Aid, non-traded goods, and growth*

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Abstract

We examine the effects of foreign aid in a small recipient country with two traded goods, one nontraded good, and two factors. Learning by doing and intersectoral knowledge spillovers contribute to endogenous growth. We obtain two main results. First, a permanent increase in untied aid raises (or lowers) the growth rate if and only if the nontraded good is more capital-intensive (or effective-labor-intensive) than the operating traded good. Second, a permanent increase in untied aid raises welfare if the nontraded good is more capital-intensive than the operating traded good; otherwise, it may raise or lower welfare.

JEL classification: F43; O41

Keywords: Aid and growth; Nontraded goods; Intersectoral knowledge spillovers; Stolper-Samuelson theorem; Small-country transfer paradox

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1 Introduction

Does foreign aid stimulate economic growth of recipient countries? After a decade of numerous empirical studies (e.g., Burnside and Dollar, 2000; Hansen and Tarp, 2001; Dalgaard et al., 2004; Rajan and Subramanian, 2008), we have yet to find any robust relationship between aid and growth. Rajan and Subramanian (2008) reported that, even after correcting for the downward endogeneity bias (i.e., tendency of low-growth countries to receive more aid), the aid/GDP ratio did not positively and significantly affect the annual growth rate of real per capita GDP averaged over ten- to forty-year periods. This was true even if the squared aid term was included (e.g., Hansen and Tarp, 2001), aid was interacted with policy measures (e.g., Burnside and Dollar, 2000), it was interacted with geographical measures (e.g., Dalgaard et al., 2004), or it was disaggregated into different categories. To make aid more effective in promoting growth, we have to explain: "what is it that offsets the transfers and subsidized credit inherent in aid and prevents it from having a robust positive effect on growth?" (Rajan and Subramanian, 2008, p. 644) The purpose of this paper is to provide a theoretical framework to answer this question.

The existing but still small body of theoretical literature emphasizes that the aid-growth relationship depends on how aid is used. Chatterjee et al. (2003) first constructed an endogenous growth model of a one-sector small open borrowing country with public as well as private capital and world capital market imperfections, and demonstrated that only tied aid for public investment raised the growth rate by raising the returns to private capital. Chatterjee and Turnovsky (2007) incorporated a labor-leisure choice into Chatterjee et al. (2003), and further showed that untied aid transferred to households lowered the growth rate because the transferred income increased leisure at the expense of labor, which in turn lowered the returns to private capital. The message from this literature is that aid should be used for productive government activities such as building infrastructure.

We intend to give a different story of aid and growth, focusing more on market interactions among sectors. Increased income from aid will usually increase the demands for goods, which will change at least the prices of nontraded goods even if the recipient country is small.[1] The effects of the price changes will be transmitted to all sectors asymmetrically depending on factor intensities. The possibility that the presence of nontraded goods distorts the aid-growth or aid-welfare relationship is examined empirically by Rajan and Subramanian (2005), and theoretically by Yano and Nugent (1999), Schweinberger (2002), and Cerra et al. (2009). Rajan and Subramanian (2005) provided empirical evidence that aid inflows caused higher nontraded goods prices and thus real exchange rate appreciation, which in turn lowered the share of more tradable industries in the manufacturing sector. Yano and Nugent (1999) developed a static model of a small open economy with two traded goods, one nontraded good, and two factors, capital and labor. They demonstrated that foreign aid in the form of capital might paradoxically reduce the recipient's welfare if the aid expanded production of the nontraded good sufficiently that the resulting fall in the price of the nontraded good decreased the excess demand for the import-competing good and hence the tariff revenue severely. Schweinberger (2002) extended Yano and Nugent (1999) to deal with imperfect capital mobility, and pointed out that the nontraded goods effect was likely to diminish with greater capital mobility. Cerra et al. (2009) constructed a one-traded-good, one-nontraded-good, two-factor neoclassical growth model, and showed that untied aid raised welfare because it did not affect the price of the nontraded good in the steady state. However, no one has constructed a theoretical model that explains how aid affects the long-run

[1] Nontraded goods account for a considerable part of developing economies. According to the World Development Indicators Online, services sectors, which are representative of nontraded goods, had an average share of 53% of GDP even in low- and middle-income countries in 2000.

aggregate growth rate through changes in the prices of nontraded goods. This paper is one attempt to do it.

To formulate the simplest endogenous growth model with at least one nontraded good as well as trade in goods, we extend Yano and Nugent (1999) to incorporate learning by doing and knowledge spillovers of the Arrow (1962)-Romer (1986) type. We follow Peletier (1998), Drugeon et al. (2003), and Naito (2006) by assuming that new knowledge created through learning by doing spills over across sectors. This assumption seems plausible in our model because workers having technological know-how can move freely among sectors.[2] As will be shown later, for a given initial condition, the equilibrium path of consumption is chosen so that the economy should be in the steady state from the initial period onward. This implies that the economy adjusts to exogenous shocks by jumping from the old steady state to the new one. In contrast to endogenous growth models with transitional dynamics (e.g., Lucas, 1988; Futagami et al., 1993), the no-transition property allows us to examine the growth and welfare effects of policy changes in an exact analytical manner. Another implication of our endogenous growth model is that the returns to capital need not equal the subjective discount rate in the steady state. Indeed, that requirement was responsible for the long-run neutrality of untied aid on the price of the nontraded good in Cerra et al. (2009).

The most difficult part of our two-traded-good (i.e., a capital good and a consumption good), one-nontraded-good, two-factor (i.e., capital and effective labor) endogenous growth model is that the economy actually tends to specialize completely in a traded good.[3] Most of the existing papers using the two-traded-good, one-nontraded-good, two-factor framework (e.g., Komiya, 1967; Peletier, 1998; Yano and Nugent, 1999; Schweinberger, 2002) simply assume that all three goods are always produced. This is equivalent to assuming that the two factor prices are determined by the two zero-profit conditions for the traded goods, and that the price of the nontraded good is pegged at its unit cost at the prevailing factor prices. However, if we allow the price of the nontraded good to be different from the above technologically-determined value, then one of the two zero-profit conditions for the traded goods becomes slack, causing the corresponding sector to shut down. Considering the cases of complete specialization, we carefully derive the supply curve of the nontraded good for each factor intensity ranking. The supply curve has both a flat region and upward-sloping regions. In the flat region, the economy is incompletely specialized, and then production allocation is indeterminate for that price. In the upward-sloping regions, the economy completely specializes in either traded good, and thus all properties of the two-by-two Heckscher-Ohlin model apply there. It is the possibility of complete specialization that permits the price of the nontraded good to change with demand shocks such as foreign aid.

Focusing mainly on the case of complete specialization, we obtain the following main results. First, a permanent increase in untied aid raises (or lowers) the growth rate if and only if the nontraded good is more capital-intensive (or effective-labor-intensive) than the operating traded good. Aid increases the excess demand for the nontraded good through the income effect, which in turn raises its price under the downward-sloping excess demand curve. This affects the returns to capital and hence the growth rate through the Stolper-Samuelson theorem. By departing from the one-sector framework of Chatterjee et al. (2003) and Chatterjee and Turnovsky (2007), we show that the aid-growth relationship depends on technological differences among sectors. This result also provides a theoretical foundation for empirical studies on aid and growth, especially for Rajan and Subramanian (2005), with an additional implication that the higher nontraded goods prices caused by aid do not necessarily lower the long-run aggregate growth rate.

Second, a permanent increase in untied aid raises welfare if the nontraded good is more capital-intensive

[2]Keller (2002) and Park (2004) gave empirical evidence for intersectoral knowledge spillovers arising from R&D.

[3]When we say "the economy completely specializes in a traded good", we mean that "the economy produces only that good out of the two traded goods, as well as the nontraded good".

than the operating traded good; otherwise, it may raise or lower welfare. It should be noted that the presence of distortions is necessary for the possibility of the small-country transfer paradox. Whereas Yano and Nugent (1999) and Schweinberger (2002) considered an import tariff as the cause of distortions, positive production externalities through learning by doing and intersectoral knowledge spillovers keep the equilibrium growth rate suboptimally low in this paper. When the nontraded good is more effective-labor-intensive than the operating traded good, aid may be paradoxically welfare-reducing because it drives down the suboptimally low growth rate even further.

It should be mentioned that our results are obtained at the cost of the inactive import-competing sector. Not only does it seem unrealistic, but it also rules out the possible effect of an import tariff on the growth rate. Future research should be devoted to improving this aspect.

The rest of this paper is organized as follows. Section 2 formulates the model and then examines how the pattern of specialization is determined. Section 3 studies the dynamics. Sections 4 and 5 investigate the growth and welfare effects of untied aid, respectively. Section 6 analyzes briefly the effects of tied aid for production subsidies. Section 7 concludes.

2 The model

Consider a small open recipient country that has three sectors. Good 1 and good 2 are traded goods, whereas good 3 is a nontraded good. In line with Yano and Nugent (1999) and Schweinberger (2002), we assume that all three goods are consumed. Moreover, good 1 can also be invested for capital accumulation, so we call it a capital good. We choose good 1 as the numeraire.

2.1 Firms

In sector $j(j = 1, 2, 3)$, the representative firm maximizes its profit $\Pi_j = p_j Y_j - rK_j - wL_j$, subject to the production function $Y_j = F_j(K_j, HL_j)$, with p_j, r, w, and H given, where p_j is the price of good j; Y_j is the output; r is the rental rate; K_j is the demand for capital; w is the wage rate; L_j is the demand for labor; and H is the effectiveness of a unit of raw labor. It is assumed that $F_j(\cdot)$ is increasing, concave, linearly homogeneous, and differentiable. Following Peletier (1998), Drugeon et al. (2003), and Naito (2006), we assume that new knowledge created through learning by doing in each sector spills over across the whole economy:

$$H(K) = hK; h > 0, \tag{1}$$

where K is the aggregate capital stock. The cost minimization problem can be rewritten in terms of effective labor: minimize $rK_j + vH_j$, subject to $Y_j \leq F_j(K_j, H_j)$, with r, v, and Y_j given, where $v \equiv w/H$ is the wage rate per unit of effective labor; and $H_j \equiv HL_j$ is the effective amount of labor demand. Under constant returns to scale, we can define the unit cost function as $c_j(r, v) \equiv \min_{a_{Kj}, a_{Hj}}\{ra_{Kj} + va_{Hj} : 1 \leq F_j(a_{Kj}, a_{Hj})\}$, where $a_{Kj} \equiv K_j/Y_j$ and $a_{Hj} \equiv H_j/Y_j$ are the demands for capital and effective labor per unit of output, respectively. As usual, $c_j(\cdot)$ is increasing, concave, linearly homogeneous, and differentiable. With the unit cost function, the first-order condition for profit maximization is given by:

$$p_j - c_j(r, v) \leq 0, Y_j \geq 0, Y_j(p_j - c_j(r, v)) = 0. \tag{2}$$

2.2 Households

Suppose that the representative household is endowed with one unit of raw labor for all periods $t \in [0, \infty)$, and K_0 units of capital in the initial period. He/she maximizes his/her utility $U = \int_0^\infty (C_t^{1-\theta} - 1)/(1 - \theta)\exp(-\rho t)dt, C_t = V(C_{1t}, C_{2t}, C_{3t})$, subject to the flow budget constraint:

$$\dot{K}_t = (r_t - \delta)K_t + v_t H_t + T_t - \textstyle\sum_{j=1}^{3} p_{jt} C_{jt}, \tag{3}$$

with $\{r_t, v_t, H_t, T_t, p_{1t}, p_{2t}, p_{3t}\}_{t=0}^{\infty}$ and K_0 given, where C is the consumption index; θ is the inverse of the elasticity of intertemporal substitution; ρ is the subjective discount rate; C_j is consumption of good j; δ is the depreciation rate; T is the untied foreign aid inflow; and a dot over a variable represents differentiation with respect to time (e.g., $\dot{K}_t \equiv dK_t/dt$). We omit the time subscripts whenever there is no confusion. It is assumed that the consumption index function $V(\cdot)$ is increasing, concave, linearly homogeneous, and differentiable. The fact that T appears in Eq. (3) implies that the government simply transfers aid to the representative household. We retain this assumption until section 5.

In each period, we can define the expenditure function as $E(p_1, p_2, p_3, C) \equiv \min_{C_1, C_2, C_3}\{\sum_j p_j C_j : C \leq V(C_1, C_2, C_3)\}$. Under linear homogeneity of $V(\cdot)$, we have $E(p_1, p_2, p_3, C) = e(p_1, p_2, p_3)C$, where $e(p_1, p_2, p_3)$ is the minimized expenditure per unit of the consumption index. Since $e(\cdot)$ is increasing, concave, linearly homogeneous, and differentiable, as usual, we have $e_j(\cdot) \equiv \partial e(\cdot)/\partial p_j > 0$ and $e_{jj}(\cdot) \equiv \partial^2 e(\cdot)/\partial p_j^2 < 0$.

With the expenditure function, our problem is to choose $\{C_t\}_{t=0}^{\infty}$ to maximize U subject to $\dot{K}_t = (r_t - \delta)K_t + v_t H_t + T_t - e(p_{1t}, p_{2t}, p_{3t})C_t$. The first-order conditions and the transversality condition are:

$$C_t^{-\theta} - \lambda_t e(p_{1t}, p_{2t}, p_{3t}) = 0,$$
$$\lambda_t(r_t - \delta) = \rho\lambda_t - \dot{\lambda}_t,$$
$$\lim_{t\to\infty} \exp(-\rho t)\lambda_t K_t = 0,$$

where λ_t is the current-value shadow price of a unit of good 1 in period t. The first one is rewritten as $\lambda_t = X_t^{-\theta}$, where $X_t \equiv e(p_{1t}, p_{2t}, p_{3t})^{1/\theta} C_t$ is interpreted as a modified consumption index. With this transformation, the Euler equation for X_t and the transversality condition are derived as:

$$\gamma_{Xt} = (1/\theta)(r_t - \delta - \rho), \tag{4}$$

$$0 = \lim_{t\to\infty} \exp\left(-\int_0^t (r_s - \delta)ds\right)K_t, \tag{5}$$

where γ_{sub} is the growth rate of the subscript (e.g., $\gamma_{Xt} \equiv \dot{X}_t/X_t$). The advantage of defining X is that the Euler equation does not contain the rates of price changes. To attain positive growth and bounded utility, we assume that $\gamma_{Ct} > 0$ and $\rho - (1 - \theta)\gamma_{Ct} > 0$.

2.3 Equilibrium

The market clearing conditions for good 3, capital, and effective labor are given by, respectively:

$$Y_3 = C_3, \tag{6}$$
$$K = \textstyle\sum_j K_j = \sum_j a_{Kj} Y_j, \tag{7}$$
$$H = \textstyle\sum_j H_j = \sum_j a_{Hj} Y_j. \tag{8}$$

From Eqs. (2), (3), (6), (7), and (8), we obtain the current account balance:

$$Y_1 - C_1 - \dot{K} - \delta K + p_2(Y_2 - C_2) + T = 0.$$

2.4 Patterns of specialization

Following Dixit and Norman (1980), we define the GDP function as:

$$\begin{aligned} G(p_1, p_2, p_3, K, H) &\equiv \max_{\{Y_j, K_j, H_j\}_{j=1}^3} \{\textstyle\sum_j p_j Y_j : F_j(K_j, H_j) \geq Y_j, K \geq \sum_j K_j, H \geq \sum_j H_j\} \\ &\equiv \min_{r,v} \{rK + vH : p_j \leq c_j(r, v)\}. \end{aligned}$$

As is well known, $G(\cdot)$ is increasing, convex, and linearly homogeneous in (p_1, p_2, p_3), whereas it is increasing, concave, and linearly homogeneous in (K, H). We solve the second problem graphically as in Dixit and Norman (1980) to see the patterns of specialization.[4] Consider the first case in which good 3 is the least capital-intensive of the three goods: $a_{Kj'}/a_{Hj'} > a_{Kj''}/a_{Hj''} > a_{K3}/a_{H3}$ for all (r, v), where traded good j' is more capital-intensive than traded good $j''(j', j'' = 1, 2, j' \neq j'')$. In Fig. 1, where r and v are measured on the horizontal and vertical axes, respectively, the unit cost curve of good j' is the steepest, whereas that of good 3 is the flattest, along any common ray from the origin. The solution should at most be on curve FAG, the southwest boundary of a set $\{(r, v) : p_j \leq c_j(r, v)$ for all $j = 1, 2\}$, where point A: $(\widetilde{r}, \widetilde{v})$ is the point at which the constraints hold with equality for both traded goods with (p_1, p_2) given. If p_3 happens to be equal to $\widetilde{p}_3 \equiv c_3(\widetilde{r}, \widetilde{v})$, then the solution is just at point A, where the economy is incompletely specialized.[5] For $p_3 \neq \widetilde{p}_3$, however, the economy cannot be incompletely specialized. If $p_3 > \widetilde{p}_3$, then curve FAG and curve $p_3 = c_3(r, v)$ intersect at a point to the northwest of point A, say point B: $(r_{j'}(p_3), v_{j'}(p_3))$, where the economy completely specializes in good j'. Intuitively, a rise in the price of the nontraded good, the most effective-labor-intensive one, tends to raise the price of effective labor relative to capital. This is more damaging to firms in sector j'', the more effective-labor-intensive of the two traded sectors, forcing them to shut down. Similarly, if $p_3 < \widetilde{p}_3$, then the solution is at a point to the southeast of point A, say point C: $(r_{j''}(p_3), v_{j''}(p_3))$, where the economy completely specializes in good j''. Our graphical analysis of the factor payment minimization problem implies that the GDP function is written as:

[4] Based on the traditional approach, Jones (1974) analyzed diagrammatically the patterns of specialization corresponding to half of our cases, and obtained the same results as ours. The difference between this paper and Jones (1974) is that our dual formulation enables us to solve explicitly for the absolute factor prices and the GDP function.

[5] An additional requirement for incomplete specialization is that the aggregate capital/effective labor ratio $K/H = 1/h$ should be such that $a_{Kj''}/a_{Hj''} > 1/h > a_{K3}/a_{H3}$ at point A.

$$G(\cdot) = g(p_3)K;$$

$$g(p_3) = \begin{cases} r_{j'}(p_3) + v_{j'}(p_3)h & \text{for } p_3 \in (\widetilde{p}_3, \overline{p}_3), \\ \widetilde{r} + \widetilde{v}h & \text{for } p_3 = \widetilde{p}_3, \\ r_{j''}(p_3) + v_{j''}(p_3)h & \text{for } p_3 \in (\underline{p_3}, \widetilde{p}_3), \end{cases}$$

where $g(p_3)$ is the maximized GDP per unit of capital; $\overline{p}_3$ and $\underline{p_3}$ are the upper and lower bounds on p_3 for production of at least two goods, respectively.[6] Since $G(\cdot)$ is increasing and convex in p_3, we have $g_3(\cdot) \equiv dg(\cdot)/dp_3 > 0$ and $g_{33}(\cdot) \equiv d^2g(\cdot)/dp_3^2 > 0$ in differentiable regions. However, it is not differentiable at $p_3 = \widetilde{p}_3$ because production allocation is indeterminate under incomplete specialization. As explained by Dixit and Norman (1980), the indeterminacy comes from the fact that we have only two equations (i.e., factor market clearing conditions) to solve for three variables (i.e., outputs). Consequently, the supply curve of good 3 per unit of capital $y_3(p_3)$ is not a function but a correspondence: it has two upward-sloping regions with complete specialization in either traded good, and a flat region with incomplete specialization. Finally, note that the Stolper-Samuelson theorem holds: an increase in p_3, the price of the most effective-labor-intensive good, raises the price of effective labor but lowers the price of capital for each pattern of specialization.

The other cases can be analyzed similarly. In the second case where good 3 is the most capital-intensive: $a_{K3}/a_{H3} > a_{Kj'}/a_{Hj'} > a_{Kj''}/a_{Hj''}$, the economy completely specializes in good j'' for $p_3 > \widetilde{p}_3$, whereas it completely specializes in good j' for $p_3 < \widetilde{p}_3$.[7] In the third case where the capital/effective labor ratio of good 3 is in the middle: $a_{Kj'}/a_{Hj'} > a_{K3}/a_{H3} > a_{Kj''}/a_{Hj''}$, curve FAG and curve $p_3 = c_3(r, v)$ intersect at two points for $p_3 > \widetilde{p}_3$. If $a_{Kj'}/a_{Hj'} > 1/h > a_{K3}/a_{H3}$ at point A, then the economy completely specializes in good j' just like the first case. If $a_{K3}/a_{H3} > 1/h > a_{Kj''}/a_{Hj''}$ at point A, then the economy completely specializes in good j'' as in the second case. For $p_3 < \widetilde{p}_3$, the solution is at point A, where the nontraded good cannot be produced. In this case, the lower bound on p_3 is set at $\underline{p_3} = 0$.

The analysis in this section has the following implications. If we assume incomplete specialization in line with the literature (e.g., Komiya, 1967; Peletier, 1998; Yano and Nugent, 1999; Schweinberger, 2002), then the factor prices and the price of the nontraded good are determined by technologies and the world prices of the two traded goods, and hence they are invariable with any demand shock. More importantly, however, if we allow for complete specialization, then the factor prices and the price of the nontraded good can be variable with demand shocks. This suggests that, as Rajan and Subramanian (2005) hypothesized, the price of the nontraded good may work as a transmission mechanism of foreign aid under complete specialization. Moreover, the price change induced by foreign aid can affect the long-run growth rate through the Stolper-Samuelson theorem. We pursue this possibility in the rest of this paper.

3 Dynamics

Before investigating the effects of foreign aid, we study the dynamics of our model. From Eqs. (3), (4), and (6), our dynamic system is given by:

[6] They are implicitly defined by $1/h = a_{K3}(r_{j'}(\overline{p}_3), v_{j'}(\overline{p}_3))/a_{H3}(r_{j'}(\overline{p}_3), v_{j'}(\overline{p}_3))$ and $a_{Kj''}(r_{j''}(\underline{p_3}), v_{j''}(\underline{p_3}))/a_{Hj''}(r_{j''}(\underline{p_3}), v_{j''}(\underline{p_3})) = 1/h$, respectively.

[7] The restriction on the aggregate capital/effective labor ratio must now be that $a_{K3}/a_{H3} > 1/h > a_{Kj'}/a_{Hj'}$ at point A. The upper and lower bounds on p_3 are now implicitly defined by $a_{K3}(r_{j''}(\overline{p}_3), v_{j''}(\overline{p}_3))/a_{H3}(r_{j''}(\overline{p}_3), v_{j''}(\overline{p}_3)) = 1/h$ and $1/h = a_{Kj'}(r_{j'}(\underline{p_3}), v_{j'}(\underline{p_3}))/a_{Hj'}(r_{j'}(\underline{p_3}), v_{j'}(\underline{p_3}))$, respectively.

$$\begin{aligned}
\gamma_{Xt} &= (1/\theta)(r(p_{3t}) - \delta - \rho), \\
\dot{K}_t &= (g(p_{3t}) - \delta)K_t + T_t - e(p_{3t})^{1-1/\theta} X_t, \\
e_3(p_{3t})e(p_{3t})^{-1/\theta} X_t &= y_3(p_{3t})K_t,
\end{aligned}$$

where $y_3(p_3)$ is the supply correspondence of good 3 per unit of capital characterized in the previous section. We assume permanent aid in the sense that the aid/capital ratio $\tau \equiv T_t/K_t$ is constant over time, so that the aid/GDP ratio, the main explanatory variable of the aid-growth regressions, is constant over time for a constant p_3. Defining $x_t \equiv X_t/K_t$ as the modified consumption index per unit of capital, the above system is reduced to:

$$\dot{x}_t = x_t[(1/\theta)(r(p_{3t}) - \delta - \rho) - (g(p_{3t}) - \delta + \tau - e(p_{3t})^{1-1/\theta} x_t)], \tag{9}$$
$$0 = e_3(p_{3t})e(p_{3t})^{-1/\theta} x_t - y_3(p_{3t}). \tag{10}$$

We define a steady state as a situation in which all variables grow at constant (perhaps zero) rates. Eq. (9) implies that p_{3t} and x_t must be constant in the steady state. From Eqs. (9) and (10), a steady state must satisfy:

$$(1/\theta)(r(p_3) - \delta - \rho) = g(p_3) - \delta + \tau - e(p_3)^{1-1/\theta} x, \tag{11}$$
$$e_3(p_3)e(p_3)^{-1/\theta} x = y_3(p_3). \tag{12}$$

Let (x^*, p_3^*) be the steady-state values of (x, p_3) solving Eqs. (11) and (12).

To examine the existence and uniqueness of the steady state, we define the compensated excess demand correspondence for good 3 per unit of capital as:

$$z_3^c(p_3, x) \equiv e_3(p_3)e(p_3)^{-1/\theta} x - y_3(p_3). \tag{13}$$

Solving Eq. (11) for x, we have $x = x(p_3)$, which captures the income effect. Substituting this expression into Eq. (13), we obtain the ordinary excess demand correspondence for good 3 per unit of capital: $z_3(p_3) \equiv z_3^c(p_3, x(p_3))$. A steady state is obtained if and only if $z_3(p_3^*) = 0$. We then have $x^* = x(p_3^*)$. Finally, all growing variables grow at the same rate: $\gamma_X^* = \gamma_K^* = \gamma_C^* = (1/\theta)(r(p_3^*) - \delta - \rho) \equiv \gamma^*$. The following lemma provides the conditions that ensure the existence and uniqueness of the steady state:[8]

Lemma 1 *Suppose that* $\lim_{p_3 \to \overline{p}_3} z_3(p_3) < 0$ *and* $z_3(p_3^1) < z_3(p_3^0)$ *for all* $p_3^1 > p_3^0$*. Then there exists the unique* $p_3^* \in (\underline{p_3}, \overline{p}_3)$ *such that* $z_3(p_3^*) = 0$.

Proof. Since $\lim_{p_3 \to \underline{p_3}} y_3(p_3) = 0$, we have $\lim_{p_3 \to \underline{p_3}} z_3(p_3) > 0$. On the other hand, the previous section has shown that $y_3(p_3)$ is continuous on $(\underline{p_3}, \overline{p}_3)$, and so is $z_3(p_3)$. From these facts and the first assumption, applying the intermediate value theorem implies the existence. The uniqueness immediately holds from the second assumption about monotonicity. ■

[8] The first condition is equivalent to $T < \lim_{p_3 \to \overline{p}_3}(C_1 + \dot{K} + \delta K + p_2 C_2)$, because Walras' law and Eqs. (7) and (8) imply that $C_1 + \dot{K} + \delta K + p_2 C_2 + p_3(C_3 - Y_3) = T$ as $p_3 \to \overline{p}_3$.

Next, we investigate the transitional dynamics around the unique steady state. Suppose first that $p_3^* \neq \widetilde{p}_3$. Then totally differentiating Eqs. (11) and (13), we obtain:

$$dz_3 = \zeta dp_3; \zeta \equiv \sigma + e_3 e^{-1}\phi,$$
$$\sigma \equiv [e_{33}e^{-1/\theta} - (1/\theta)e_3^2 e^{-1/\theta-1}]x - g_{33} < 0, \phi \equiv g_3 - (1-1/\theta)e^{-1/\theta}e_3 x - (1/\theta)r_3,$$

where $r_3(\cdot) \equiv dr(\cdot)/dp_3$. In this expression, ζ gives the slope of the ordinary excess demand curve for good 3 per unit of capital against p_3 in differentiable regions. As usual, it consists of the substitution effect, which is necessarily negative, and the income effect. The second assumption in Lemma 1 is equivalent to assuming that $\zeta < 0$.

Linearizing Eqs. (9) and (10) around the steady state, and eliminating $p_3 - p_3^*$, local dynamics are characterized by a single differential equation for x :

$$\dot{x} = a^*(x - x^*); a^* \equiv x^*(e^{*1-1/\theta} + \phi^* e_3^* e^{*-1/\theta}/\sigma^*) = x^* e^{*1-1/\theta}\zeta^*/\sigma^*.$$

Since $\zeta^* < 0$, the dynamics of x are unstable around x^*. This implies that, for the given initial condition K_0, X_0 must be chosen such that $x_t = x^*$ for all $t \in [0, \infty)$. Therefore, the dynamic system has no transitional dynamics.

When $p_3^* = \widetilde{p}_3$, on the other hand, x^* is determined only by Eq. (11), and then the supply of good 3 is determined by its demand through Eq. (12). Linearizing the only dynamic equation (9) with p_3 fixed at $p_3 = \widetilde{p}_3$, we obtain $\dot{x} = x^* e^{*1-1/\theta}(x - x^*)$, again implying no transitional dynamics. To sum up:

Proposition 1 *Suppose that the assumptions in Lemma 1 hold. Then the economy is always in the unique steady state along the equilibrium path.*

It is learning by doing and intersectoral spillovers that are responsible for the no-transition property. They keep the returns to capital constant, and minimize the number of state variables to one. That is why our multi-sector model has the simple dynamics just like the one-sector Arrow (1962)-Romer (1986) model.

4 Growth effect of untied aid

We know from the previous section that, with the aid/capital ratio constant over time, the economy always grows at the following constant rate:

$$\gamma(p_3) \equiv (1/\theta)(r(p_3) - \delta - \rho), \tag{14}$$

where we suppress the asterisks representing the steady state. Because p_3 is invariable with any demand shock if $p_3 = \widetilde{p}_3$ in the original equilibrium, we focus on the case in which $p_3 \neq \widetilde{p}_3$ and thus the economy completely specializes in either traded good in the original equilibrium. From Eqs. (3), (6), and (14), our model is formulated as:

$$\gamma(p_3)K + \delta K + e(p_3)C = g(p_3)K + T, \tag{15}$$
$$e_3(p_3)C = g_3(p_3)K. \tag{16}$$

In each period, with K predetermined and with T exogenous, Eqs. (15) and (16) are solved for p_3 and C. Since all K, C, and T grow at the same rate $\gamma(p_3)$, the equilibrium vale of p_3 solving the above system is indeed constant over time.

First, we see how an exogenous increase in T affects p_3. Totally differentiating Eqs. (15) and (16), we have:

$$edC = dT - (e_3C - g_3K)dp_3 - \gamma_3 K dp_3, \tag{17}$$

$$0 = (e_{33}C - g_{33}K)dp_3 + e_3 dC, \tag{18}$$

where $\gamma_3(\cdot) \equiv d\gamma(\cdot)/dp_3 = (1/\theta)r_3(\cdot)$. In the right-hand side of Eq. (17), the first and second terms represent the direct transfer effect and the terms of trade effect, respectively, which are common in the static transfer literature. The third term stands for the indirect growth effect: for example, a rise in the growth rate leaves less income for consumption. Note that the second term now becomes zero since good 3 is nontraded from Eq. (16). Substituting Eq. (17) into Eq. (18), the change in the ordinary excess demand correspondence for good 3 is expressed as:

$$0 = \Delta dp_3 + e_3 e^{-1} dT; \Delta \equiv e_{33}C - g_{33}K - e_3 e^{-1}(e_3C - g_3K + \gamma_3 K),$$

where Δ indicates the slope of the ordinary excess demand curve for good 3 against p_3. Simple calculation shows that $\Delta = K\zeta < 0$ as expected. Consequently, we obtain:

$$dp_3/dT = -e_3 e^{-1}/\Delta > 0.$$

An increase in foreign aid increases the demand for good 3, thereby raising its price under the downward-sloping excess demand curve, a very intuitive result.

We know from section 2 that the relationship between p_3 and the factor prices obeys the Stolper-Samuelson theorem. Formally, totally differentiating the zero-profit conditions $p_j = c_j(r, v)$ for the operating traded good $j (j = 1$ or $2)$ and the nontraded good, and solving them for changes in factor prices, we obtain $dr = -a_{Hj}dp_3/a$ and $dv = a_{Kj}dp_3/a$, where $a \equiv a_{Kj}a_{H3} - a_{Hj}a_{K3} >$ (or $<$)0 if and only if $a_{Kj}/a_{Hj} >$ (or $<$)a_{K3}/a_{H3}, that is, the operating traded good is more capital-intensive (or effective-labor-intensive) than the nontraded good. Combining our results, we obtain:

$$\frac{d\gamma}{dT} = \gamma_3 \frac{dp_3}{dT} = \frac{1}{\theta} r_3 \frac{dp_3}{dT} = \frac{1}{\theta}\left(-\frac{a_{Hj}}{a}\right)\frac{dp_3}{dT} \left\{ \begin{array}{c} > \\ < \end{array} \right\} 0 \Leftrightarrow \frac{a_{Kj}}{a_{Hj}} \left\{ \begin{array}{c} < \\ > \end{array} \right\} \frac{a_{K3}}{a_{H3}}.$$

Proposition 2 .

1. *Suppose that $p_3 = \widetilde{p}_3$ in the original equilibrium. Then a permanent increase in untied aid does not affect the growth rate.*

2. *Suppose that $p_3 \neq \widetilde{p}_3$ in the original equilibrium. Then a permanent increase in untied aid raises (or lowers) the growth rate if and only if the nontraded good is more capital-intensive (or effective-labor-intensive) than the operating traded good.*

This proposition gives one theoretical explanation why we fail to find a robust positive relationship between aid and growth as in Rajan and Subramanian (2008). Aid may not affect the growth rate when a country is incompletely specialized. Furthermore, aid may even lower the growth rate as suggested by Rajan and Subramanian (2005) when the nontraded good is more effective-labor-intensive than the operating traded good. If we observe that nontraded goods such as services are more labor-intensive than exported goods such as manufacturing, which does not seem unusual, then our model predicts that aid lowers growth.[9]

5 Welfare effect of untied aid

Remembering that the consumption index in period t is given by $C_t = C_0 \exp(\gamma t)$, the utility function is rewritten as $U = [1/(1-\theta)]\{C_0^{1-\theta}/[\rho - (1-\theta)\gamma] - 1/\rho\}$, where $\rho - (1-\theta)\gamma > 0$ from the assumption for bounded utility. Total differentiation gives $dU = \{C_0^{1-\theta}/[\rho-(1-\theta)\gamma]\}\{dC_0/C_0 + d\gamma/[\rho-(1-\theta)\gamma]\}$, showing that welfare is increasing in the initial consumption index and the growth rate.

As for the consumption index, Eqs. (16) and (17) yield $edC = (1 - K d\gamma/dT)dT$. If $p_3 = \widetilde{p}_3$ in the original equilibrium, then $d\gamma = 0$ and thus welfare rises trivially from increasing aid.[10] If $p_3 \neq \widetilde{p}_3$ in the original equilibrium, however, then a change in the growth rate has the opposing effect on the consumption index. Noting from Eq. (15) that $eC = K(g - \delta - \gamma + \tau)$, we obtain:

$$\frac{dU}{dT_0} = \frac{C_0^{1-\theta}}{\rho - (1-\theta)\gamma}\left\{\frac{1}{K_0(g-\delta-\gamma+\tau)} + \frac{d\gamma}{dT_0}\left[\frac{1}{\rho-(1-\theta)\gamma} - \frac{1}{g-\delta-\gamma+\tau}\right]\right\}.$$

As $g - \delta - \gamma + \tau = r + vh - \delta - \gamma + \tau = \rho - (1-\theta)\gamma + vh + \tau > \rho - (1-\theta)\gamma > 0$, it turns out that the direct growth effect on welfare always outweighs its indirect effect through the initial consumption index. Combined with Proposition 2, the welfare effect is summarized in the following proposition:

Proposition 3 .

1. *Suppose that $p_3 = \widetilde{p}_3$ in the original equilibrium. Then a permanent increase in untied aid raises welfare.*
2. *Suppose that $p_3 \neq \widetilde{p}_3$ in the original equilibrium.*
   - (a) *Suppose that the nontraded good is more capital-intensive than the operating traded good. Then a permanent increase in untied aid raises welfare.*
   - (b) *Suppose that the nontraded good is more effective-labor-intensive than the operating traded good. Then a permanent increase in untied aid raises (or lowers) welfare if and only if $(d\gamma/dT_0)\{1/[\rho - (1-\theta)\gamma] - 1/(g-\delta-\gamma+\tau)\} >$ (or $<$) $-1/[K_0(g-\delta-\gamma+\tau)]$.*

The key to understanding why aid may either raise or lower welfare is that the equilibrium growth rate falls short of the optimal growth rate in our model with learning by doing and intersectoral knowledge spillovers. This is because, in competitive equilibrium, people care only about the private rate of return to capital r, ignoring the fact that their investment socially earns an additional return vh through increasing the effectiveness of labor. When the nontraded good is more capital-intensive than the operating traded good, aid brings the equilibrium growth rate closer to the optimal one, thereby improving resource allocation. When the nontraded good is more effective-labor-intensive than the operating traded good, however, aid

[9] Using the annual data between 1948 and 1969 for the United States, Burgess (1976) estimated that nontraded goods (services and structures) were more labor-intensive than traded goods (durables and nondurables).

[10] With $p_3 = \widetilde{p}_3$ unchanged, the aid-induced increase in the demand for the nontraded good brings about the same increase in its output. The outputs of the traded goods change according to Eqs. (7) and (8).

pulls the equilibrium growth rate further away from the optimal one, which may cause the small-country transfer paradox.

6 Tied aid for production subsidies

Although our main focus is on untied aid, our model can also be applied to tied aid. Suppose that, instead of transferring aid to the representative household, the government is forced to use it for subsidizing production of the three goods. Then the government budget constraint is given by $s_1 Y_1 + s_2 p_2 Y_2 + s_3 p_3 Y_3 = T$, where s_j is the rate of an ad-valorem production subsidy to sector j. With the producer price of good j changed to $(1 + s_j)p_j$, and with T removed from Eq. (3), we obtain the same current account balance as in section 2.

We restrict our attention to the case in which the original equilibrium is at point B in Fig. 1, where good 3 is the least capital-intensive and the economy completely specializes in the most capital-intensive good j'. First, an infinitesimal aid-induced rise in $s_{j''}$ to the imported good j'' shifts curve $p_{j''} = c_{j''}(r, v)$ outward and then moves the incomplete specialization point A to the northwest along curve $p_{j'} = c_{j'}(r, v)$, but it does not affect the equilibrium point B and hence the growth rate. This is because the imported good is actually not produced. Second, a rise in $s_{j'}$ to the exported good j' shifts curve $p_{j'} = c_{j'}(r, v)$ outward, moving point B as well as point A to the southeast along curve $p_3 = c_3(r, v)$ and $p_{j''} = c_{j''}(r, v)$, respectively. This is not the end of the story, however: the decreased supply of good 3 raises p_3, whose amount depends inversely on the degree of substitution. Since the former effect raises r but the latter effect lowers it, the total growth effect is ambiguous. Third, a rise in s_3 to the nontraded good 3 shifts curve $p_3 = c_3(r, v)$ outward, pushing point B to the northwest along curve $p_{j'} = c_{j'}(r, v)$. However, the increased supply of good 3 lowers p_3, pulling the equilibrium point back toward the original one. Since the fall in p_3 increases the equilibrium quantity of demand and hence supply of good 3, $(1 + s_3)p_3$ should still be higher than the original level. Therefore, the growth rate falls due to the Stolper-Samuelson theorem.

In contrast to the one-sector framework of Chatterjee et al. (2003) and Chatterjee and Turnovsky (2007), tying aid to stimulating production of each single sector does not necessarily promote growth in our three- (or actually two-) sector model. This is because the returns to capital depend on the Stolper-Samuelson theorem and the endogenous price of the nontraded good. One way to avoid this difficulty is to raise subsidies to all sectors at the same rate. In Fig. 1, this moves the equilibrium point to the northeast along ray OB without changing the supply of good 3 and hence p_3. These proportional subsidies raise the growth rate by enabling the economy to behave like a one-sector model.

7 Concluding remarks

Our model is admittedly simplified especially because it leads to no production of the import-competing good, which seems unlikely in practice at the present level of aggregation. Some effort will be required to make the model more realistic. Still, our model has interesting policy implications. First, factor intensity ranking can be a determinant of the aid-growth relationship. For example, suppose that good 1, good 2, and good 3 are manufacturing, agriculture, and service, respectively. When the nontraded service is more effective-labor-intensive than the traded good a developing country exports, aid slows down aggregate growth as Rajan and Subramanian (2005) argued. Otherwise, however, aid raises growth even if it is untied. Second, endogenous growth based on externalities can give rise to the small-country transfer paradox. In the literature on this issue (e.g., Yano and Nugent, 1999; Schweinberger, 2002), a decrease in tariff revenue caused by import substitution is essential for the welfare-reducing aid. In contrast, we show that aid may lower welfare even under free trade in our dynamic small open economy. When aid lowers growth, its welfare effect is at best ambiguous. This backs up Easterly's (2003, p. 40) view that: "In virtually no other field of economics do economists and policymakers promise such large welfare benefits for modest policy interventions as 'we' do in aid and growth. The macroeconomic evidence does not support these claims."

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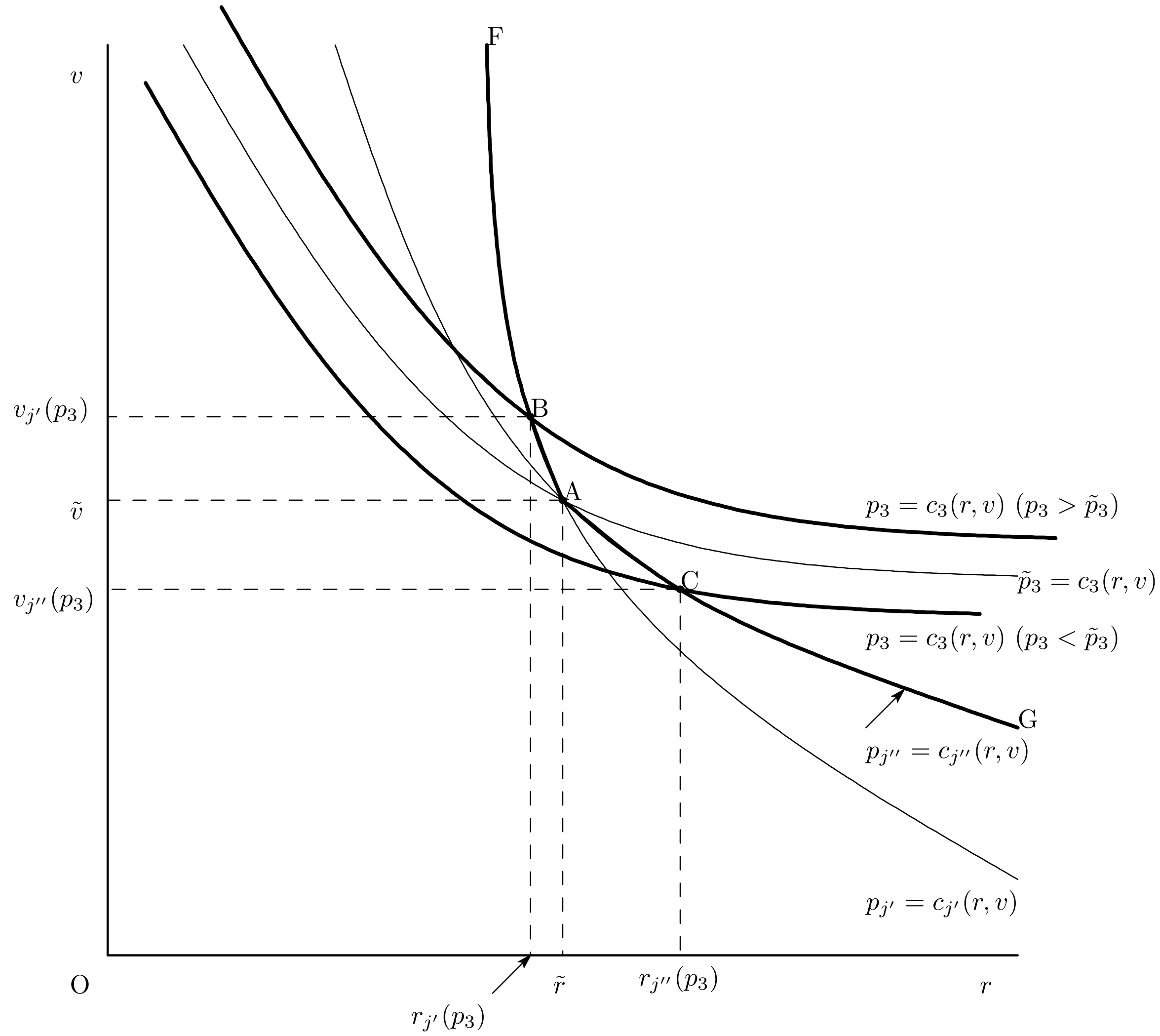

Fig. 1. Equilibrium factor prices: $a_{Kj'}/a_{Hj'} > a_{Kj''}/a_{Hj''} > a_{K3}/a_{H3}$.