An Eaton–Kortum model of trade and growth*

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Abstract

We combine a multi-country, continuum-good Ricardian model of Eaton and Kortum (2002) with a multi-country AK model of Acemoglu and Ventura (2002) to examine how trade liberalization affects countries' growth rates and extensive margins of trade over time. Focusing on the three-country case, we obtain three main results. First, a permanent fall in any trade cost raises the balanced growth rate. Second, trade liberalization increases the liberalizing countries' long-run fractions of exported varieties to all destinations. Third, the long-run effects of trade liberalization are different from its short-run effects, which can reverse the welfare implications of the static Eaton-Kortum model.

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1 Introduction

We are now witnessing a revival of the Ricardian model of international trade.[1] This revival is led by Eaton and Kortum (2002), who extend a two-country, continuum-good Ricardian model of Dornbusch et al. (1977) to an arbitrary number of countries by assuming that each country's productivity of each variety in the unit interval is randomly drawn from a country-specific probability distribution. One of the remarkable features of the Eaton-Kortum model is that it enables us to analyze the effects of trade liberalization on the extensive margins of trade (i.e., the numbers or fractions of varieties each country imports from and exports to other countries) under high levels of asymmetry across countries.[2] However, their static formulation overlooks the fact that countries tend to export more and more varieties as they grow faster than the rest of the world (e.g., Hummels and Klenow, 2005; Kehoe and Ruhl, 2013). Allowing for different growth paths across countries may help explain the evolution of the extensive margins that the static framework cannot describe. The purpose of this paper is to develop an Eaton-Kortum model of trade and growth to examine how trade liberalization affects countries' growth rates and extensive margins of trade over time.

To model the interactions between trade and growth, we have to decide which growth mechanism to choose, factor accumulation or research and development (R&D). According to cross-country development accounting exercises reviewed by Caselli (2005), only 34 to 39 percent of cross-country income variation can be explained by variation in factors; the rest comes from variation in total factor productivity (TFP). This suggests that R&D affects countries' growth more than factor accumulation, although the latter still explains around one third of income differences. In spite of the empirical weakness, we use a multi-country AK model of Acemoglu and Ventura (2002), one of the accumulation-based endogenous growth models.[3] This is because the model demonstrates that deterioration in the terms of trade for faster-growing countries brings about a long-run convergence of growth rates across (arbitrarily asymmetric) countries and the resulting stable world income distribution even without international knowledge spillovers.[4] The feature is desirable for studying the short- and long-run impacts of trade liberalization, which are not necessarily symmetric across countries. Naito (2012) introduces international competition a la Dornbusch et al. (1977) in place of product differentiation in the two-country version of Acemoglu and Ventura (2002), and investigates the effects of unilateral trade liberalization on countries' growth rates, extensive margins, and welfare. While obtaining clear-cut results, the two-country model cannot be applied to problems involving more than two countries. For example, considering a preferential trade agreement between two countries requires at least one outsider

[1] See Matsuyama (2008) and Eaton and Kortum (2012) for reviews of recent developments in the Ricardian trade theory.

[2] Although Melitz (2003) studies the same topic in a multi-country monopolistic competition model with firm heterogeneity, his analysis is based on the assumption of symmetric countries having a common productivity distribution. Baldwin and Robert-Nicoud (2008) embed the Melitz framework into an R&D-based endogenous growth model with expanding product variety, but their analysis is also subject to the symmetry assumption.

[3] The AK model is a class of endogenous growth models based on factor accumulation, where the marginal product of capital is (at least directly) independent of the size of capital stock. As explained in famous growth textbooks such as Barro and Sala-i-Martin (2004, Chapter 4) and Acemoglu (2009, Chapter 11), there are two ways to formulate this: (i) technologies for producing the capital good are constant returns to scale in capital; or (ii) technologies for producing the capital good are constant returns to scale in capital and labor, but the efficiency of labor is proportional to some nonrival input such as knowledge represented by the aggregate capital stock (e.g., Romer, 1986) or public services financed by a flat-rate income tax (e.g., Barro, 1990). In type (ii), since benefits of the nonrival input spread over all workers at the same time, the resulting long-run growth rate is increasing in the size of population. This counterfactual feature, called the scale effect, also applies to the earlier R&D-based endogenous growth models such as Romer (1990) and Aghion and Howitt (1992), although there are several attempts to eliminate it in the literature (e.g., Jones, 1995; Peretto, 1998; Dinopoulos and Thompson, 1998; see also Jones, 1999, for review). However, our model falls into type (i), and thus is not subject to the scale effect, because the final good technologies are constant returns to scale in intermediate goods, which in turn are constant returns to scale in capital, without nonrival input anywhere.

[4] In contrast, a two-country R&D-based endogenous growth model of Feenstra (1996) shows that trade in goods leads to a divergence of growth rates in the absence of international knowledge spillovers.

country.[5] By combining Eaton and Kortum (2002) with Acemoglu and Ventura (2002), we formulate a model of endogenous growth and extensive margins which is applicable to multi-country problems.

After providing a general multi-country model, we focus on the three-country case to understand the mechanics of our model more deeply. We obtain the following main results. First, a permanent fall in any trade cost raises the balanced growth rate. For example, a fall in country 1's import trade cost from country 2 increases its growth potential by allowing it to import varieties from country 2 more cheaply. Since this raises the relative rental rates and hence the terms of trade of countries 2 and 3 against country 1 through the above-mentioned stabilization mechanism, all countries grow at a higher balanced growth rate in the long run. This result is consistent with the recent sophisticated empirical research finding the positive relationship between trade liberalization and economic growth (e.g., Wacziarg and Welch, 2008; Estevadeordal and Taylor, 2013).[6]

Second, trade liberalization increases the liberalizing countries' long-run fractions of exported varieties to all destinations. Country 1's unilateral trade liberalization for imports from country 2 shifts its extensive margins of imports away from country 3 to country 2. Not only that, it also increases country 1's extensive margins of exports to all destinations because of its decreased long-run rental rates relative to countries 2 and 3. The result also applies to preferential trade liberalization between countries 1 and 2, with their long-run bilateral terms of trade unchanged: it increases their extensive margins of exports to both inside and outside destinations through their decreased long-run rental rates relative to country 3. Moreover, the fact that their fractions of domestic varieties and those of imported varieties from country 3 decrease means that both trade creation and trade diversion at the extensive margins occur as a result of such preferential trade liberalization. This result provides one theoretical explanation for why faster-growing countries export increasingly more varieties (e.g., Hummels and Klenow, 2005; Kehoe and Ruhl, 2013).

Third, the long-run effects of trade liberalization are different from its short-run effects, which can reverse the welfare implications of the static Eaton-Kortum model. In particular, starting from near the symmetric BGP, both trade liberalization schemes mentioned above raise the growth rates in countries 1 and 2 but lower the growth rate in country 3 in the initial period. The last result implies that country 3's rate of return to capital and hence its welfare fall in the static Eaton-Kortum model. In our dynamic model, however, even country 3's overall welfare can rise through accelerated long-run growth. The present result is remarkable in the context of regional trade agreements. It is well known that a free trade agreement between two countries raises a third country's welfare only if the tariff complementarity effect works, that is, each member country voluntarily lowers its optimal external tariff at the same time as the establishment of the FTA (e.g., Bagwell and Staiger, 1999; Ornelas, 2005). In contrast, in our model, welfare of a nonmember country can rise without member countries lowering their external trade costs. Finally, it should be emphasized that the difference between the long- and short-run impacts of trade liberalization emerges only in the case of more

[5]Dinopoulos and Syropoulos (1997) construct a three-country R&D-based endogenous growth model to see the long-run growth and welfare effects of unilateral, bilateral, and multilateral trade liberalization. Due to the presence of imperfect competition and nontraded goods, reductions in trade costs do not always raise countries' long-run growth. However, they do not address the evolution of trade patterns.

[6]Wacziarg and Welch (2008) collect a data set of 136 countries during 1950-1998 including the famous Sachs and Warner (1995) openness dummy. Their fixed-effects regressions reveal that, when a country changes from "closed" to "open", its annual growth rate of real GDP per capita goes up by 1.42 percentage points on average. Estevadeordal and Taylor (2013) construct a data set of 75 countries for two fifteen-year periods, 1975-1989 and 1990-2004, and divide the countries into two groups, liberalizers (i.e., countries whose size of tariff reduction during 1985-2000 was above the median) and nonliberalizers. Applying difference-in-differences regressions, they find that the change in the growth rate of liberalizers between the two periods is, on average, 0.72 percentage points larger than nonliberalizers. It should be noted that those results just show the average growth effect of trade liberalization: as Wacziarg and Welch (2008, Table 7) admit, in a subsample of 24 developing countries, eight of them experienced a slower growth after liberalization.

than two countries. This highlights the importance of departing from the two-country model of Naito (2012).

The rest of this paper is organized as follows. Section 2 first sets up the multi-country model, and then concentrate on the three-country case. Section 3 examines analytically the long-run effects of unilateral and bilateral trade liberalization, and compare them with the short-run effects. Section 4 concludes.

2 The model

Consider a world with $N(\geq 2)$ countries. In each country $j(=1,...,N)$, there is only one nontradable final good, which can either be consumed or invested for capital accumulation.[7] The final good is produced from a continuum of tradable intermediate goods indexed by $i(\in[0,1])$ under constant returns to scale and perfect competition.[8] Each variety i in turn can be produced using nontradable capital under constant returns to scale and perfect competition. Capital is the only primary factor.[9]

2.1 Households

The representative household in country j maximizes its utility $U_j = \int_0^\infty \ln C_{jt} \exp(-\rho_j t)dt$, subject to the budget constraint:

$$p_{jt}^Y(C_{jt} + \dot{K}_{jt} + \delta_j K_{jt}) = r_{jt}K_{jt}, \tag{1}$$

with $\{p_{jt}^Y, r_{jt}\}_{t=0}^\infty$ and K_{j0} given, where C_j is consumption; ρ_j is the subjective discount rate; p_j^Y is the price of the final good; K_j is the stock of capital; δ_j is the depreciation rate of capital; r_j is the rental rate of capital; and a dot over a variable represents differentiation with respect to time t (e.g., $\dot{K}_{jt} \equiv dK_{jt}/dt$). We omit the time subscripts whenever there is no confusion. Under the logarithmic instantaneous utility function, it is optimal for the representative household to keep the consumption/capital ratio constant at ρ_j for all periods, which immediately implies that capital always grows at the same (but not necessarily constant) rate as consumption given by the Euler equation:[10]

$$\dot{K}_{jt}/K_{jt} = \dot{C}_{jt}/C_{jt} = r_{jt}/p_{jt}^Y - \delta_j - \rho_j \forall t \in [0,\infty). \tag{2}$$

We call this common growth rate "the growth rate in country j".

2.2 Final good firms

The representative final good firm in country j maximizes its profit $\Pi_j^Y = p_j^Y Y_j - \int_0^1 p_j(i)x_j(i)di$, subject to the production function $Y_j = B_j(\int_0^1 x_j(i)^{(\sigma_j-1)/\sigma_j}di)^{\sigma_j/(\sigma_j-1)}; \sigma_j > 1$, with p_j^Y and $\{p_j(i)\}_{i=0}^1$ given, where

[7] This looks different from Acemoglu and Ventura (2002), where a capital good sector is distinct from a consumption good sector. However, since the two final good sectors have the same technologies except for a TFP parameter in the capital good sector, the relative price of the two final goods is fixed by the TFP parameter, and hence their two-final-good model effectively behaves like a one-final-good model.

[8] In Acemoglu and Ventura (2002), the final good technologies are Cobb-Couglas in capital and a CES composite of the intermediate goods. Introducing the direct use of capital in this way will not qualitatively affect our results.

[9] The AK model seems unrealistic in that no labor is explicitly used in the production process. However, as Barro and Sala-i-Martin (2004) argue, the "K" can be interpreted as including both physical and human capital (representing effective labor). In fact, if the final good technologies are constant returns to scale in physical and human capital, and the final good is used for consumption and investment in physical and human capital, the two types of capital always grow at the same rate, so the two-capital model behaves like the one-capital AK model.

[10] We reach the former statement by integrating the budget constraint (1) from $s = t$ to infinity, and using the Euler equation and the transversality condition.

Y_j is the supply of the final good; $p_j(i)$ is the demand price of variety i; $x_j(i)$ is the demand for variety i; B_j is the productivity in the final good sector; and σ_j is the elasticity of substitution between any two varieties. The minimized cost is written as:

$$\int_0^1 p_j(i)x_j(i)di = q_jY_j; q_j(\{p_j(i)\}_{i=0}^1) \equiv B_j^{-1}(\int_0^1 p_j(i)^{1-\sigma_j}di)^{1/(1-\sigma_j)}, \tag{3}$$

where $q_j(\cdot)$ is the unit cost function, working as the price index of intermediate goods. With Eq. (3), the first-order condition for profit maximization, also implying zero profit, is given by:

$$p_j^Y = q_j. \tag{4}$$

2.3 Intermediate good firms

Our formulation of the intermediate good sector is based on Eaton and Kortum (2002). The representative intermediate good firm producing variety i in country j maximizes its profit $\Pi^x(i_j) = p(i_j)x(i_j) - r_jK^x(i_j)$, subject to the production function $x(i_j) = K^x(i_j)/a_j(i_j)$, with $p(i_j)$ and r_j given, where $p(i_j)$ is the supply price of variety i; $x(i_j)$ is the supply of variety i; $K^x(i_j)$ is the demand for capital in producing variety i; and $a_j(i_j)$ is the unit capital requirement of variety i; and a subscript after i indicates the country producing variety i. If variety i is actually produced in country j, then its supply price should be equal to its unit cost, which results in zero profit:

$$x(i_j) > 0 \Rightarrow p(i_j) = r_ja_j(i_j), i_j \in I_j \subset [0, 1], \tag{5}$$

where I_j is the set of varieties produced in country j.

We consider iceberg trade costs: one has to ship $\tau_{nj}(\geq 1)$ units of each variety in country j to deliver one unit of that variety to country n. It is assumed that $\tau_{jj} = 1\forall j, \tau_{nj} > 1\forall j \neq n, \tau_{nj} \leq \tau_{nk}\tau_{kj}\forall j, k, n$. If $j \neq n$, then we call τ_{nj} country n's import trade cost from country j, or country j's export trade cost to country n. The unit cost of producing variety i in country j and delivering it to country n is expressed as:

$$p_{nj}(i) = \tau_{nj}r_ja_j(i), i \in [0, 1], j, n = 1, ..., N.$$

Since the representative final good firm in country n buys variety i from the cheapest source, its demand price is given by:

$$p_n(i) = \min\{\{p_{nj}(i)\}_{j=1}^N\}.$$

Let A_j denote an independent and identically distributed random variable for $a_j(i)$. Following Eaton and Kortum (2002), we impose a Fréchet distribution on country j's productivity of capital in each variety $1/A_j$:

$$F_j(z) \equiv \Pr(1/A_j \leq z) \equiv \exp(-b_jz^{-\theta}); b_j > 0, \theta > 1.$$

The parameter b_j captures country j's overall state of intermediate good technology: the higher b_j is, the higher $1/A_j$ tends to be. On the other hand, the parameter θ indicates (the inverse of) variability of the productivity distribution. The fact that θ is common to all countries will give us a useful representation of the intermediate good price index function (3). With $F_j(z)$, the distributions of $P_{nj} = \tau_{nj}r_jA_j$ and

$P_n = \min\{\{P_{nj}\}_{j=1}^N\}$, the i.i.d. random variables for $p_{nj}(i)$ and $p_n(i)$, respectively, are expressed in the following simple forms:

$$G_{nj}(p) \equiv \Pr(P_{nj} \leq p) = 1 - \exp(-p^\theta b_j(\tau_{nj} r_j)^{-\theta}),$$
$$G_n(p) \equiv \Pr(P_n \leq p) = 1 - \exp(-p^\theta \Phi_n); \Phi_n \equiv \textstyle\sum_{j=1}^N b_j(\tau_{nj} r_j)^{-\theta}.$$

Moreover, these price distributions provide us with three well-known properties:

Lemma 1 (Eaton and Kortum (2002)) .

1. *The probability that country n buys a variety from country j is:*

$$\pi_{nj}(\{\tau_{nk} r_k\}_{k=1}^N) \equiv b_j(\tau_{nj} r_j)^{-\theta}/\Phi_n = b_j(\tau_{nj} r_j)^{-\theta}/[\textstyle\sum_{k=1}^N b_k(\tau_{nk} r_k)^{-\theta}]; \qquad (6)$$
$$\textstyle\sum_{j=1}^N \pi_{nj}(\cdot) = 1 \forall n.$$

2. *The conditional distribution of P_{nj}, given that country n buys a variety from country j, is the same as $G_n(p)$ for all j.*

3. *The intermediate good price index function (3) for country n is rewritten as:*

$$Q_n(\{\tau_{nj} r_j\}_{j=1}^N) \equiv c_n \Phi_n^{-1/\theta} = c_n[\textstyle\sum_{j=1}^N b_j(\tau_{nj} r_j)^{-\theta}]^{-1/\theta}; \qquad (7)$$
$$c_n \equiv B_n^{-1}\Gamma(1 + (1-\sigma_n)/\theta)^{1/(1-\sigma_n)}, 1 + (1-\sigma_n)/\theta > 0,$$

where $\Gamma(1 + (1-\sigma_n)/\theta)$ is the Gamma function.

Lemma 1 has some implications. First, π_{nj} also shows the fraction of varieties country n buys from country j. This is because the probability π_{nj} applies to a large number of varieties in the unit interval. If $j \neq n$, then π_{nj} is called country n's extensive margin of imports from country j, or country j's extensive margin of exports to country n. Second, π_{nj} is homogeneous of degree zero, whereas Q_n is homogeneous of degree one, in the source countries' rental rates multiplied by country n's trade costs. The assumption of common θ is responsible for the convenient result.

2.4 Markets

If variety i is actually produced in country j and delivered to country n, then its demand price is expressed as:

$$p_n(i_j) = \tau_{nj} p(i_j) = \tau_{nj} r_j a_j(i_j), i_j \in I_{nj} \subset I_j, \qquad (8)$$

where I_{nj} is the set of varieties country n buys from country j. In each country, the market-clearing conditions for the final good, capital, and the intermediate goods are given by:

$$Y_j = C_j + \dot{K}_j + \delta_j K_j, \tag{9}$$

$$K_j = \int_{I_j} K^x(i_j) di_j, \tag{10}$$

$$x(i_j) = \textstyle\sum_{n=1}^{N} \tau_{nj} x_n(i_j), i_j \in I_j. \tag{11}$$

Finally, summing up Eqs. (1), (3), (4), and (5) for all countries, and considering Eq. (8), we obtain Walras' law: the sum of the values of excess demands for the three types of markets is identically zero.

2.5 Dynamic system

Before deriving the dynamic system, it is worth mentioning two key relationships. First, from Eqs. (4) and (7), the rate of return to capital gross of depreciation in the Euler equation (2) for country n is rewritten as:

$$r_n/p_n^Y = r_n/Q_n(\{\tau_{nj} r_j\}_{j=1}^N) = 1/Q_n(\{\tau_{nj} r_j/r_n\}_{j=1}^N), \tag{12}$$

where linear homogeneity of $Q_n(\{\tau_{nj} r_j\}_{j=1}^N)$ is used. This means that the growth rate in country n is decreasing in $\tau_{nj} r_j/r_n$. This intuitively makes sense: a fall in τ_{nj} or a rise in r_n/r_j, with the latter indicating an improvement in country n's terms of trade $p(i_n)/p(i_j) = (r_n/r_j)a_n(i_n)/a_j(i_j)$, raises its growth rate. Second, the cost share of varieties country n buys from country j is equal to the fraction of varieties country n buys from country j (6):

$$\int_{I_{nj}} p_n(i_j) x_n(i_j) di_j/(Q_n Y_n) = \pi_{nj}(\{\tau_{nk} r_k\}_{k=1}^N) = \pi_{nj}(\{\tau_{nk} r_k/r_n\}_{k=1}^N). \tag{13}$$

The first equality holds because, from property 2 of Lemma 1, the conditional expectation of the expenditure for a variety, given that country n buys it from country j, is the same as $Q_n Y_n$ for all varieties.[11] The second equality follows from the fact that $\pi_{nj}(\{\tau_{nk} r_k\}_{k=1}^N)$ is homogeneous of degree zero. Eq. (13) implies that all adjustments in the cost shares occur at the extensive margins. Note that country n's cost shares depend on the same ratios $\tau_{nk} r_k/r_n$ as its rate of return to capital.

Let us choose capital in the last country N as the numeraire: $r_N \equiv 1$. And, let $\kappa_j \equiv K_j/K_N$ denote the relative supply of capital in country j to country N. Then our model is dramatically reduced to just two types of equations:[12]

$$\dot{\kappa}_j = \kappa_j(\gamma_j(\{\tau_{jn} r_n/r_j\}_{n=1}^N) - \gamma_N(\{\tau_{Nn} r_n\}_{n=1}^N)), j = 1, ..., N-1; \tag{14}$$

$$\gamma_j(\{\tau_{jn} r_n/r_j\}_{n=1}^N) \equiv \dot{C}_j/C_j = 1/Q_j(\{\tau_{jn} r_n/r_j\}_{n=1}^N) - \delta_j - \rho_j,$$

$$\kappa_j = \textstyle\sum_{n=1}^{N} \pi_{nj}(\{\tau_{nk} r_k/r_n\}_{k=1}^N)\kappa_n/(r_j/r_n), j = 1, ..., N-1, \tag{15}$$

[11] Deriving country n's demand for variety i from Eq. (3), and multiplying it by its demand price, the expenditure for variety i is expressed as $p_n(i)x_n(i) = B_n^{\sigma_n - 1} q_n^{\sigma_n} Y_n p_n(i)^{1-\sigma_n}$. On the other hand, the conditional expectation of $P_{nj}^{1-\sigma_n}$, given that country n buys a variety from country j, is calculated as $\int_0^\infty p^{1-\sigma_n}(dG_n(p)/dp)dp = (B_n Q_n)^{1-\sigma_n}$. Combining these results, the conditional expectation of the expenditure for variety i_j, given that country n buys it from country j, is $B_n^{\sigma_n - 1} Q_n^{\sigma_n} Y_n (B_n Q_n)^{1-\sigma_n} = Q_n Y_n \forall i_j \in I_{nj} \forall j$.

[12] We obtain Eq. (14) from Eqs. (2), (12), and the definition of κ_j. Eq. (15) is obtained by rewriting Eq. (10) using Eqs. (1), (4), (8), (9), (11), and (13).

where γ_j is the growth rate in country j. Eq. (14) states that κ_j evolves according to the difference between the growth rates in countries j and N. On the other hand, Eq. (15) can be interpreted as the capital market-clearing condition in country j relative to country N.[13] This corresponds to the labor market-clearing condition (21) of Eaton and Kortum (2002). Our model is different from Eaton and Kortum (2002) in that factor supplies and hence factor prices are endogenously changing over time. For all $t \in [0, \infty)$, with $\{\tau_{jn}\}_{j,n=1}^{N}$ exogenous and with $\{\kappa_{jt}\}_{j=1}^{N-1}$ predetermined, Eq. (15) determines $\{r_{jt}\}_{j=1}^{N-1}$, and then Eq. (14) determines $\{\dot{\kappa}_{jt}\}_{j=1}^{N-1}$.

Before proceeding, we calculate changes in Q_n, π_{nj}, and γ_j contained in our dynamic system. First, from Eq. (7), we obtain:

$$dQ_n/Q_n = \textstyle\sum_{j=1}^{N} \pi_{nj}(d\tau_{nj}/\tau_{nj} + dr_j/r_j). \tag{16}$$

Interestingly, the partial elasticity of Q_n with respect to $\tau_{nj} r_j$ is just π_{nj}. Second, substituting $\Phi_n = (Q_n/c_n)^{-\theta}$ from Eq. (7) into Eq. (6), and using Eq. (16), $d\pi_{nj}/\pi_{nj}$ is calculated as:

$$d\pi_{nj}/\pi_{nj} = -\theta \textstyle\sum_{k \neq j} \pi_{nk}(d\tau_{nj}/\tau_{nj} + dr_j/r_j - d\tau_{nk}/\tau_{nk} - dr_k/r_k). \tag{17}$$

The usual substitution effects work here: the higher $\tau_{nj} r_j$ is and/or the lower $\tau_{nk} r_k$ is for $k \neq j$, the less likely country n is to buy varieties from country j. Third, totally differentiating $\gamma_j = r_j/p_j^Y - \delta_j - \rho_j$ from Eq. (2), and using Eqs. (4) and (16), we obtain:

$$d\gamma_j = -\Gamma_j \textstyle\sum_{n \neq j} \pi_{jn}(d\tau_{jn}/\tau_{jn} + dr_n/r_n - dr_j/r_j); \Gamma_j \equiv \gamma_j + \delta_j + \rho_j. \tag{18}$$

This is totally consistent with the discussion right after Eq. (12), with n and j interchanged. Finally, letting $n = j$ in Eq. (17), replacing n with k in Eq. (18), and comparing the two equations, we reach the following lemma:

Lemma 2 *The growth rate in country j rises if and only if its fraction of domestic varieties decreases:*

$$d\gamma_j/(d\pi_{jj}/\pi_{jj}) = -\Gamma_j/\theta < 0.$$

Arkolakis et al. (2012) find that each country's welfare is always negatively related to its domestic expenditure share in a wide class of trade models including the static Eaton-Kortum and Melitz models. Lemma 2 provides a similar negative relationship between each country's growth rate and its domestic expenditure share: a country grows faster whenever it becomes more open (i.e., has a smaller fraction of domestic varieties), and vice versa.

2.6 Three-country model

In the rest of this paper, we focus on the case where $N = 3$. This is the minimum number of countries that allows us to consider regional trade agreements. For example, a preferential trade agreement between countries 1 and 2 can be expressed by reducing τ_{12} and τ_{21} but keeping τ_{13} and τ_{23} (as well as τ_{31} and τ_{32}) unchanged. For $N = 3$, Eqs. (14) and (15) are restated as follows:

[13] Eq. (15) is equivalent to $r_j K_j = \sum_{n=1}^{N} \pi_{nj} r_n K_n$. This equation implies two further results. First, it is rewritten as $\sum_{n \neq j} \pi_{jn} r_j K_j = \sum_{n \neq j} \pi_{nj} r_n K_n$. The latter equation shows country j's balanced trade condition (i.e., value of imports = value of exports). Second, summing up the former equation for $j = 1, ..., N-1$, we obtain the same equation for $j = N$. This means that country N's capital market-clearing condition is indeed redundant.

$$
\begin{aligned}
\dot{\kappa}_1 &= \kappa_1(\gamma_1(1, \tau_{12}r_2/r_1, \tau_{13}/r_1) - \gamma_3(\tau_{31}r_1, \tau_{32}r_2, 1)), && (19)\\
\dot{\kappa}_2 &= \kappa_2(\gamma_2(\tau_{21}r_1/r_2, 1, \tau_{23}/r_2) - \gamma_3(\tau_{31}r_1, \tau_{32}r_2, 1)), && (20)\\
\kappa_1 &= \pi_{11}(1, \tau_{12}r_2/r_1, \tau_{13}/r_1)\kappa_1 + \pi_{21}(\tau_{21}r_1/r_2, 1, \tau_{23}/r_2)\kappa_2/(r_1/r_2)\\
&\quad + \pi_{31}(\tau_{31}r_1, \tau_{32}r_2, 1)/r_1, && (21)\\
\kappa_2 &= \pi_{12}(1, \tau_{12}r_2/r_1, \tau_{13}/r_1)\kappa_1/(r_2/r_1) + \pi_{22}(\tau_{21}r_1/r_2, 1, \tau_{23}/r_2)\kappa_2\\
&\quad + \pi_{32}(\tau_{31}r_1, \tau_{32}r_2, 1)/r_2. && (22)
\end{aligned}
$$

We define a balanced growth path (BGP) as a path along which all variables grow at constant rates. From Eqs. (19) and (20), for both $\dot{\kappa}_1/\kappa_1$ and $\dot{\kappa}_2/\kappa_2$ to be constant, r_1 and r_2 should be constant. Then Eqs. (21) and (22) imply that κ_1 and κ_2 should be constant. With $\dot{\kappa}_1 = \dot{\kappa}_2 = 0$, a BGP is implicitly determined by:

$$
\begin{aligned}
0 &= \gamma_1(1, \tau_{12}r_2^*/r_1^*, \tau_{13}/r_1^*) - \gamma_3(\tau_{31}r_1^*, \tau_{32}r_2^*, 1), && (23)\\
0 &= \gamma_2(\tau_{21}r_1^*/r_2^*, 1, \tau_{23}/r_2^*) - \gamma_3(\tau_{31}r_1^*, \tau_{32}r_2^*, 1), && (24)\\
\kappa_1^* &= \pi_{11}(1, \tau_{12}r_2^*/r_1^*, \tau_{13}/r_1^*)\kappa_1^* + \pi_{21}(\tau_{21}r_1^*/r_2^*, 1, \tau_{23}/r_2^*)\kappa_2^*/(r_1^*/r_2^*)\\
&\quad + \pi_{31}(\tau_{31}r_1^*, \tau_{32}r_2^*, 1)/r_1^*, && (25)\\
\kappa_2^* &= \pi_{12}(1, \tau_{12}r_2^*/r_1^*, \tau_{13}/r_1^*)\kappa_1^*/(r_2^*/r_1^*) + \pi_{22}(\tau_{21}r_1^*/r_2^*, 1, \tau_{23}/r_2^*)\kappa_2^*\\
&\quad + \pi_{32}(\tau_{31}r_1^*, \tau_{32}r_2^*, 1)/r_2^*, && (26)
\end{aligned}
$$

where an asterisk over a variable represents a BGP. Eqs. (23) and (24) determine r_1^* and r_2^*, and then Eqs. (25) and (26) determine κ_1^* and κ_2^*. Let us call the common growth rate satisfying Eqs. (23) and (24) "the balanced growth rate". Note that the long-run rental rates are not determined by the capital market-clearing conditions, but by the balanced growth conditions. This will make the long-run effects of trade liberalization different from the short-run effects corresponding to the static Eaton-Kortum model.

The existence, uniqueness, and stability of a BGP is discussed in Appendix C. Simply stated, there exists a unique BGP that is locally stable unless the three countries are extremely different. Here we just assume that a unique and stable BGP exists.

3 Long-run effects of trade liberalization

3.1 Balanced growth rate

In this section, we examine the long-run impacts of trade liberalization analytically. Substituting Eq. (18) into the totally differentiated forms of Eqs. (23) and (24), we have:

$$
\begin{aligned}
a_{11}^* dr_1^*/r_1^* + a_{12}^* dr_2^*/r_2^* &= \Gamma_1^*(\pi_{12}^* d\tau_{12}/\tau_{12} + \pi_{13}^* d\tau_{13}/\tau_{13}) - \Gamma_3^*(\pi_{31}^* d\tau_{31}/\tau_{31} + \pi_{32}^* d\tau_{32}/\tau_{32}),\\
a_{21}^* dr_1^*/r_1^* + a_{22}^* dr_2^*/r_2^* &= \Gamma_2^*(\pi_{21}^* d\tau_{21}/\tau_{21} + \pi_{23}^* d\tau_{23}/\tau_{23}) - \Gamma_3^*(\pi_{31}^* d\tau_{31}/\tau_{31} + \pi_{32}^* d\tau_{32}/\tau_{32});
\end{aligned}
$$

$$
\begin{aligned}
a_{11}^* &\equiv \Gamma_1^*(\pi_{12}^* + \pi_{13}^*) + \Gamma_3^*\pi_{31}^* > 0,\\
a_{12}^* &\equiv -\Gamma_1^*\pi_{12}^* + \Gamma_3^*\pi_{32}^*,\\
a_{21}^* &\equiv -\Gamma_2^*\pi_{21}^* + \Gamma_3^*\pi_{31}^*,\\
a_{22}^* &\equiv \Gamma_2^*(\pi_{21}^* + \pi_{23}^*) + \Gamma_3^*\pi_{32}^* > 0,\\
a^* &\equiv a_{11}^*a_{22}^* - a_{12}^*a_{21}^*\\
&= \Gamma_1^*\Gamma_2^*[\pi_{12}^*\pi_{23}^* + \pi_{13}^*(\pi_{21}^* + \pi_{23}^*)] + \Gamma_2^*\Gamma_3^*[\pi_{23}^*\pi_{31}^* + \pi_{21}^*(\pi_{32}^* + \pi_{31}^*)]\\
&+ \Gamma_3^*\Gamma_1^*[\pi_{31}^*\pi_{12}^* + \pi_{32}^*(\pi_{13}^* + \pi_{12}^*)] > 0.
\end{aligned}
$$

The coefficients in the left-hand sides of these equations represent how $\gamma_1 - \gamma_3$ and $\gamma_2 - \gamma_3$ respond to the rental rates, respectively. For example, a_{11}^* is always positive because a rise in r_1 raises γ_1 but lowers γ_3. On the other hand, since a rise in r_2 lowers both γ_1 and γ_3, the sign of a_{12}^* is ambiguous, depending on the openness of countries 1 and 3 against country 2. Consider what happens when τ_{12} falls. Since this raises country 1's growth potential, r_1 tends to fall for $\gamma_1 - \gamma_3$ to go back to zero. Moreover, the fall in r_1 affects $\gamma_2 - \gamma_3$, which in turn causes r_2 to change. Solving the above equations for dr_1^*/r_1^* and dr_2^*/r_2^*, we obtain:

$$
\begin{aligned}
dr_1^*/r_1^* &= (1/a^*)[a_{22}^*\Gamma_1^*(\pi_{12}^*d\tau_{12}/\tau_{12} + \pi_{13}^*d\tau_{13}/\tau_{13})\\
&- a_{12}^*\Gamma_2^*(\pi_{21}^*d\tau_{21}/\tau_{21} + \pi_{23}^*d\tau_{23}/\tau_{23})\\
&- (a_{22}^* - a_{12}^*)\Gamma_3^*(\pi_{31}^*d\tau_{31}/\tau_{31} + \pi_{32}^*d\tau_{32}/\tau_{32})], \qquad (27)\\
dr_2^*/r_2^* &= (1/a^*)[-a_{21}^*\Gamma_1^*(\pi_{12}^*d\tau_{12}/\tau_{12} + \pi_{13}^*d\tau_{13}/\tau_{13})\\
&+ a_{11}^*\Gamma_2^*(\pi_{21}^*d\tau_{21}/\tau_{21} + \pi_{23}^*d\tau_{23}/\tau_{23})\\
&- (a_{11}^* - a_{21}^*)\Gamma_3^*(\pi_{31}^*d\tau_{31}/\tau_{31} + \pi_{32}^*d\tau_{32}/\tau_{32})]. \qquad (28)
\end{aligned}
$$

As expected, a fall in either τ_{12} or τ_{13} lowers r_1^*, whereas its effect on r_2^* is ambiguous. Similarly, a fall in either τ_{21} or τ_{23} lowers r_2^*, whereas its effect on r_1^* is ambiguous. Finally, since $a_{22}^* - a_{12}^* = \Gamma_2^*(\pi_{21}^* + \pi_{23}^*) + \Gamma_1^*\pi_{12}^* > 0$ and $a_{11}^* - a_{21}^* = \Gamma_1^*(\pi_{12}^* + \pi_{13}^*) + \Gamma_2^*\pi_{21}^* > 0$, a fall in either τ_{31} or τ_{32} raises both r_1^* and r_2^*.

Substituting Eqs. (27) and (28) back into Eq. (18) for country 3, the change in the balanced growth rate is generally expressed as:

$$
\begin{aligned}
d\gamma_1^* &= d\gamma_2^* = d\gamma_3^*\\
&= -(\Gamma_1^*\Gamma_2^*\Gamma_3^*/a^*)\\
&\times \{[\pi_{23}^*\pi_{31}^* + \pi_{21}^*(\pi_{32}^* + \pi_{31}^*)](\pi_{12}^*d\tau_{12}/\tau_{12} + \pi_{13}^*d\tau_{13}/\tau_{13})\\
&+ [\pi_{31}^*\pi_{12}^* + \pi_{32}^*(\pi_{13}^* + \pi_{12}^*)](\pi_{21}^*d\tau_{21}/\tau_{21} + \pi_{23}^*d\tau_{23}/\tau_{23})\\
&+ [\pi_{12}^*\pi_{23}^* + \pi_{13}^*(\pi_{21}^* + \pi_{23}^*)](\pi_{31}^*d\tau_{31}/\tau_{31} + \pi_{32}^*d\tau_{32}/\tau_{32})\}.
\end{aligned}
$$

Proposition 1 *For all $j, n = 1, 2, 3, n \neq j$, a permanent fall in τ_{jn} raises the balanced growth rate.*

This is a very powerful result: regardless of the signs and sizes of a_{12}^* and a_{21}^*, trade liberalization anywhere does raise the balanced growth rate. This is because trade liberalization in one country not only raises its own growth potential, but also raises the terms of trade of the other countries against the liberalizing country. For example, a fall in τ_{12} lowers r_1^*, which raises the terms of trade of country 3 against country 1. Also, since $(dr_1^*/r_1^* - dr_2^*/r_2^*)/(d\tau_{12}/\tau_{12}) = (1/a^*)(a_{22}^* + a_{21}^*)\Gamma_1^*\pi_{12}^* > 0$ from Eqs. (27) and (28), it raises the terms of trade of country 2 against country 1. These terms of trade improvements for the non-liberalizing countries contribute to the higher balanced growth rate. This proposition implies that the positive long-run growth effect of unilateral trade liberalization in Naito (2012) still holds in a three-country case, although we will see in section 3.3 that it might not be true for some country in the short run. On the other hand, our robust result is different from the three-country R&D-based endogenous growth model of Dinopoulos and Syropoulos (1997), where unilateral trade liberalization can either raise or lower the long-run growth rate of the liberalizing country depending on whether the growth intensity (summarizing the productivity in production and R&D and expenditure share) of its export sector is larger or smaller than its nontraded sector.

3.2 Fractions of domestic and traded varieties

3.2.1 Unilateral trade liberalization

Although Lemma 2 and Proposition 1 readily imply that the long-run fractions of domestic varieties all decrease with a fall in any trade cost, it is difficult to draw general conclusions about changes in the long-run fractions of traded varieties by simply substituting Eqs. (27) and (28) into Eq. (17), so we first consider a fall in one trade cost τ_{12} at a time. In view of Eq. (17), we need information about changes in $r_1^*, r_2^*, r_1^*/r_2^*, \tau_{12}r_2^*$, and $\tau_{12}r_2^*/r_1^*$. We already know from section 3.1 that a fall in τ_{12} lowers r_1^* and r_1^*/r_2^*, but its effect on r_2^* is ambiguous depending on the sign of a_{21}^*. For changes in $\tau_{12}r_2^*$ and $\tau_{12}r_2^*/r_1^*$, we obtain:

$$
\begin{aligned}
&(d\tau_{12}/\tau_{12} + dr_2^*/r_2^*)/(d\tau_{12}/\tau_{12}) \\
&= (1/a^*)\{\Gamma_1^*\Gamma_2^*(\pi_{12}^* + \pi_{13}^*)(\pi_{21}^* + \pi_{23}^*) + \Gamma_2^*\Gamma_3^*[\pi_{23}^*\pi_{31}^* + \pi_{21}^*(\pi_{32}^* + \pi_{31}^*)] \\
&+ \Gamma_3^*\Gamma_1^*\pi_{32}^*(\pi_{13}^* + \pi_{12}^*)\} \\
&> 0, \\
&(d\tau_{12}/\tau_{12} + dr_2^*/r_2^* - dr_1^*/r_1^*)/(d\tau_{12}/\tau_{12}) \\
&= (1/a^*)\{\Gamma_1^*\Gamma_2^*\pi_{13}^*(\pi_{21}^* + \pi_{23}^*) + \Gamma_2^*\Gamma_3^*[\pi_{23}^*\pi_{31}^* + \pi_{21}^*(\pi_{32}^* + \pi_{31}^*)] \\
&+ \Gamma_3^*\Gamma_1^*\pi_{32}^*\pi_{13}^*\} \\
&> 0.
\end{aligned}
$$

Hence, a fall in τ_{12} lowers $\tau_{12}r_2^*$ and $\tau_{12}r_2^*/r_1^*$ as well as r_1^* and r_1^*/r_2^*. Combining this with Eq. (17), we reach the following proposition:

Proposition 2 *A permanent fall in τ_{12} increases π_{12}^*, π_{21}^*, and π_{31}^*, whereas it decreases $\pi_{11}^*, \pi_{13}^*, \pi_{22}^*$, and π_{33}^*.*

Although one might be disappointed that we cannot find the directions of changes in all of the nine fractions of varieties, this proposition actually has rich implications. First, in country 1, some varieties bought from countries 1 and 3 are replaced by those from country 2. This is because the fall in τ_{12} makes country 2's varieties cheaper than the rest for the liberalizing country. Second, and more interestingly, country 1's fractions of exported varieties to all destinations increase. For either country 2 or 3, the falls in r_1^* and r_1^*/r_2^* caused by the fall in τ_{12} make it cheaper to import varieties from country 1 than the other sources. In this sense, import promotion also works as export promotion at the extensive margins.

3.2.2 Bilateral trade liberalization

We next see the case where τ_{12} and τ_{21} are reduced at the same time. It is still true from Lemma 2 and Proposition 1 that π_{11}^*, π_{22}^*, and π_{33}^* decrease. For traded varieties, we have to use Eq. (17), (27), and (28). Taking the difference between Eqs. (27) and (28), with $d\tau_{13} = d\tau_{23} = d\tau_{31} = d\tau_{32} = 0$, we have:

$$a^*(dr_1^*/r_1^* - dr_2^*/r_2^*) = (a_{22}^* + a_{21}^*)\Gamma_1^*\pi_{12}^* d\tau_{12}/\tau_{12} - (a_{11}^* + a_{12}^*)\Gamma_2^*\pi_{21}^* d\tau_{21}/\tau_{21},$$

where $a_{22}^* + a_{21}^* > 0$ and $a_{11}^* + a_{12}^* > 0$. We consider a particular type of bilateral trade liberalization such that r_1^*/r_2^* and hence the long-run bilateral terms of trade between countries 1 and 2 should be unchanged. Setting the left-hand side of the above equation equal to zero, we obtain:

$$(d\tau_{21}/\tau_{21})/(d\tau_{12}/\tau_{12})|_{dr_1^*/r_1^* = dr_2^*/r_2^*} = [(a_{22}^* + a_{21}^*)\Gamma_1^*\pi_{12}^*]/[(a_{11}^* + a_{12}^*)\Gamma_2^*\pi_{21}^*]. \tag{29}$$

Under the liberalization rule (29), the rate of change in r_1^* (and also r_2^*) is calculated as:

$$(dr_1^*/r_1^*)/(d\tau_{12}/\tau_{12})|_{dr_1^*/r_1^* = dr_2^*/r_2^*} = \Gamma_1^*\pi_{12}^*/(a_{11}^* + a_{12}^*) > 0.$$

This means that r_1^* and r_2^* always fall at the same rate. Then Eq. (17) implies the following proposition:

Proposition 3 *Permanent falls in τ_{12} and τ_{21}, with r_1^*/r_2^* unchanged, increase $\pi_{12}^*, \pi_{21}^*, \pi_{31}^*$, and π_{32}^*, whereas they decrease $\pi_{11}^*, \pi_{13}^*, \pi_{22}^*, \pi_{23}^*$, and π_{33}^*.*

This proposition is stronger than the previous one in that the directions of changes in all of the nine fractions of varieties are unambiguously determined under bilateral trade liberalization with the long-run bilateral terms of trade unchanged. In fact, Proposition 3 can be seen as just a composite of Proposition 2 applied to τ_{12} and τ_{21}: a preferential trade agreement between countries 1 and 2 shifts their import demands away from the outsider to each insider, and also increases their fractions of exported varieties to both inside and outside destinations. Moreover, the fractions of domestic varieties decrease in all of the three countries. This indicates that the preferential trade agreement brings about trade creation (i.e., π_{11}^* and π_{22}^* are replaced by π_{12}^* and π_{21}^*, respectively) as well as trade diversion (i.e., π_{13}^* and π_{23}^* are replaced by π_{12}^* and π_{21}^*, respectively). Finally, the fact that country 3's fractions of exported varieties decrease does not mean that the non-member country loses from the preferential trade agreement. It actually enjoys the higher balanced growth rate along with the improved terms of trade against both countries 1 and 2.

3.3 Comparison with short-run effects

To compare the long-run effects of unilateral trade liberalization with its short-run effects, we investigate analytically the impacts of a change in τ_{12} on main endogenous variables in the initial period in Appendix A. A fall in τ_{12} makes country 1 substitute country 2's varieties for its own, thereby raising r_2. Although it is unclear if r_1 falls or not, $r_1/r_2, \tau_{12}r_2$, and $\tau_{12}r_2/r_1$ necessarily fall. Since country 2's terms of trade against the other two countries improve, its growth rate γ_2 necessarily rises. The directions of changes in γ_1 and γ_3

are generally ambiguous due to the ambiguous change in r_1. Finally, although π_{12} rises whereas π_{22} and π_{32} fall, the directions of changes in the other six fractions of varieties remain ambiguous.

To find clearer results analytically, we focus on a special case where the old BGP is symmetric across countries. Fortunately, the short- and long-run effects of unilateral and bilateral trade liberalization on the growth rates and fractions of varieties are all identified in Appendix B and summarized in Table 1.

In the short run, unilateral trade liberalization lowers r_1. Although this tends to lower γ_1 but raise γ_3, these effects are outweighed by the fall in $\tau_{12}r_2/r_1$ and the rise in r_2, thereby raising γ_1 but lowering γ_3, respectively. Due to the fall in $\tau_{12}r_2/r_1$, country 1 buys more of country 2's varieties but less of the others. On the other hand, the price changes induce both countries 2 and 3 to buy less of country 2's varieties but more of the others. Since countries 1 and 2 start to grow faster than country 3, both κ_1 and κ_2 increase, which in turn lowers both r_1 and r_2. In the long run, r_2 goes back to its old BGP value. The balanced growth rate necessarily rises. The directions of changes in the fractions of varieties in the long run are the same as in the short run, except π_{23} and π_{33}: they decrease from their old BGP values because country 3's cost advantage over country 2 disappears in the long run.

Unlike unilateral trade liberalization, bilateral trade liberalization with the long-run bilateral terms of trade unchanged raises not only r_2 but also r_1 in the short run. This is because, for both countries 1 and 2, the decreased demands for their domestic varieties are more than compensated by the increased demands for each other's varieties. Since country 3's terms of trade against all other countries deteriorate, country 3 starts to grow more slowly, whereas countries 1 and 2 start to grow faster, than the old BGP just like unilateral trade liberalization. Due to the strong direct effects of the falls in τ_{12} and τ_{21}, both trade creation and diversion occur even in the short run. On the other hand, reflecting the price changes, country 3 imports less from both countries 1 and 2. The gaps in the short-run growth rates pull both r_1 and r_2 down below their old BGP values, which in turn raises the balanced growth rate. Since country 3's varieties become relatively more expensive, country 3 imports more from both countries 1 and 2 in the long run.

There are many variables whose long-run responses are opposite to their short-run responses representing the static model of Eaton and Kortum (2002): γ_3, π_{23}, and π_{33} for unilateral trade liberalization; or $r_1, r_2, \gamma_3, \pi_{31}, \pi_{32}$, and π_{33} for bilateral trade liberalization. The most remarkable result is stated in the following proposition:

Proposition 4 *Starting from the symmetric BGP, both trade liberalization schemes specified in Propositions 2 and 3 raise γ_1 and γ_2 but lower γ_3 in the initial period.*

As long as we are around the symmetric BGP, trade liberalization, whether unilateral or bilateral, must lower the growth rate in a non-liberalizing country (i.e., country 3) in the short run. In the two-country model of Naito (2012), a fall in τ_{12} improves the terms of trade of country 2 against country 1, which in turn raises the growth rate in country 2 in the initial period. It is true that a fall in τ_{12} still improves country 2's terms of trade against all other countries even in our three-country model, thereby pushing up country 2's growth rate in the short run. However, this fact implies that country 3's terms of trade against country 2 must deteriorate, which pulls down country 3's growth rate in the short run. It is the presence of a third country that is responsible for the new result.

Propositions 1 and 4 have welfare implications that differentiate our model from Eaton and Kortum (2002). If $\dot{K}_j + \delta_j K_j = 0$ in Eq. (1), then our model reduces to the static Eaton-Kortum model, where country j's welfare is given by $\ln C_j = \ln((r_j/p_j^Y)K_j)$, and the equilibrium rental rates are determined by Eqs. (21) and (22). The fact that the initial growth rate in country 3 falls from its old BGP value in our model means that r_3/p_3^Y and hence country 3's welfare must fall in the static model. In contrast, the liberalization-induced rise in the balanced growth rate in our model can raise country 3's overall utility, reversing the pessimistic view from the static Eaton-Kortum model.

Our Eaton-Kortum model of trade and growth adds great value to Eaton and Kortum's own dynamic extensions of their static model. Eaton and Kortum (1999) provide a multi-country R&D-based growth model with one tradable homogeneous final good and a continuum of nontradable differentiated intermediate goods, where the highest-quality idea among domestically created and internationally diffused ideas is adopted in each intermediate good sector of each country. Although the paper studies the relationship between the degree of diffusion and productivity growth, it does not consider the growth effect of trade.[14] Eaton and Kortum (2001a) combine the static trade model of Eaton and Kortum (2002) with the R&D-based growth model of Eaton and Kortum (1999) in the absence of international technology diffusion. Assuming an exogenous growth rate of population which is common to all countries, they show that the long-run growth rate of technology in each country is equal to the population growth rate and does not depend on trade costs due to their semi-endogenous growth specification.[15] Nor do they examine how countries' growth rates evolve depending on the trade costs during the transition. Eaton and Kortum (2001b) embed the Eaton and Kortum (2002) structure in a two-sector neoclassical growth model with a common exogenous growth rate of technology to explain some stylized facts concerning trade in capital goods. While focusing on the BGP with exogenous growth, they do not analyze the long- or short-run growth effect of trade. Unlike those papers, our endogenous growth model with tractable transitional dynamics reveals that any trade liberalization scheme does affect the growth rates of all countries not only in the short run but also in the long run.

4 Concluding remarks

Our three-country, continuum-good Ricardian model of trade and growth with endogenous extensive margins has some policy implications. First, trade liberalization, be it unilateral, bilateral, or multilateral, raises global growth. This is because it raises the growth potential of the liberalizing countries, which in turn raises the relative rental rates and hence the terms of trade of the non-liberalizing countries against the liberalizing ones. This supports the recent empirical research such as Wacziarg and Welch (2008) and Estevadeordal and Taylor (2013) reporting the positive relationship between trade liberalization and economic growth. Second, import promotion acts as export promotion at the extensive margins. The falling long-run rental rates of the liberalizing countries relative to the non-liberalizing ones arising from their faster growth make it cheaper to buy varieties from the liberalizing countries. This explains the mechanism underlying the empirical evidence on economic growth and extensive margins of exports found by Hummels and Klenow (2005) and Kehoe and Ruhl (2013). Third, the difference between the short- and long-run effects of trade liberalization opens up the possibility that its welfare effects in the static Eaton-Kortum model can be reversed in a positive direction. This contributes to the literature on regional trade agreements: unlike Bagwell and Staiger (1999) and Ornelas (2005), where a free trade agreement between two countries benefits a third country only if each member country voluntarily lowers its optimal external tariff, welfare of the third country can rise without adjustments in their external trade costs in our model.

[14] In fact, due to Walras' law, the final good becomes nontraded even if tradable because all other goods and factors are nontradable.

[15] Jones (1995) assumes that technology A grows according to $\dot{A} = \delta A^{\phi} L_A$, where δ is the productivity in R&D; ϕ is the degree of knowledge spillovers in R&D; and L_A is the size of R&D workers. When $\phi < 1$, we have $\dot{A}/A = (\dot{L}_A/L_A)/(1-\phi)$ at the BGP, meaning that the long-run growth rate of technology is proportional to the growth rate of population. This case is called semi-endogenous growth. Eaton and Kortum (2001a) consider a special case where $\phi = 0$.

There are some directions for future research. First, replacing iceberg trade costs with import tariffs will somewhat complicate the effect of trade liberalization on welfare of the liberalizing countries. By the standard optimal tariff argument, a fall in a country's import tariff will partly lower its welfare in the short run through decreased tariff revenue associated with deteriorated terms of trade. However, since the tariff reduction raises the balanced growth rate just like a fall in the corresponding iceberg trade cost, the liberalizing country's welfare partly rises in the long run. As long as countries' subjective discount rates are sufficiently low that the long-run welfare gains from faster growth outweigh the possible short-run welfare losses from decreased tariff revenue, not only the non-liberalizing but also the liberalizing countries will still gain from tariff reductions. Second, it will be interesting to perform counterfactual experiments based on structurally estimated parameters. Since our model works for an arbitrary number of countries, we could do this in a world with much more than three countries as Eaton and Kortum (2002) do. Even then, the qualitative results obtained in our dynamic three-country model provide a benchmark against which quantitative results will be evaluated.

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Appendix A. Short-run effects of a change in τ_{12}

Substituting Eq. (17) into the logarithmically differentiated forms of Eqs. (21) and (22) with $d\kappa_j/\kappa_j = 0 \forall j$, we obtain:

$$
\begin{aligned}
c_{11} dr_1/r_1 + c_{12} dr_2/r_2 = & -\theta\pi_{11} r_1 \kappa_1 \pi_{12} d\tau_{12}/\tau_{12} - \theta\pi_{11} r_1 \kappa_1 \pi_{13} d\tau_{13}/\tau_{13} \\
& + \theta\pi_{21} r_2 \kappa_2 (1-\pi_{21}) d\tau_{21}/\tau_{21} - \theta\pi_{21} r_2 \kappa_2 \pi_{23} d\tau_{23}/\tau_{23} \\
& + \theta\pi_{31}(1-\pi_{31}) d\tau_{31}/\tau_{31} - \theta\pi_{31}\pi_{32} d\tau_{32}/\tau_{32}, \\
c_{21} dr_1/r_1 + c_{22} dr_2/r_2 = & \, \theta\pi_{12} r_1 \kappa_1 (1-\pi_{12}) d\tau_{12}/\tau_{12} - \theta\pi_{12} r_1 \kappa_1 \pi_{13} d\tau_{13}/\tau_{13} \\
& - \theta\pi_{22} r_2 \kappa_2 \pi_{21} d\tau_{21}/\tau_{21} - \theta\pi_{22} r_2 \kappa_2 \pi_{23} d\tau_{23}/\tau_{23} \\
& - \theta\pi_{32}\pi_{31} d\tau_{31}/\tau_{31} + \theta\pi_{32}(1-\pi_{32}) d\tau_{32}/\tau_{32};
\end{aligned}
$$

$$
\begin{aligned}
c_{11} &\equiv -\{\theta[\pi_{11}r_1\kappa_1(1-\pi_{11})+\pi_{21}r_2\kappa_2(1-\pi_{21})+\pi_{31}(1-\pi_{31})]+(1-\pi_{11})r_1\kappa_1\}<0,\\
c_{12} &\equiv \theta(\pi_{11}r_1\kappa_1\pi_{12}+\pi_{21}r_2\kappa_2\pi_{22}+\pi_{31}\pi_{32})+\pi_{21}r_2\kappa_2>0,\\
c_{21} &\equiv \theta(\pi_{22}r_2\kappa_2\pi_{21}+\pi_{12}r_1\kappa_1\pi_{11}+\pi_{32}\pi_{31})+\pi_{12}r_1\kappa_1>0,\\
c_{22} &\equiv -\{\theta[\pi_{22}r_2\kappa_2(1-\pi_{22})+\pi_{12}r_1\kappa_1(1-\pi_{12})+\pi_{32}(1-\pi_{32})]+(1-\pi_{22})r_2\kappa_2\}<0,\\
c &\equiv c_{11}c_{22}-c_{12}c_{21}>0.
\end{aligned}
$$

Focusing on a change in τ_{12}, its short-run effects on r_1 and r_2 are given by:

$$
\begin{aligned}
&(dr_1/r_1)/(d\tau_{12}/\tau_{12})\\
&=(\theta\pi_{12}r_1\kappa_1/c)\\
&\times\{\theta[\pi_{22}r_2\kappa_2(\pi_{11}\pi_{23}-\pi_{21}\pi_{13})+\pi_{32}(\pi_{11}\pi_{33}-\pi_{31}\pi_{13})]+r_2\kappa_2(\pi_{11}\pi_{23}-\pi_{21}\pi_{13})\},\\
&(dr_2/r_2)/(d\tau_{12}/\tau_{12})\\
&=(\theta\pi_{12}r_1\kappa_1/c)\{-[\theta(\pi_{11}r_1\kappa_1\pi_{13}+\pi_{21}r_2\kappa_2\pi_{23}+\pi_{31}\pi_{33})+\pi_{13}r_1\kappa_1]\pi_{11}+c_{11}\pi_{13}\}<0.
\end{aligned}
$$

The rates of changes in $r_1/r_2, \tau_{12}r_2$, and $\tau_{12}r_2/r_1$ are then calculated as:

$$
\begin{aligned}
&(dr_1/r_1-dr_2/r_2)/(d\tau_{12}/\tau_{12})\\
&=(\theta\pi_{12}r_1\kappa_1/c)\{\pi_{11}[\theta(\pi_{22}r_2\kappa_2\pi_{23}+\pi_{12}r_1\kappa_1\pi_{13}+\pi_{32}\pi_{33})+\pi_{32}]\\
&+(1-\pi_{12})[\theta(\pi_{11}r_1\kappa_1\pi_{13}+\pi_{21}r_2\kappa_2\pi_{23}+\pi_{31}\pi_{33})+\pi_{31}]\}\\
&>0,\\
&(d\tau_{12}/\tau_{12}+dr_2/r_2)/(d\tau_{12}/\tau_{12})\\
&=(1/c)\{[\theta(\pi_{11}r_1\kappa_1\pi_{13}+\pi_{21}r_2\kappa_2\pi_{23}+\pi_{31}\pi_{33})+\pi_{13}r_1\kappa_1]\\
&\times[\theta(\pi_{22}r_2\kappa_2\pi_{21}+\pi_{32}\pi_{31})+\pi_{21}r_2\kappa_2]-c_{11}[\theta(\pi_{22}r_2\kappa_2\pi_{23}+\pi_{32}\pi_{33})+\pi_{23}r_2\kappa_2]\}\\
&>0,\\
&(d\tau_{12}/\tau_{12}+dr_2/r_2-dr_1/r_1)/(d\tau_{12}/\tau_{12})\\
&=(1/c)\{[\theta(\pi_{22}r_2\kappa_2\pi_{23}+\pi_{12}r_1\kappa_1\pi_{13}+\pi_{32}\pi_{33})+\pi_{32}][\theta(\pi_{21}r_2\kappa_2\pi_{22}+\pi_{31}\pi_{32})+\pi_{21}r_2\kappa_2]\\
&+[\theta(\pi_{11}r_1\kappa_1\pi_{13}+\pi_{21}r_2\kappa_2\pi_{23}+\pi_{31}\pi_{33})+\pi_{31}]\\
&\times\{\theta[\pi_{22}r_2\kappa_2(1-\pi_{22})+\pi_{32}(1-\pi_{32})]+(1-\pi_{22})r_2\kappa_2\}\}\\
&>0.
\end{aligned}
$$

From these results and Eq. (18), the short-run effects of a change in τ_{12} on the growth rates are obtained as:

$$
\begin{aligned}
& d\gamma_1/(d\tau_{12}/\tau_{12}) \\
& = -\Gamma_1\{\pi_{12}(d\tau_{12}/\tau_{12} + dr_2/r_2 - dr_1/r_1)/(d\tau_{12}/\tau_{12}) + \pi_{13}[-(dr_1/r_1)/(d\tau_{12}/\tau_{12})]\}, \\
& d\gamma_2/(d\tau_{12}/\tau_{12}) \\
& = -\Gamma_2\{\pi_{21}(dr_1/r_1 - dr_2/r_2)/(d\tau_{12}/\tau_{12}) + \pi_{23}[-(dr_2/r_2)/(d\tau_{12}/\tau_{12})]\} < 0, \\
& d\gamma_3/(d\tau_{12}/\tau_{12}) \\
& = -\Gamma_3[\pi_{31}(dr_1/r_1)/(d\tau_{12}/\tau_{12}) + \pi_{32}(dr_2/r_2)/(d\tau_{12}/\tau_{12})].
\end{aligned}
$$

Finally, in view of Lemma 2 and Eq. (17), only three out of nine fractions of varieties have definite signs: $(d\pi_{12}/\pi_{12})/(d\tau_{12}/\tau_{12}) < 0, (d\pi_{22}/\pi_{22})/(d\tau_{12}/\tau_{12}) > 0, (d\pi_{32}/\pi_{32})/(d\tau_{12}/\tau_{12}) > 0$.

Appendix B. Comparative dynamics around the symmetric BGP

Suppose that all parameters are symmetric across countries: $\rho_j = \rho, \delta_j = \delta, B_j = B, \sigma_j = \sigma, b_j = b \forall j, \tau_{nj} = \tau > 1 \forall n, j, j \neq n$. Then we can easily see from Eq. (7) that $r_1^* = r_2^* = 1$ solve Eqs. (23) and (24). Since the balanced growth rates as well as the depreciation and subjective discount rates are equal across countries, we have $\Gamma_j^* = \Gamma^* \forall j$. Moreover, Eq. (6) implies that:

$$
\begin{aligned}
& \pi_{nj}^* = \tau^{-\theta}/(1 + 2\tau^{-\theta}) \equiv \pi^* < 1/3 \forall n, j, j \neq n, \\
& \pi_{nn}^* = 1 - 2\pi^* > 1/3 \forall n.
\end{aligned}
$$

Finally, Eqs. (25) and (26) are solved for $\kappa_1^* = \kappa_2^* = 1$. The following comparative dynamics are evaluated at this symmetric BGP.

Unilateral trade liberalization

We first examine the directions of changes in the long run fractions of varieties which are not covered in Proposition 2. From Eqs. (27) and (28), we have:

$$
\begin{aligned}
a^* = a_{11}^* a_{22}^* - a_{12}^* a_{21}^* &= (3\Gamma^* \pi^*)^2 - 0^2 = 9\Gamma^{*2}\pi^{*2}, \\
(dr_1^*/r_1^*)/(d\tau_{12}/\tau_{12}) &= (1/a^*)(a_{22}^* \Gamma_1^* \pi_{12}^*) = 3\Gamma^{*2}\pi^{*2}/a^* = 1/3, \\
(dr_2^*/r_2^*)/(d\tau_{12}/\tau_{12}) &= (1/a^*)(-a_{21}^* \Gamma_1^* \pi_{12}^*) = 0.
\end{aligned}
$$

Then Eq. (17) implies that $(d\pi_{23}^*/\pi_{23}^*)/(d\tau_{12}/\tau_{12}) > 0$ and $(d\pi_{32}^*/\pi_{32}^*)/(d\tau_{12}/\tau_{12}) > 0$.

In the short run, the coefficients $c_{11}, c_{12}, c_{21}, c_{22}$, and $c = c_{11}c_{22} - c_{12}c_{21}$ at the symmetric BGP are calculated as:

$$
\begin{aligned}
c_{11}^* &= c_{22}^* = -\{\theta[(1-2\pi^*)2\pi^* + \pi^*(1-\pi^*) + \pi^*(1-\pi^*)] + 2\pi^*\} = -2\pi^*[\theta(2-3\pi^*)+1],\\
c_{12}^* &= c_{21}^* = \theta[(1-2\pi^*)\pi^* + \pi^*(1-2\pi^*) + \pi^{*2}] + \pi^* = \pi^*[\theta(2-3\pi^*)+1],\\
c^* &= c_{11}^*c_{22}^* - c_{12}^*c_{21}^* = (-2c_{12}^*)^2 - c_{12}^{*2} = 3c_{12}^{*2}.
\end{aligned}
$$

Then $(dr_1/r_1)/(d\tau_{12}/\tau_{12})$ and $(dr_2/r_2)/(d\tau_{12}/\tau_{12})$ are respectively given by:

$$
\begin{aligned}
(dr_1/r_1)/(d\tau_{12}/\tau_{12}) &= (\theta\pi^*c_{12}^*/c^*)(1-3\pi^*) > 0,\\
(dr_2/r_2)/(d\tau_{12}/\tau_{12}) &= -\theta\pi^*c_{12}^*/c^*.
\end{aligned}
$$

From Appendix A, the signs of $d\gamma_1/(d\tau_{12}/\tau_{12})$ and $d\gamma_3/(d\tau_{12}/\tau_{12})$ at the symmetric BGP are entirely determined by the signs of $(d\tau_{12}/\tau_{12}+dr_2/r_2-dr_1/r_1)/(d\tau_{12}/\tau_{12})-(dr_1/r_1)/(d\tau_{12}/\tau_{12})$ and $(dr_1/r_1)/(d\tau_{12}/\tau_{12})+(dr_2/r_2)/(d\tau_{12}/\tau_{12})$, respectively. Since they are calculated as $(d\tau_{12}/\tau_{12} + dr_2/r_2 - dr_1/r_1)/(d\tau_{12}/\tau_{12}) - (dr_1/r_1)/(d\tau_{12}/\tau_{12}) = (3c_{12}^*\pi^*/c^*)[\theta(1-\pi^*)+1] > 0$ and $(dr_1/r_1)/(d\tau_{12}/\tau_{12}) + (dr_2/r_2)/(d\tau_{12}/\tau_{12}) = -3\theta\pi^{*2}c_{12}^*/c^* < 0$, we have $d\gamma_1/(d\tau_{12}/\tau_{12}) < 0$ and $d\gamma_3/(d\tau_{12}/\tau_{12}) > 0$. The last inequality means that, around the symmetric BGP, a fall in τ_{12} indeed lowers γ_3 in the short run.

For the fractions of varieties, we immediately know that $(d\pi_{11}/\pi_{11})/(d\tau_{12}/\tau_{12}) > 0$ and $(d\pi_{33}/\pi_{33})/(d\tau_{12}/\tau_{12}) < 0$ from Lemma 2, and that $(d\pi_{13}/\pi_{13})/(d\tau_{12}/\tau_{12}) > 0, (d\pi_{21}/\pi_{21})/(d\tau_{12}/\tau_{12}) < 0$, and $(d\pi_{31}/\pi_{31})/(d\tau_{12}/\tau_{12}) < 0$ from Eq. (17). Moreover, we obtain $(d\pi_{23}/\pi_{23})/(d\tau_{12}/\tau_{12}) < 0$ from $\pi^*(dr_1/r_1)/(d\tau_{12}/\tau_{12}) + (1-2\pi^*)(dr_2/r_2)/(d\tau_{12}/\tau_{12}) = (\theta\pi^*c_{12}^*/c^*)[3\pi^*(1-\pi^*)-1] < 0$.

Bilateral trade liberalization

For bilateral trade liberalization, we only have to examine its short-run effects around the symmetric BGP. First of all, Eq. (29) reduces to $(d\tau_{21}/\tau_{21})/(d\tau_{12}/\tau_{12})|_{dr_1^*/r_1^*=dr_2^*/r_2^*} = [(3\Gamma^*\pi^*+0)\Gamma^*\pi^*]/[(3\Gamma^*\pi^*+0)\Gamma^*\pi^*] = 1$, meaning that τ_{21} is reduced by the same rate as τ_{12} in order to keep r_1^*/r_2^* constant. Then, noting that $-\theta\pi_{11}^*r_1^*\kappa_1^*\pi_{12}^*d\tau_{12}/\tau_{12} + \theta\pi_{21}^*r_2^*\kappa_2^*(1-\pi_{21}^*)d\tau_{21}/\tau_{21} = [-\theta(1-2\pi^*)\pi^* + \theta\pi^*(1-\pi^*)]d\tau_{12}/\tau_{12} = \theta\pi^{*2}d\tau_{12}/\tau_{12}$ and $\theta\pi_{12}^*r_1^*\kappa_1^*(1-\pi_{12}^*)d\tau_{12}/\tau_{12} - \theta\pi_{22}^*r_2^*\kappa_2^*\pi_{21}^*d\tau_{21}/\tau_{21} = \theta\pi^{*2}d\tau_{12}/\tau_{12}$, the rates of changes in r_1 and r_2 are respectively given by:

$$
\begin{aligned}
(dr_1/r_1)/(d\tau_{12}/\tau_{12})|_{dr_1^*/r_1^*=dr_2^*/r_2^*} &= -3\theta\pi^{*2}c_{12}^*/c^* < 0,\\
(dr_2/r_2)/(d\tau_{12}/\tau_{12})|_{dr_1^*/r_1^*=dr_2^*/r_2^*} &= -3\theta\pi^{*2}c_{12}^*/c^* = (dr_1/r_1)/(d\tau_{12}/\tau_{12})|_{dr_1^*/r_1^*=dr_2^*/r_2^*} < 0.
\end{aligned}
$$

In contrast to its long-run effects, bilateral trade liberalization raises both r_1 and r_2, with r_1/r_2 unchanged even in the short run. Therefore, $\gamma_1(1, \tau_{12}r_2/r_1, \tau_{13}/r_1)$ and $\gamma_2(\tau_{21}r_1/r_2, 1, \tau_{23}/r_2)$ rise whereas $\gamma_3(\tau_{31}r_1, \tau_{32}r_2, 1)$ falls in the short run.

For the fractions of varieties, we immediately know that $(d\pi_{11}/\pi_{11})/(d\tau_{12}/\tau_{12})| > 0, (d\pi_{22}/\pi_{22})/(d\tau_{12}/\tau_{12})| > 0$, and $(d\pi_{33}/\pi_{33})/(d\tau_{12}/\tau_{12})| < 0$ from Lemma 2, and that

$(d\pi_{31}/\pi_{31})/(d\tau_{12}/\tau_{12})| > 0$ and $(d\pi_{32}/\pi_{32})/(d\tau_{12}/\tau_{12})| > 0$ from Eq. (17), where the condition that $dr_1^*/r_1^* = dr_2^*/r_2^*$ after each vertical line is omitted to save space. Moreover, since $(d\tau_{12}/\tau_{12}+dr_2/r_2)/(d\tau_{12}/\tau_{12})| = 1+(dr_2/r_2)/(d\tau_{12}/\tau_{12})| = (3c_{12}^*\pi^*/c^*)[2\theta(1-2\pi^*)+1] > 0$ and $(d\tau_{21}/\tau_{21}+dr_1/r_1)/(d\tau_{12}/\tau_{12})| = (d\tau_{12}/\tau_{12}+$

$dr_2/r_2)/(d\tau_{12}/\tau_{12})| > 0$, Eq. (17) implies that $(d\pi_{12}/\pi_{12})/(d\tau_{12}/\tau_{12})| < 0$ and $(d\pi_{21}/\pi_{21})/(d\tau_{12}/\tau_{12})| < 0$. Finally, we obtain $(d\pi_{13}/\pi_{13})/(d\tau_{12}/\tau_{12})| > 0$ and $(d\pi_{23}/\pi_{23})/(d\tau_{12}/\tau_{12})| > 0$ from $(1-2\pi^*)(dr_1/r_1)/(d\tau_{12}/\tau_{12})| + \pi^*(d\tau_{12}/\tau_{12} + dr_2/r_2)/(d\tau_{12}/\tau_{12})| = (3\pi^{*2}c^*_{12}/c^*)[\theta(1-2\pi^*)+1] > 0$ and $\pi^*(d\tau_{21}/\tau_{21} + dr_1/r_1)/(d\tau_{12}/\tau_{12})| + (1-2\pi^*)(dr_2/r_2)/(d\tau_{12}/\tau_{12})| = (1-2\pi^*)(dr_1/r_1)/(d\tau_{12}/\tau_{12})| + \pi^*(d\tau_{12}/\tau_{12} + dr_2/r_2)/(d\tau_{12}/\tau_{12})| > 0$.

Appendix C. The existence, uniqueness, and stability of a BGP

Consider a three-country dynamic system consisting of Eqs. (19)-(22). Its BGP is defined by Eqs. (23)-(26).

We first show that there exists a BGP, where $r^*_j \in (0,\infty)\forall j = 1, 2$. (If it is true, then Eqs. (25) and (26) are uniquely solved for positive and finite κ^*_1 and κ^*_2.) In view of Eq. (7), as r_1 approaches zero with $r_2 \in (0,\infty)$ given, $Q_2(\tau_{21}r_1/r_2, 1, \tau_{23}/r_2)$ and $Q_3(\tau_{31}r_1, \tau_{32}r_2, 1)$ approach zero whereas $Q_1(1, \tau_{12}r_2/r_1, \tau_{13}/r_1)$ remains positive. This implies that $\lim_{r_1\to 0}(\gamma_1(1, \tau_{12}r_2/r_1, \tau_{13}/r_1) - \gamma_3(\tau_{31}r_1, \tau_{32}r_2, 1)) = -\infty < 0$. On the other hand, since $Q_1(1, \tau_{12}r_2/r_1, \tau_{13}/r_1)$ approaches zero whereas $Q_2(\tau_{21}r_1/r_2, 1, \tau_{23}/r_2)$ and $Q_3(\tau_{31}r_1, \tau_{32}r_2, 1)$ are positive as r_1 approaches infinity, we have $\lim_{r_1\to\infty}(\gamma_1(1, \tau_{12}r_2/r_1, \tau_{13}/r_1) - \gamma_3(\tau_{31}r_1, \tau_{32}r_2, 1)) = \infty > 0$. Thus, from the intermediate value theorem, there exists $r_1 = R_1(r_2) \in (0,\infty)$ such that $\gamma_1(1, \tau_{12}r_2/r_1, \tau_{13}/r_1) - \gamma_3(\tau_{31}r_1, \tau_{32}r_2, 1) = 0$. Similarly, with $r_1 \in (0,\infty)$ given, there exists $r_2 = R_2(r_1) \in (0,\infty)$ such that $\gamma_2(\tau_{21}r_1/r_2, 1, \tau_{23}/r_2) - \gamma_3(\tau_{31}r_1, \tau_{32}r_2, 1) = 0$. Solving $r_1 = R_1(r_2)$ and $r_2 = R_2(r_1)$, we obtain r^*_1 and r^*_2, which are positive and finite.

Fig. 1 illustrates the determination of r^*_1 and r^*_2 in a particular case. In the (r_1, r_2)-plane, both curves $r_1 = R_1(r_2)$ and $r_2 = R_2(r_1)$ are positively sloped, and the former is steeper than the latter. These graphs intersect only once at point A: (r^*_1, r^*_2), which gives a unique BGP. To understand when this is the case, we turn to mathematics. Substituting Eq. (18) with $d\tau_{jn}/\tau_{jn} = 0\forall j, n$ into the totally differentiated forms of Eqs. (23) and (24), we obtain:

$$0 = d\gamma^*_1 - d\gamma^*_3 = a^*_{11}dr^*_1/r^*_1 + a^*_{12}dr^*_2/r^*_2, \tag{C.1}$$

$$0 = d\gamma^*_2 - d\gamma^*_3 = a^*_{21}dr^*_1/r^*_1 + a^*_{22}dr^*_2/r^*_2, \tag{C.2}$$

where $a^*_{11}, a^*_{12}, a^*_{21}$, and a^*_{22} are defined in section 3.1. Since a rise in r_1 raises γ_1 but lowers γ_3, a^*_{11} is always positive. This ensures that $R_1(\cdot)$ is single-valued, and that $\gamma_1 - \gamma_3 > 0$ if and only if $r_1 > R_1(r_2)$. On the other hand, a^*_{12} is negative if and only if a rise in r_2 lowers γ_1 by more than the fall in γ_3. This is likely to occur when π^*_{12} is larger than π^*_{32}, that is, country 1 is more open to country 2 than country 3. By the same reasoning, a^*_{22} is always positive, but a^*_{21} can either be positive or negative. Eqs. (C.1) and (C.2) are rewritten as, respectively:

$$dr^*_2/dr^*_1|_{r_1=R_1(r_2)} = -(r^*_2/r^*_1)a^*_{11}/a^*_{12},$$

$$dr^*_2/dr^*_1|_{r_2=R_2(r_1)} = -(r^*_2/r^*_1)a^*_{21}/a^*_{22}.$$

These expressions give us some information about the shapes of curves $r_1 = R_1(r_2)$ and $r_2 = R_2(r_1)$. First, curve $r_1 = R_1(r_2)$ is positively sloped if and only if a^*_{12} is negative. Similarly, curve $r_2 = R_2(r_1)$ is positively sloped if and only if a^*_{21} is negative. Fig. 1 corresponds to the case where both a^*_{12} and a^*_{21} are negative. In this case, a rise in r_2 from a point on curve $r_1 = R_1(r_2)$ pulls $\gamma_1 - \gamma_3$ down from zero. Then

r_1 has to rise so that $\gamma_1 - \gamma_3$ should go back to zero. Second, curve $r_1 = R_1(r_2)$ becomes more vertical, the smaller is a_{12}^* in absolute value. Similarly, curve $r_2 = R_2(r_1)$ becomes more horizontal, the smaller is a_{21}^* in absolute value. This implies that, the more similar the three countries are, the steeper curve $r_1 = R_1(r_2)$ is whereas the flatter curve $r_2 = R_2(r_1)$ is, and the more likely a BGP is to be unique. Third, taking the difference between the slopes at an intersection, we have:

$$dr_2^*/dr_1^*|_{r_1=R_1(r_2)} - dr_2^*/dr_1^*|_{r_2=R_2(r_1)} = -(r_2^*/r_1^*)a^*/(a_{12}^*a_{22}^*),$$

where $a^* \equiv a_{11}^*a_{22}^* - a_{12}^*a_{21}^* > 0$. This means that curve $r_1 = R_1(r_2)$ crosses curve $r_2 = R_2(r_1)$ from below (i.e., $dr_2^*/dr_1^*|_{r_1=R_1(r_2)} > dr_2^*/dr_1^*|_{r_2=R_2(r_1)}$) if and only if the former is positively sloped (i.e., $dr_2^*/dr_1^*|_{r_1=R_1(r_2)} > 0$). Fig. 1 applies to this case.

To study transitional dynamics, we first see how r_1 and r_2 respond to κ_1 and κ_2 in each period. Logarithmically differentiating Eqs. (21) and (22), and using Eq. (17) with $d\tau_{jn}/\tau_{jn} = 0 \forall j, n$, we obtain:

$$\begin{aligned}
c_{11}dr_1/r_1 + c_{12}dr_2/r_2 &= (1-\pi_{11})r_1\kappa_1 d\kappa_1/\kappa_1 - \pi_{21}r_2\kappa_2 d\kappa_2/\kappa_2, \\
c_{21}dr_1/r_1 + c_{22}dr_2/r_2 &= -\pi_{12}r_1\kappa_1 d\kappa_1/\kappa_1 + (1-\pi_{22})r_2\kappa_2 d\kappa_2/\kappa_2,
\end{aligned}$$

where c_{11}, c_{12}, c_{21}, and c_{22} are defined in Appendix A. The relative demand for capital in country 1 to country 3, the right-hand side of Eq. (21), is decreasing in r_1 but is increasing in r_2. Similarly, the relative demand for capital in country 2 to country 3 is decreasing in r_2 but is increasing in r_1. Suppose, for example, that the relative supply of capital in country 1 to country 3 κ_1 increases. This directly tends to lower r_1 but raise r_2 from Eqs. (21) and (22), respectively. Not only that, it indirectly tends to raise r_1 through the increase in its relative demand caused by the rise in r_2, and also tends to lower r_2 through a similar demand substitution. The total effects of changes in κ_1 and κ_2 on r_1 and r_2 are obtained by solving the above two equations for dr_1/r_1 and dr_2/r_2:

$$dr_1/r_1 = (r_1\kappa_1/c)e_{11}d\kappa_1/\kappa_1 + (r_2\kappa_2/c)e_{12}d\kappa_2/\kappa_2, \tag{C.3}$$
$$dr_2/r_2 = (r_1\kappa_1/c)e_{21}d\kappa_1/\kappa_1 + (r_2\kappa_2/c)e_{22}d\kappa_2/\kappa_2; \tag{C.4}$$

$$\begin{aligned}
c &\equiv c_{11}c_{22} - c_{12}c_{21} > 0, \\
e_{11} &\equiv (1-\pi_{11})c_{22} + c_{12}\pi_{12} \\
&= -\pi_{12}[\theta(\pi_{22}r_2\kappa_2\pi_{23} + \pi_{12}r_1\kappa_1\pi_{13} + \pi_{32}\pi_{33}) + \pi_{23}r_2\kappa_2] + \pi_{13}c_{22} < 0, \\
e_{12} &\equiv -\pi_{21}c_{22} - c_{12}(1-\pi_{22}) \\
&= \theta[\pi_{12}r_1\kappa_1(\pi_{21}\pi_{13} - \pi_{11}\pi_{23}) + \pi_{32}(\pi_{21}\pi_{33} - \pi_{31}\pi_{23})], \\
e_{21} &\equiv -c_{11}\pi_{12} - (1-\pi_{11})c_{21} \\
&= \theta[\pi_{21}r_2\kappa_2(\pi_{23}\pi_{12} - \pi_{13}\pi_{22}) + \pi_{31}(\pi_{33}\pi_{12} - \pi_{13}\pi_{32})], \\
e_{22} &\equiv c_{11}(1-\pi_{22}) + \pi_{21}c_{21} \\
&= -[\theta(\pi_{11}r_1\kappa_1\pi_{13} + \pi_{21}r_2\kappa_2\pi_{23} + \pi_{31}\pi_{33}) + \pi_{13}r_1\kappa_1]\pi_{21} + c_{11}\pi_{23} < 0.
\end{aligned}$$

As a result, an increase in κ_1 always lowers r_1, but its total effect on r_2 is ambiguous. Similarly, an increase in κ_2 always lowers r_2, but its total effect on r_1 is ambiguous.

We turn to the dynamic system. Linearizing Eqs. (19) and (20) around the BGP, using Eqs. (23), (24), (C.1), (C.2), (C.3), and (C.4), and noting that $d\kappa_j/\kappa_j = \ln\kappa_j - \ln\kappa_j^*$ and $\dot{\kappa}_j/\kappa_j = d(\ln\kappa_j - \ln\kappa_j^*)/dt$, the linearized dynamic system is given by:

$$\begin{bmatrix} d(\ln\kappa_1 - \ln\kappa_1^*)/dt \\ d(\ln\kappa_2 - \ln\kappa_2^*)/dt \end{bmatrix} = J^* \begin{bmatrix} \ln\kappa_1 - \ln\kappa_1^* \\ \ln\kappa_2 - \ln\kappa_2^* \end{bmatrix}; J^* \equiv \begin{bmatrix} j_{11}^* & j_{12}^* \\ j_{21}^* & j_{22}^* \end{bmatrix},$$

$$\begin{aligned} j_{11}^* &\equiv (r_1^*\kappa_1^*/c^*)(a_{11}^*e_{11}^* + a_{12}^*e_{21}^*), \\ j_{12}^* &\equiv (r_2^*\kappa_2^*/c^*)(a_{11}^*e_{12}^* + a_{12}^*e_{22}^*), \\ j_{21}^* &\equiv (r_1^*\kappa_1^*/c^*)(a_{21}^*e_{11}^* + a_{22}^*e_{21}^*), \\ j_{22}^* &\equiv (r_2^*\kappa_2^*/c^*)(a_{21}^*e_{12}^* + a_{22}^*e_{22}^*), \end{aligned}$$

where all components of the Jacobian matrix J^* are evaluated at the BGP. Since both κ_1 and κ_2 are state variables, local stability requires that both eigenvalues associated with J^* should have negative real parts. In our two-dimensional system, the condition is equivalent to $\mathrm{tr}J^* = j_{11}^* + j_{22}^* < 0$ and $\det J^* = j_{11}^*j_{22}^* - j_{12}^*j_{21}^* > 0$. The trace and determinant of J^* are calculated as:

$$\begin{aligned} \mathrm{tr}J^* &= (1/c^*)[r_1^*\kappa_1^*(a_{11}^*e_{11}^* + a_{12}^*e_{21}^*) + r_2^*\kappa_2^*(a_{21}^*e_{12}^* + a_{22}^*e_{22}^*)], \\ \det J^* &= (r_1^*\kappa_1^*r_2^*\kappa_2^*/c^{*2})a^*(e_{11}^*e_{22}^* - e_{12}^*e_{21}^*). \end{aligned}$$

Since it is easily verified that $e_{11}e_{22} - e_{12}e_{21} = [(1-\pi_{11})(1-\pi_{22}) - \pi_{12}\pi_{21}]c > 0$, we conclude that a balanced growth path is locally stable if and only if:

$$r_1^*\kappa_1^*(a_{11}^*e_{11}^* + a_{12}^*e_{21}^*) + r_2^*\kappa_2^*(a_{21}^*e_{12}^* + a_{22}^*e_{22}^*) < 0.$$

To interpret this condition, consider the case where $a_{12}^*, a_{21}^*, e_{12}^*$, and e_{21}^* are close to zero in absolute values. Suppose, for example, that countries 1 and 2 are so small relative to country 3 in the initial period: $\kappa_{10} < \kappa_1^*$ and $\kappa_{20} < \kappa_2^*$. Then, reflecting the scarcity of capital, r_{10} and r_{20} should be higher than r_1^* and r_2^*, respectively (because $e_{11} < 0$ and $e_{22} < 0$ in Eqs. (C.3) and (C.4)). With the terms of trade of the first two countries being high, they start to grow faster than the last country (because $a_{11}^* > 0$ and $a_{22}^* > 0$ in Eqs. (C.1) and (C.2)). Since this increases κ_{1t} and κ_{2t} toward κ_1^* and κ_2^*, respectively, r_{1t} and r_{2t} fall toward r_1^* and r_2^*, respectively. Because of the diminishing terms of trade for countries 1 and 2, their growth advantages over country 3 are going to disappear in the long run.

More generally, an ambiguity arises due to the "cross effects" $a_{12}^*, a_{21}^*, e_{12}^*$, and e_{21}^*. They result from the assumption of more than two countries: a similar two-country model of Naito (2012) does not have such cross effects because there is only one relative supply of capital. Acemoglu and Ventura (2002) stress the same stability logic as the above paragraph in their continuum-country model, seeming to ignore the cross effects. However, even if the cross effects are not negligible, our dynamic system is still stable as long as the "own effects" mentioned in the previous paragraph are dominant.

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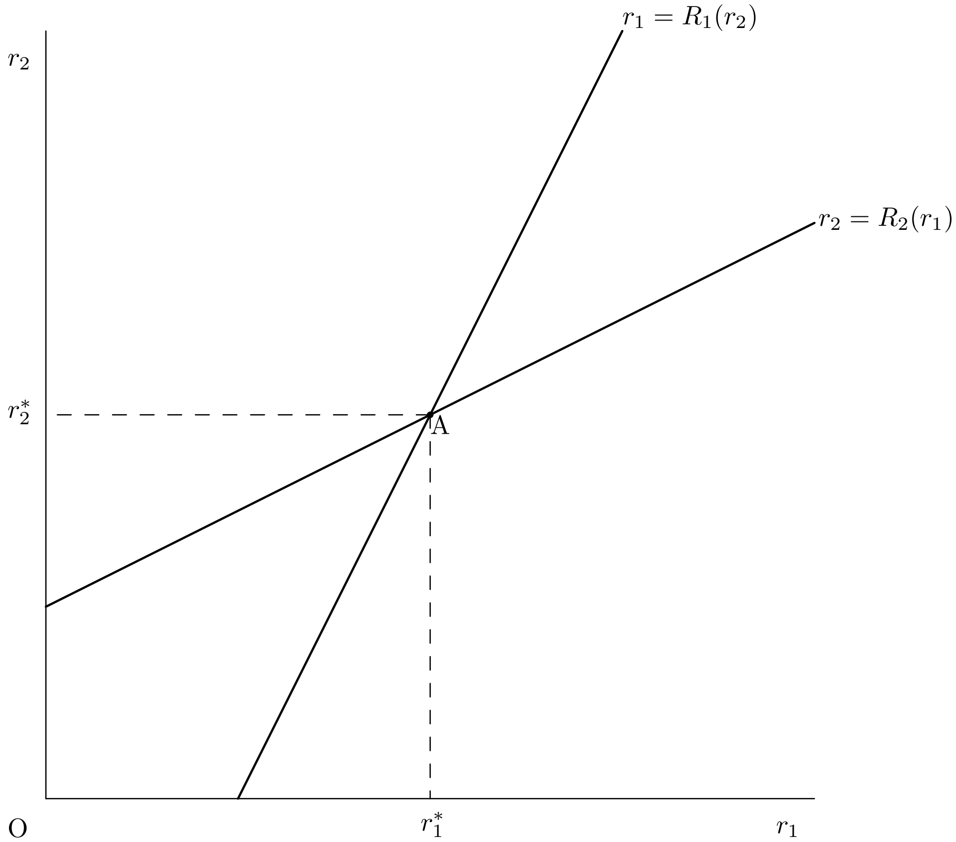

Fig. 1. Rental rates at the balanced growth path: $a_{12}^* < 0, a_{21}^* < 0$.

$\tau_{12} \downarrow$	r_1	r_2	γ_1	γ_2	γ_3	π_{11}	π_{12}	π_{13}	π_{21}	π_{22}	π_{23}	π_{31}	π_{32}	π_{33}
short-run	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$
long-run	$\downarrow$	0	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$

(a) Unilateral trade liberalization

$\tau_{12} \downarrow$, $\tau_{21} \downarrow$	r_1	r_2	γ_1	γ_2	γ_3	π_{11}	π_{12}	π_{13}	π_{21}	π_{22}	π_{23}	π_{31}	π_{32}	π_{33}
short-run	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$
long-run	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$

(b) Bilateral trade liberalization

Table 1: Effects of trade liberalization around the symmetric BGP