SURVEY ON THE LIFE OF BUILDINGS IN JAPAN

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ABSTRACT

This paper discusses a method for the estimation of a building's life time, and presents the results of a field survey, implemented by the authors, on the actual state of a building's life time. The life time estimation method discussed here is based on the reliability theory and life table method. The field survey included wooden residential houses, reinforced concrete apartment houses and office buildings, and estimated their average lives.

1. INTRODUCTION

There may be many ways of estimating a building's life time, such as averaging the lives of demolished buildings, calculating it from material deteriorating speed and so on. As for the average life of a human, the life table is widely used, and the method of making it will be also effective to buildings. This is why we decided to study making a life table for buildings. We think that the ledgers prepared for fixed property taxes are very good as a data source for buildings life tables, because they include details of each building, such as built year, area, structure and so on. The ledgers of demolished ones are also available.

2. BASICS OF RELIABILITY THEORIES

If you want to examine the life of some item, you will get a group of specimens, put them into a testing environment and record the times when failures occur. In an ordinary life test, specimens are put into a test simultaneously, and many of the theories for analyzing life data are effective under this condition. Life data of buildings mentioned here, or generally life table style data, do not satisfy this condition, and we will discuss this later. Now let us take a general view of the basics about the theories of life data analysis. Some definitions of functions are as follows:

\( R(t) \): reliability function, which represents the remaining rate of an item at time \( t \).
\( F(t) \): unreliability function or accumulated failure rate.
\( f(t) \): failure probability density function.
\( \lambda(t) \): failure rate at time \( t \).

In this paper, \( R(t) \) represents the remaining rate of a cohort of buildings, which is an imaginary group of buildings or houses built simultaneously at a supposed time. These functions have the following relations.

\[
F(t) = 1 - R(t)
\]
\[
f(t) = \frac{dF(t)}{dt} = -\frac{dR(t)}{dt}
\]
\[
\lambda(t) = \frac{f(t)}{R(t)} = -\frac{1}{R(t)} \frac{dR(t)}{dt}
\]
\[
\therefore R(t) = \exp \left\{ - \int \lambda(t) dt \right\}
\]

Sometimes \( f(t) \) or \( R(t) \) follows some stochastic distribution functions like "Normal distribution", "Log-normal distribution" or "Weibull's distribution". These are as follows:

Normal distribution:

\[
f(t) = \frac{1}{\sqrt{2\pi}} \exp \left\{ -\frac{(t - \mu)^2}{\sigma^2} \right\}
\]
\[
R(t) = 1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} \exp \left\{ -\frac{(x - \mu)^2}{\sigma^2} \right\} dx
\]

Log-normal distribution:

\[
f(t) = \frac{1}{\sqrt{2\pi t}} \exp \left\{ -\frac{(\ln t - \mu)^2}{\sigma^2} \right\}
\]
\[
R(t) = 1 - \frac{1}{\sqrt{2\pi t}} \int_{0}^{t} \frac{1}{x} \exp \left\{ -\frac{(\ln x - \mu)^2}{\sigma^2} \right\} dx
\]

Weibull's distribution (3 parameters)

\[
f(t) = \frac{m}{\eta} \left( \frac{t - \delta}{\eta} \right)^{m-1} \exp \left\{ - \left( \frac{t - \delta}{\eta} \right)^m \right\}
\]
\[
R(t) = \exp \left\{ - \left( \frac{t - \delta}{\eta} \right)^m \right\}
\]

3. LIFE TABLE DATA

Table 1 shows an example of building life data (or life table data) from the ledgers of fixed property tax. This data on wooden residential houses in Japan, contains the number of remaining units at Jan. 1st of 1987 and the number of demolished ones during the year of 1987. The ages shown in the left columns are counted from the newly built time (year) of buildings. The figures shown here are the maximum age of
the group, i.e. age "1" means from age "0" up to under age "1", and "2" means "1" to "2". In other words, the group of houses built during the year of 1986 is regarded as having age "1" at the beginning of 1987, that of 1985 having age "2" and so on. These data were collected from 48 major cities from Hokkaido to Kyushu, including Sapporo, Aomori, Yokohama, Niigata, Osaka, Hiroshima, Takamatsu and Fukuoka. But the data from Tokyo and Nagoya relates to only one ward for each city.

Table 1. Wooden Residential Houses Data for Life Table (1987)

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<th>Age</th>
<th>Remaining Number</th>
<th>Demolished Number</th>
<th>Age</th>
<th>Remaining Number</th>
<th>Demolished Number</th>
<th>Age</th>
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</table>

![Figure 1. Model of Building Life Data](image)

Strictly speaking, units in a same year old group do not have the same age. For example, an unit built on Jan. 1st of 1980 and another built on Dec. 31 of 1980 are grouped in the same 7 year old group here, but their ages differ by almost one year from each other. Figure 1. shows a model of life table data. A sloping line from bottom left to upper right represents a unit built at a point of time which the horizontal axis represents. A vertical line drawn at the point \( K_1 \) is the time of observation. The sloped lines (or units) crossing this vertical line are remaining or "living" units at time \( K_1 \). \( K_0 \) is the end of the period for counting demolished or "dead" units. The \( i \) year old group's number of demolished units is represented by cut lines included in the dotted area \( T_i \). The vertical axis represents age while the horizontal does time. The projection of a sloped line onto the vertical axis means the aging of a unit. Then the
length of a cut sloped line is proportional to the age of a demolished unit. It will be clear from this model that each unit demolished in $T_i$ has a different beginning and end of life, and then has a different age. The scale in the statistical method for life estimation should be "age", then this type of data should be arranged according to the age axis, not to the time axis.

![Figure 2. Transformation from the time axis plot to the age axis plot](image)

Figure 2. shows the transformation of building life data expression from the time axis to the age axis. Here, a unit is shown as a horizontal line, while it was a sloped line in figure 1. In the time axis, the end point of each line is aligned vertically, i.e. is simultaneous, but in the age axis the beginning point is vertical.

4. LIFE TIME ESTIMATION METHOD OF BUILDINGS

![Figure 3a. Model of Ordinary Life Test](image)  ![Figure 3b. Model of Building Life Data](image)

Figure 3a shows a model for ordinary data of life test, and figure 3b the life table data. In figure 3a, the remaining number at time $t_1$ of the specimens is $N_2$, while $N_1$ is the remaining number at time $t_0$ and this is the total of specimens. The remaining probability at time $t_1$, exactly of the period from time $t_0$ up to $t_1$ and let it be $Pr\{ t_1 | t_0 \}$, is
Generally, the remaining rate from time $t_{i-1}$ up to $t_i$, $Pr\{t_i|t_{i-1}\}$, is

$$Pr\{t_i|t_{i-1}\} = \frac{N_i - d_i}{N_i}$$

Then the remaining probability from the beginning of test $t_0$ up to $t_i$, $Pr\{t_i|t_0\}$ is

$$Pr\{t_i|t_0\} = Pr\{t_i|t_0\} \cdot Pr\{t_2|t_1\} \cdots Pr\{t_{i-2}|t_{i-1}\} \cdot Pr\{t_1|t_{i-1}\}$$

$$= \frac{N_1 - d_1}{N_1} \cdot \frac{N_2 - d_2}{N_2} \cdots \frac{N_{i-1} - d_{i-1}}{N_{i-1}} \cdot \frac{N_i - d_i}{N_i}$$

$$= \frac{N_i - d_i}{N_i}$$

In the life table model shown in figure 1, let the remaining number of any newly built year group at time $K_i$ be $N_i$, and the number of units newly built between time $K_i$ and $K_0$ be $N_{i+1}$, and let the demolished number of any group between $K_i$ and $K_0$ be $d_i$. $N_i$ and $d_i$ are obtained as field data, as for this paper, from the ledgers. As shown in figure 3b, the vertical line at $K_i$ in figure 1 is expressed as a sloped line along the age axis. The remaining probability through the area $T_i$ in figure 3b is that from age (not time) $t_0$ up to time (not age) $K_0$, and can be expressed as $Pr\{K_0|t_0\}$. As for $T_i$, the remaining probability through it is that of from time $K_i$ up to time $K_0$, and expressed as $Pr\{K_0|K_i\}$. Using remaining numbers and demolished numbers, each can be expressed as follows;

$$Pr\{K_0|t_0\} \cong \frac{N_1 - d_1}{N_1}$$

$$Pr\{K_0|K_i\} \cong \frac{N_i - d_i}{N_i}$$

Assume that a remaining function $R(t)$ exists. Figure 4 contains two areas $T_i$ and $T_{i'}$ of figure 3b. In $T_i$, the remaining probability of a unit (shown as a line of height $x$) from age $t_0$ (the vertical line on the left side) to time $K_0$ (the sloped line on the right side) is $R(u,x)$. $T_i$ is a triangle and suppose that the number of units in it is enough, the remaining probability through $T_i$ can be expressed as

$$Pr\{K_0|t_0\} = \int_0^1 R(u,x)dx$$

and the right side of the equation is approximated using the trapezoidal rule as shown
\[
\int_0^1 R(u_1 x)dx \cong \frac{1}{2} \left\{ R(0) + R(1) \right\} \\
= \frac{1}{2} \left\{ 1 + R(t_i) \right\} \quad (\because R(0) = 1)
\]

Then

\[
\frac{1}{2} \left\{ 1 + R(t_i) \right\} \cong \frac{N_1 - d_i}{N_i} \\
R(t_i) \cong \frac{N_1 - 2d_i}{N_i}
\]

In \( T_i \) \((i \geq 2)\), the remaining probability of a unit (at height \( x \)) from time \( K_0 \) (the sloped line on the left side) to time \( K_i \) (the sloped line on the right side) is

\[
\frac{R(t_i - u_i (1-x))}{R(t_{i-1} - u_i (1-x))}
\]

because this unit has lived until time \( t_{i-1} - u_i (1-x) \) and lives up to \( t_i - u_i (1-x) \).

Then

\[
\Pr\{K_0|K_{i-1}\} \cong \int_0^1 \frac{R(t_i - u_i (1-x))}{R(t_{i-1} - u_i (1-x))} dx
\]

Suppose that the period is small enough and the failure rate \( \lambda_i(t) \) can be regarded as constant \((\lambda(t) \cong \hat{\lambda})\), then \( R(t) \) is approximated by an exponential distribution, i.e. \( R(t) \cong \exp(-\hat{\lambda} t) \). Then

\[
\frac{R(t_i - u_i (1-x))}{R(t_{i-1} - u_i (1-x))} \cong \frac{\exp(-\hat{\lambda} t_i)}{\exp(-\hat{\lambda} t_{i-1})} \\
= \frac{R(t_i)}{R(t_{i-1})}
\]

So this equation does not contain \( x \),

\[
\Pr\{K_0|K_{i-1}\} \cong \frac{R(t_i)}{R(t_{i-1})}
\]

Then
\[ \frac{R(t_i)}{R(t_{i-1})} \approx \frac{N_i - d_i}{N_i} \]
\[ R(t_i) \approx R(t_{i-1}) \cdot \frac{N_i - d_i}{N_i}, \quad (i \geq 2) \]

Let \( R_i \) be the estimation of \( R(t_i) \),

\[ R_i = \frac{N_i - 2d_i}{N_i} \]
\[ R_i = R_1 \prod_{s=2}^{i} \frac{N_s - d_s}{N_s}, \quad (i \geq 2) \]

5. LIFE TIME ESTIMATION OF JAPANESE HOUSES AND BUILDINGS

The following figures show the results of the remaining rate analyses of Japanese houses and office buildings using above-mentioned equations. Figure 5 is for wooden residential houses, and figure 6 is for reinforced concrete (r.c.) apartment houses, this curve is extended by fitting parametric distribution (Weibull’s distribution) by the least square method. Figure 7 is for office buildings of r.c. structure.

6. CONCLUSION

We proposed a life time estimation method using data like the life table. If average life can be defined as the expected years for a half of the cohort demolish, that of Japanese wooden residential houses is estimated at 38.2 years in 1987 and r.c. offices at 34.8. As for r.c. apartment houses, though the data on old buildings is not sufficient, the extrapolated curve shows that their average life would be 50.6 years.
7. REFERENCES
