in each country. In spite of the collaborative design of a lesson, its enactment was quite different. The analysis of the resources (namely lesson plan, curricula, and textbooks) allows us to identify the elements that yield the differences between the two lessons.

INTRODUCTION

Since a decade, researchers try to understand mathematics teachers’ work and their evolution through the analysis of resources the teachers use or develop for their teaching practices. This approach, called documentational approach to didactics (Gueudet & Trouche, 2009), focuses on the use of resources. Resources and their use, or the document should be very different according to the educational contexts or the institutions because they are culturally rooted. As teaching practices differ across contexts, the roles of resources, such as a textbook, are also different. We consider that some resources for teachers constitute a didactic infrastructure (Chevallard, 2009) that strongly supports and shapes the lesson design.

In this paper, we carry out a comparative analysis of the resources or of the didactic infrastructure for the student-teachers used in the design and implementation of mathematics lessons in two countries, Switzerland and Japan, and investigate related cultural factors that yield the differences of teaching practices identified in our previous comparative study of lessons. The resources we will analyse are, in particular, the lesson plans developed during a project-based student and teacher exchange programme (called PEERS project), as well as the mathematics textbooks and the national or regional curricula which are principal references for the student-teachers.

In what follows, we first present, as a context, the PEERS project and the differences we have identified in a comparative analysis of lessons, and then carry out an analysis of resources, in order to identify the cultural elements that yield such differences.

COLLABORATIVE DESIGN OF A LESSON

We briefly describe our project of the collaborative design of a mathematical lesson, which allows us to mention the collected data, and the results of comparative analysis in our previous study (Clivaz & Miyakawa, to appear).
PEERS Project

PEERS project is a student and professor exchange program carried out by Lausanne University of Teacher Education (HEP Vaud). It consists of student exchanges around a jointly defined research project by a group of students from HEP Vaud in association with a group of students from the partner university. Each PEERS is supervised by a teacher-researcher of each institution, combining face-to-face (one week in fall and another week in spring) with distance online collaborative work phases. PEERS with Joetsu University of Education (the previous affiliation of the first author) was supervised by the two authors of this paper.

In our PEERS, the group of students and professors first met through Skype meetings in fall 2017, and decided on the general theme of PEERS and the mathematical theme: the collaborative development of a problem solving geometry lesson for grade 4 pupils following a lesson study process (Hart, Alston, & Murata, 2011). The group spent one week in Joetsu in October 2017 to design a task, to study the topic and to plan the lesson together. At the end of the week, a first draft of the lesson plan was ready. During the winter, the two groups developed their lessons separately and taught them several times in each country. The Japanese group spent one week in Lausanne in February 2018. During this week, the group observed the last Swiss lesson, watched the video of the last Japanese lesson, and discussed the differences and commonalities.

The problem the group selected for designing a lesson was the one in the Swiss textbook (Danalet, Dumas, Studer, & Villars-Kneubühler, 1999). The question is: “Divide a square into several squares, but not more than 20. Find as many solutions as possible”. The lesson plan by the Swiss students is available on the websites of Lausanne Laboratory Lesson Study (www.hepl.ch/3LS).

![Figure 1. Some of the possible solutions for 4, 6 and 7](image)

Differences between two “same” lessons

In our previous study, we carried out a comparative analysis of the lessons implemented in each country. Even though the task was initially designed collaboratively in the face-to-face workshops in Japan, its implementations in Switzerland and Japan were very different. We found explicit differences, in particular, in the structures of lesson described with the phases (see Figure 2) and in the teacher’s way of validating pupil’s answers found in each of the corresponding phases.

Concerning the structure, one big difference is that the sharing phase in the Japanese lesson, which is often called *neriage*, is longer than in the Swiss lesson, and includes not only the collective work that the teacher manages in the whole classroom setting, but also the group work, that is to say, the sharing or *neriage* in the Japanese lesson.
aims not only to share pupil’s answers in the classroom but also to develop mathematical ideas in the collective setting. Another difference is that there was no phase for synthesis (or *matome*) in the Swiss lesson.

![Fig 2. Structure of the two lessons](image)

The teacher’s validation of pupil’s answers was also one of the biggest differences between the lessons from the two countries. It was the principal and recurring difficulties for Swiss students as well as Japanese students when designing, teaching and discussing the lesson. In the Swiss lesson, the main concern of the Swiss team was to deal with many pupils coming to the teacher during the group research phase (the orange part in Figure 2) to ask him/her: “is this correct?” During this phase, the Swiss teacher takes care of pupils one by one in front of the board and tells the pupils whether or not the solution is correct. The way of validation of the solutions by the Swiss team is an evaluation rather than a validation. In comparison, the Japanese teacher moves from one group to another and asks questions such as “are they really all squares? Could you think about it?”, and leaves pupils to make decisions by themselves. This characteristic can be found also in the sharing phase (in bronze colour in Figure 2).

**A COMPARATIVE ANALYSIS OF RESOURCES**

We carried out a comparative analysis of the resources developed during collaborative design and used while teaching by the two groups of student-teachers, and investigated the cultural factors that yield the differences in teaching practices.

**Lesson plans**

The lesson plan was a principal resource that the Swiss and Japanese student-teachers developed when designing and implementing a lesson. At the first phase of collaborative design of a lesson, they wrote a draft of lesson plan together; then they finished writing a lesson plan separately in each side and revised it according to the implemented lessons.

In the final versions of lesson plan, one may find some similarities and differences. Both lesson plans include the goals of the lesson, the table showing the chronological progression of classroom teaching, as well as the plan of board writing for which the Swiss team followed the Japanese style lesson plan. Both lesson consists of different
phases or moments including introduction, research, sharing, and synthesis. However, one may find several differences in the detail. The Japanese lesson plan, which follows more or less the ordinary format of Japanese lesson plan (e.g. Fernandez & Yoshida, 2001; Miyakawa & Winsløw, 2013), provides a much more precise explanation, not only on the goal and idea of the designed lesson, but also on the objective of the experimentation itself, saying “through the teaching practices with the Swiss task in Japanese school and the analysis of possibilities of the lesson, we also aim to gain a new insight into the development of teaching material”. On the contrary, the table showing the chronological progression of classroom teaching is much more precise in the Swiss lesson plan and provides a precise list of teacher’s actions and pupil’s actions.

One of the critical aspects that yield the differences between the two implemented lessons was the collectivity in the classroom activity. The collective dimension is more often referred to in the Japanese lesson plan than in the Swiss lesson plan. While the description of the chronological teaching progression in Japanese lesson plan is short, several remarks are given to promote the collective development of problem solving in the whole classroom setting, for example, in the column of “Teacher’s supports”:

- “When a wrong answer is given, (the teacher) takes it to the whole class, and check why it is wrong, accordingly”;
- “In order to share the succeeded cases in the whole class, make pupils stick the origami on the blackboard”;
- “In order to share pupil’s idea with the whole class, project the origami by the video projector”.

The sense of collectivity in the mathematics classroom, as one may see it in the “structured problem solving lesson” in Japan (Stigler & Hiebert, 1999), is a principal factor that shapes the structure of the Japanese lesson, which explains the relative long time spent on neriage or sharing, compared to Swiss lessons (see Figure 2), as well as the teacher’s way of validating pupil’s answer, not individually but rather collectively.

**National or regional curricula**

When writing the lesson plan, both teams referred to the national or regional curricula. The goals of the lesson in the Swiss lesson plan were cited from the regional curriculum (Table 1). MSN in the table means “Mathématique et Science de la nature” (Mathematics and natural sciences), and the numbers show the cycle (first digit) and the domain (second digit; 1: space; 5: modelling). Table 1 includes the goals related to solving geometric problems.

<table>
<thead>
<tr>
<th>elements for problems solving (MSN21 in connection with MSN25): solving geometric problems</th>
</tr>
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<tbody>
<tr>
<td>o ... by imagining and by using visual representations (codes, diagrams, graphics, tables, ...)</td>
</tr>
<tr>
<td>o ... by sorting and organizing data</td>
</tr>
<tr>
<td>o ... by communicating its results and interpretations</td>
</tr>
<tr>
<td>o ... by asking questions and by defining a framework of study</td>
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</tbody>
</table>
... by mobilizing ... mathematical tools
- MSN 21
  - Pose and solve problems to structure the plan
    - ... by representing plane figures ... using sketch ...
  - Decomposition of a plan surface into elementary surfaces, and recomposition

<table>
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<tr>
<th>Table 1. The goals of lesson given in the Swiss lesson plan</th>
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| In contrast, the Japanese team situated this lesson not in the domain of geometry, but in the domain of function (relation of numerical quantities), in which pupils are required to identify the pattern behind quantity changes. The goal given in the Japanese lesson plan was “Be able to discover different ways to divide a square, identify their pattern, and apply it”.
| This choice of goal is due to the goals of mathematics teaching given in the national curriculum. The goal of Grade 4 geometry in the Japanese national curriculum is more specific to some geometrical concepts and does not conform to the problem chosen in the project. In fact, the Japanese national curriculum published in 2009 says:
| “Through the analysis of geometrical figures by focusing on their components and their positions, be able to understand the plane figures such as the parallelogram and the rhombus, as well as the solid figures such as the rectangular parallelepiped.”
| This difference of teaching goal would be another reason why the Japanese lesson had the synthesis phase (matome), while the Swiss lesson did not. The Japanese student-teachers are always required to teach specific mathematical concepts or methods, through the problem solving activities. They therefore could not attain their goal without having synthesis phase wherein the pattern of increase of the number of squares is summarized and discussed. In contrast, the Swiss team could arrive at their goal without having it, since their objective is mainly to experience the problem solving activities mentioned in Table 1, and not necessarily to learn a specific concept or method.
| Textbooks
| Another resource the student-teachers used for the collaborative design of a lesson was the textbooks. The activity of the division of squares was taken from a Swiss textbook. One characteristic of this Swiss mathematics textbook (Danalet et al., 1999) is that it consists of a collection of different problems, often without explicit mathematical knowledge to learn (at least for pupils) and without any suggested order to build a teaching sequence. The problem solving activity for the Swiss team is built on the notion of focusing on the process of resolution, not so much on the acquisition of a specific mathematical knowledge. This would be one of the reasons why the lesson implemented by the Swiss student-teacher did not allocate time for the synthesis.
| In contrast, a chapter of the Japanese textbook consists of an amalgam of different elements such as problem-situations, summaries of specific mathematical knowledge to learn, and exercises. The Japanese student-teachers tried to make explicit in the lesson plan specific mathematical knowledge as an objective, although they adopted,
as a problem-situation of the designed lesson, the one from the Swiss textbook. This is also due to the structure of Japanese problem solving lesson (Stigler & Hiebert, 1999), including summary of mathematical content (synthesis phase), which the Japanese team tried to follow in their lesson. The idea of problem solving for Japanese team remains in the structure of lesson instead of the objective of mathematics teaching.

DISCUSSION AND CONCLUSION

The cross-cultural comparative analysis of resources in addition to the analysis of lessons allow us to elucidate the characteristics of classroom teaching practices (e.g., lesson structure, teacher’s instruction) as well as the “theory” (or the cultural factors) that shapes such practices. Such a “theory” is difficult to identify, because it is often shared within the teachers’ community without asking its validity. In our case, the curricular documents (the national or regional curricula and the textbooks) highlighted the perspective of mathematics teaching of each country, emphasizing either the mathematical contents (Japan) or the problem solving skills (Switzerland). The lesson plans allowed us to identify the idea of problem solving with respect to the collective, which yields the differences of implemented lessons between the two countries.

References


