For Online Publication

Appendix to “Network-motivated Lending Decisions: A Rationale for Forbearance Lending”

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Abstract

This online appendix includes supplemental materials for “Network-motivated Lending Decisions: A Rationale for Forbearance Lending” by Ogura, Okui, and Saito. Online Appendix Online Appendix 1: contains the proofs of the propositions. Online Appendix Online Appendix 2: presents a numerical example for the theoretical model. Online Appendix Online Appendix 3: derives the bias property of the OLS estimator applied to the spatial autoregressive model. Online Appendix Online Appendix 4: presents the formula for standard errors for the OLS estimator, which are adjusted to address the problem that the demand influence coefficients are estimated regressors. Online Appendix Online Appendix 5: presents additional empirical results to examine the robustness of the empirical results in the main text.

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Online Appendix 1: Proofs of propositions

A1.1 Full solution for the monopolistic-bank case and useful lemmas

The real profit of the monopolistic bank, i.e., the total profit from all operating firms in the entire network, before subtracting the costs of refinancing \( \sum_{i \in S} (1 + \rho)F_i \), is:

\[
\Pi(e) = \sum_{i \in S} \pi_i(e) = \frac{v_f}{p_i \theta}, \tag{A.1}
\]

where \( \pi_i(e) \) is the profit of firm \( i \) before repayment of \( (1 + \rho)F_i \), and \( e \equiv (e_1, e_2, \cdots, e_n)' \).

The demand influence coefficient in the main text is:

\[
v' = 1' \sum_{k=0}^{\infty} Q^k. \tag{A.2}
\]

By substituting the f.o.c. of firm \( i \), (10) in the main text into \( p^j \) in the definition of the \((i, j)\)th element of \( Q \), \( q_{ij} = e_i w_{ij} p_i^{1-\theta} p_j^\theta / p_j \), we obtain the following expression.

\[
Q = \left( \frac{\theta - 1}{\theta} \right)^\theta P_\theta E W' \hat{P}_\theta, \tag{A.3}
\]

where \( P_\theta \) is an \( n \times n \) diagonal matrix of which the \( i \)th diagonal element is \( p_i^{1-\theta} \) (we assume that \( p_i^{1-\theta} = 0 \) if \( e_i = 0 \)), \( E \) is an \( n \times n \) diagonal matrix of which the \( i \)th diagonal element is \( e_i \), i.e., \( e = E1 \), and \( \hat{P}_\theta \) is an \( n \times n \) diagonal matrix of which the \( i \)th diagonal element is \( p_i^{\theta - 1} \) if \( e_i = 1 \) or 0 otherwise. Note that \( \hat{P}_\theta P_\theta = P_\theta \hat{P}_\theta = E \). Substituting (A.3) into the expression (A.2) gives:

\[
v' = 1' P_\theta A^{-1} \hat{P}_\theta, \quad \text{where} \quad A \equiv I - \left( \frac{\theta - 1}{\theta} \right)^\theta E W'. \tag{A.4}
\]

The price vector is given by the following expression.

\[
p'_\theta \equiv 1' P_\theta = w_0 E B^{-1} \left( \frac{\theta - 1}{\theta p_0} \right)^{\theta - 1}, \tag{A.5}
\]

\[
\text{where} \quad B \equiv I - \left( \frac{\theta - 1}{\theta} \right)^{\theta - 1} E W E. \tag{A.6}
\]

The vector of the final demand for each firm \((n \times 1)\) is:

\[
f = R p_\theta^{\theta - 1} P_\theta e. \tag{A.7}
\]

Substituting (A.4) and (A.7) into (A.1) gives the following expression.

\[
\Pi(e) = \frac{R p_\theta^{\theta - 2}}{\theta} \cdot 1' P_\theta A^{-1} e. \tag{A.8}
\]
\( p_c \) is expressed by:

\[
p_c = (1'P_\theta 1)^{\frac{1}{1-\sigma}} = (w_0'EB^{-1}1)^{\frac{1}{1-\sigma}} \cdot \frac{p_0\theta}{\theta - 1},
\]

(A.9)

We list useful results as lemmas. Let \( E_1 \) be the \( n \times n \) diagonal matrix whose \( i \)-th diagonal element is \( e_i = 1 \) for \( i \in S \), \( e_i = 0 \) for \( i \in S^c \setminus \{z\} \), and \( e_z = 1 \). Let \( E_0 \) be the \( n \times n \) diagonal matrix whose \( i \)-th diagonal element is \( e_i = 1 \) for \( i \in S \), \( e_i = 0 \) for \( i \in S^c \setminus \{z\} \), and \( e_z = 1 \). Let \( p_{ce_z} \) be the value of \( p_c \) under \( E_{e_z} \). Similarly, let \( P_{\theta e_z} \) and \( p_{\theta e_z} \) be the value of \( P_\theta \) and \( p_\theta \) under \( E_{e_z} \), respectively. First, we show that the price of each product and CPI are nonincreasing in the number of operating firms. This holds because of the love-of-variety assumption in our product function. A higher variety of inputs improves the productivity and reduces product prices.

**Lemma 1** \( p_i^{1-\theta} \) \( (i = 1, 2, \cdots, n) \) is nondecreasing in the number of operating firms. Namely, each product price is nonincreasing in the number of operating firms.

**Proof.** From (A.5), we have:

\[
p_{\theta 1} - p_{\theta 0} \propto w_0'(E_1B_1^{-1} - E_0B_0^{-1})
\]

\[
= w_0'[E_1B_1^{-1}(B_0 - B_1)B_0^{-1} + (E_1 - E_0)B_0^{-1}] \geq 0
\]

(A.10)

"\( \geq 0 \)" indicates that every element of a matrix is nonnegative. This holds because all elements in \( B_{e_z}, B_{e_z}^{-1}, E_1 - E_0 \) and \( w_0 \) are nonnegative by definition. As

\[
B_0 - B_1 \propto E_1WE_1 - E_0WE_0
\]

\[
= (E_1 - E_0)WE_1 + E_0W(E_1 - E_0) \geq 0,
\]

(A.11)

every element of the first term in the bracket is also nonnegative. Thus,

\[
p_{\theta 1} \geq p_{\theta 0}.
\]

(A.12)

**Lemma 2** \( p_c \) is nonincreasing in the number of operating firms.

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Proof. From (A.9),

\[ p_{c1}^{1-\theta} - p_{c0}^{1-\theta} = 1'(E_1 B_1^{-1} E_1 - E_0 B_0^{-1} E_0)w_0 \]

\[ = 1'[(E_1 - E_0)B_1^{-1}E_1 + E_0B_1^{-1}(B_0 - B_1)B_0^{-1}E_1 + E_0B_0^{-1}(E_1 - E_0)]w_0 \]

\[ > 0. \quad (A.13) \]

The last inequality comes from the fact that all elements in $B_{e_x}, E_1 - E_0$ and $w_0$ are nonnegative by definition, and all elements in $B_0 - B_1$ are nonnegative by (A.11). This implies $p_{c1} < p_{c0}$ because $\theta > 1. \quad \square$

**Lemma 3** $v_i p_i^{1-\theta}$ $(i = 1, 2, \cdots, n)$, i.e., each element of the vector $1'P_\theta A^{-1}E$ is nondecreasing in the number of operating firms.

**Proof.**

\[ 1'P_{\theta1}A_1^{-1}E_1 - 1'P_{\theta0}A_0^{-1}E_0 = 1'(P_{\theta1} - P_{\theta0})A_1^{-1}E_1 \]

\[ + 1'P_{\theta0}(A_1^{-1}(A_0 - A_1)A_0^{-1}E_1 + A_0^{-1}(E_1 - E_0)), \quad (A.14) \]

where

\[ A_0 - A_1 = \left( \frac{\theta - 1}{\theta} \right)^{\theta} (E_1 - E_0)W' \geq 0. \quad (A.15) \]

Every element of the first term is nonnegative by Lemma 1 and because every element of $A_{e_x}, A_{e_x}^{-1}$ and $P_{\theta e_x}$ is nonnegative. Every element of the second term is also nonnegative because all elements of $E_1 - E_0, P_{\theta e_x}, A_{e_x}$, and $A_{e_x}^{-1}$ are nonnegative and $\theta > 1. \quad \square$

**Lemma 4** The profit before repayment of an operating firm $i$, $\pi_i(e)$ $(i = 1, 2, \cdots, n)$, is nondecreasing in the number of operating firms if $1 < \theta \leq 2$.

**Proof.** From (A.8), the row vector $\pi(e)$ of which the $i$th element is $\pi_i(e)$ $(i = 1, 2, \cdots, n)$ is given by:

\[ \pi(e) = \frac{R e^{\theta - 2}}{\theta} E A^{-1}p_\theta. \quad (A.16) \]
Let us denote $\mathbf{E}_0^1$ and $\mathbf{E}_1^1$ by $\mathbf{e}_0$ and $\mathbf{e}_1$, respectively. It is sufficient to show that every element of the following vector is nonnegative.

$$
\pi(\mathbf{e}_1) - \pi(\mathbf{e}_0) = \frac{R^{\theta-2}}{\theta} \mathbf{E}_1^1 A_{\theta}^{-1} p_{\theta_1} - \frac{R^{\theta-2}}{\theta} \mathbf{E}_0^0 A_{\theta}^{-1} p_{\theta_0}
= \frac{R^{\theta-2}}{\theta} \{\mathbf{E}_1^1 A_{\theta}^{-1} p_{\theta_1} - \mathbf{E}_0^0 A_{\theta}^{-1} p_{\theta_0}\}, \quad (A.17)
+ \frac{R}{\theta} (R^{\theta-2} - R^{\theta-2}) \mathbf{E}_0^0 A_{\theta}^{-1} p_{\theta_0}. \quad (A.18)
$$

Every element of the first term (A.17) is nonnegative by Lemma 3. The second term is positive if $1 < \theta < 2$, zero if $\theta = 2$, and negative if $\theta > 2$ by Lemma 2. Thus, both terms are nonnegative if $1 < \theta \leq 2$. □

**Lemma 5** The monopolistic-bank profit before subtracting the costs of refinancing, $\Pi(\mathbf{e})$, is increasing in the number of operating firms if $1 < \theta \leq 2$.

**Proof.** By the definition (A.1) and Lemma 4, $\Pi(\mathbf{e})$ is increasing in the number of operating firms if $1 < \theta \leq 2$. □

**Lemma 6** The set of $\mathbf{e}$ equipped with a partial order, where the order for each pair of $\mathbf{e}$ and $\hat{\mathbf{e}} \in \{0,1\}^n$ is defined by $\mathbf{e} \geq \hat{\mathbf{e}}$ if $e_i \geq \hat{e}_i$ for any $i = 1, 2, \cdots, n$, is a complete lattice.

**Proof.** Let us define the lattice operations for $\mathbf{e}, \hat{\mathbf{e}} \in \{0,1\}^n$,

$$
\mathbf{e} \lor \hat{\mathbf{e}} \equiv (\max[e_1, \hat{e}_1], \max[e_2, \hat{e}_2], \cdots, \max[e_n, \hat{e}_n]),
\mathbf{e} \land \hat{\mathbf{e}} \equiv (\min[e_1, \hat{e}_1], \min[e_2, \hat{e}_2], \cdots, \min[e_n, \hat{e}_n]).
$$

There exist $\mathbf{e} \lor \hat{\mathbf{e}}$ and $\mathbf{e} \land \hat{\mathbf{e}} \in \{0,1\}^n$ for any pair of $\mathbf{e}$ and $\hat{\mathbf{e}} \in \{0,1\}^n$. Thus, the set of $\mathbf{e}$ with the partial order is a lattice. Moreover, it is a complete lattice because there exist the supremum $\lor \mathbb{S}$ and the infimum $\land \mathbb{S}$ for any subset $\mathbb{S} \subseteq \{0,1\}^n$. □

**A1.2 Existence of the solution for profit maximization by the monopolistic bank.**

Let us consider the following recursive algorithm to find a local maximum of the monopolistic-bank profit. We start with an initial value of $\mathbf{e} = 1$. The bank decides to refinance $(1 + \rho)F_i$ for an
operating firm $i$ ($e_i = 1$) if its marginal contribution to the bank profit exceeds the refinancing cost, i.e.,

$$
\Pi(e) - \Pi(e_{-i}) \geq (1 + \rho)F_i,
$$

(A.19)

where $e_{-i}$ is the vector whose element is the same as the initial value $e$ except that the $i$th element $e_i$ is set at zero. The bank sets $e_i = 1$ if this inequality holds or 0 otherwise. The bank conducts this firm-by-firm evaluation. We denote the resulting updated vector by $\hat{e}$. The bank repeats this procedure. We can express this procedure by the following recursive map:

$$
\hat{e} = \mathcal{H}(e),
$$

(A.20)

where the $i$th element of $\hat{e}$ ($n \times 1$) is given by:

$$
\hat{e}_i = \mathbb{1} [\Pi(e) - \Pi(e_{-1}) - (1 + \rho)F_i \geq 0].
$$

(A.21)

The fixed point of this recursive form is a local maximum. The next lemma is a required component for establishing the existence of such a fixed point.

**Lemma 7** The profit difference $\Pi(e_{1}) - \Pi(e_{0})$ resulting from switching $e_i$ ($i = 1, 2, \ldots, n$) from 0 to 1 is nondecreasing in $e_j$ ($j \neq i$) if $1 < \theta \leq 2$.

**Proof.** Let $E_{11}$ and $E_{01}$ be $E_1$ and $E_0$ when $y \in S$, $y \neq z$, respectively. Likewise, let $E_{10}$ and $E_{00}$ be $E_1$ and $E_0$ when $y \in S^c$, $y \neq z$, respectively. We denote $e_{11} = E_{11}1, e_{01} = E_{01}1, e_{10} = E_{10}1$, and $e_{00} = E_{00}1$. We denote the price matrix $P_\theta$ and the consumer price index $p_c$ corresponding to each of the four cases by the subscript 11 (firms $z$ and $y$ operate), 01 (firm $z$ does not operate but firm $y$ operates), 10 (firm $z$ operates but firm $y$ does not) and 00 (both do not operate), respectively. It is sufficient to show that the following difference is nonnegative.
\[ \Pi(e_{11}) - \Pi(e_{01}) - \{\Pi(e_{10}) - \Pi(e_{00})\} \]

\[ = \frac{R p_{c11}^{\theta - 2}}{\theta} 1'P_{\theta 11}A_{11}^{-1}(e_{11} - e_{01}) - \frac{R p_{e10}^{\theta - 2}}{\theta} 1'P_{\theta 10}A_{10}^{-1}(e_{10} - e_{00}), \quad (A.22) \]

\[ + \frac{R p_{c11}^{\theta - 2}}{\theta} 1'P_{\theta 11}(A_{11}^{-1} - A_{01}^{-1})e_{01} - \frac{R p_{e10}^{\theta - 2}}{\theta} 1'P_{\theta 10}(A_{10}^{-1} - A_{00}^{-1})e_{00}, \quad (A.23) \]

\[ + \frac{R p_{c11}^{\theta - 2}}{\theta} 1'(P_{\theta 11} - P_{\theta 01})A_{01}^{-1}e_{01} - \frac{R p_{e10}^{\theta - 2}}{\theta} 1'(P_{\theta 10} - P_{\theta 00})A_{00}^{-1}e_{00}. \quad (A.24) \]

\[ + \frac{R}{\theta} (p_{c11}^{\theta - 2} - p_{e10}^{\theta - 2}) 1'P_{\theta 01}A_{01}^{-1}e_{01} - \frac{R}{\theta} (p_{c10}^{\theta - 2} - p_{e10}^{\theta - 2}) 1'P_{\theta 00}A_{00}^{-1}e_{00}. \quad (A.25) \]

The term (A.22) can be expanded as follows:

\[ \frac{R p_{c11}^{\theta - 2}}{\theta} 1'P_{\theta 11}A_{11}^{-1}(e_{11} - e_{01}) - \frac{R p_{e10}^{\theta - 2}}{\theta} 1'P_{\theta 10}A_{10}^{-1}(e_{10} - e_{00}) \]

\[ = \frac{R}{\theta} (p_{c11}^{\theta - 2} - p_{e10}^{\theta - 2}) 1'P_{\theta 11}A_{11}^{-1}(e_{11} - e_{01}) + \frac{R p_{e10}^{\theta - 2}}{\theta} 1'(P_{\theta 11} - P_{\theta 01})A_{11}^{-1}(e_{11} - e_{01}) \]

\[ + \frac{R p_{c10}^{\theta - 2}}{\theta} 1'P_{\theta 10}A_{10}^{-1}(A_{10} - A_{11})A_{11}^{-1}(e_{11} - e_{01}) + \frac{R p_{e10}^{\theta - 2}}{\theta} 1'P_{\theta 01}A_{01}^{-1}(e_{11} - e_{01} - e_{10} + e_{00}). \]

The first term is nonnegative if \( 1 < \theta \leq 2 \) by Lemma 2. The second term is also nonnegative by Lemma 1. The third term is also nonnegative by (A.15). The fourth term is zero. Thus, the term (A.22) is nonnegative.

The term (A.23) can be expanded as follows.

\[ \frac{R p_{c11}^{\theta - 2}}{\theta} 1'P_{\theta 11}(A_{11}^{-1} - A_{01}^{-1})e_{01} - \frac{R p_{e10}^{\theta - 2}}{\theta} 1'P_{\theta 10}(A_{10}^{-1} - A_{00}^{-1})e_{00} \]

\[ = \frac{R}{\theta} (p_{c11}^{\theta - 2} - p_{e10}^{\theta - 2}) 1'P_{\theta 11}(A_{11}^{-1} - A_{01}^{-1})e_{01} + \frac{R p_{e10}^{\theta - 2}}{\theta} 1'(P_{\theta 11} - P_{\theta 10})(A_{11}^{-1} - A_{01}^{-1})e_{01} \]

\[ + \frac{R p_{c10}^{\theta - 2}}{\theta} 1'P_{\theta 10}(A_{10}^{-1} - A_{01}^{-1} - A_{11}^{-1} + A_{00}^{-1})e_{01} + \frac{R p_{e10}^{\theta - 2}}{\theta} 1'P_{\theta 01}(A_{11}^{-1} - A_{00}^{-1})(e_{01} - e_{00}). \]

Each element of \( A_{11}^{-1} - A_{01}^{-1} \) is nonnegative by (A.15) because \( A_{11}^{-1} - A_{01}^{-1} = A_{11}^{-1}(A_{01} - A_{11})A_{01}^{-1} \). Likewise, every element of \( A_{10}^{-1} - A_{00}^{-1} \) is nonnegative. Therefore, the last term is nonnegative. The first term is nonnegative by Lemma 2 if \( 1 < \theta \leq 2 \). The second term is nonnegative by Lemma 1. The contents of the parentheses of the third term is:

\[ A_{11}^{-1} - A_{01}^{-1} - A_{10}^{-1} + A_{00}^{-1} = A_{11}^{-1}(A_{01} - A_{11})A_{01}^{-1} - A_{10}^{-1}(A_{00} - A_{10})A_{00}^{-1}. \]

As

\[ A_{01} - A_{11} = \left( \frac{\theta - 1}{\theta} \right) (E_{11} - E_{10})W' = \left( \frac{\theta - 1}{\theta} \right) (E_{10} - E_{00})W' = A_{00} - A_{10} \equiv C, \]
the previous expression is equal to:

\[
(A_{11}^{-1} - A_{10}^{-1})CA_{01}^{-1} + A_{01}^{-1}C(A_{01}^{-1} - A_{00}^{-1})
\]

\[
= A_{11}^{-1}(A_{10} - A_{11})A_{10}^{-1}CA_{01}^{-1} + A_{01}^{-1}CA_{01}^{-1}(A_{00} - A_{01})A_{00}^{-1}.
\]

Each element of this expression is nonnegative by (A.15). Thus, the third term is nonnegative.

These observations establish that the term (A.23) is nonnegative.

The term (A.24) can be expanded as follows.

\[
\frac{R_p e_{11}^{-2}}{\theta}E_1'(P_{\theta 11} - P_{\theta 01})A_{01}^{-1}e_{01} - \frac{R_p e_{10}^{-2}}{\theta}E_1'(P_{\theta 10} - P_{\theta 00})A_{00}^{-1}e_{00}
\]

\[
\frac{R}{\theta}(e_{11}^{-2} - e_{10}^{-2})E_1'(P_{\theta 11} - P_{\theta 01})A_{01}^{-1}e_{01} + \frac{R_p e_{10}^{-2}}{\theta}E_1'(P_{\theta 11} - P_{\theta 01} + P_{\theta 00})A_{01}^{-1}e_{01}
\]

\[
+ \frac{R_p e_{10}^{-2}}{\theta}E_1'(P_{\theta 10} - P_{\theta 00})A_{01}^{-1}(A_{00} - A_{01})A_{00}^{-1}e_{01} + \frac{R_p e_{10}^{-2}}{\theta}E_1'(P_{\theta 10} - P_{\theta 00})A_{00}^{-1}(e_{01} - e_{00}).
\]

The first term is nonnegative if \(1 < \theta \leq 2\) by Lemmas 1 and 2. The third term is also nonnegative by Lemmas 1 and (A.15). The fourth term is also positive by Lemma 1. The contents of the parentheses of the second term after substituting the definition (A.5) is:

\[
(P_{\theta 11} - P_{\theta 01} - P_{\theta 10} + P_{\theta 00})E_1
\]

\[
\times \left[ B_{11}^{-1}(E_{11} - E_{01}) - B_{10}^{-1}(E_{10} - E_{00}) + \left(\frac{\theta - 1}{\theta}\right)^{-1}B_{11}^{-1}(E_{11}WE_{11} - E_{01}WE_{01})B_{01}^{-1}
\]

\[
+ \left(\frac{\theta - 1}{\theta}\right)^{-1}B_{10}^{-1}(E_{10}WE_{10} - E_{00}WE_{00})B_{00}^{-1} \right] w_0
\]

\[
= [(B_{11}^{-1} - B_{10}^{-1})(E_{11} - E_{01})
\]

\[
+ \left(\frac{\theta - 1}{\theta}\right)^{-1}(B_{11}^{-1} - B_{10}^{-1})(E_{11}WE_{11} - E_{01}WE_{01})B_{01}^{-1}
\]

\[
+ \left(\frac{\theta - 1}{\theta}\right)^{-1}B_{10}^{-1}(E_{11}WE_{11} - E_{01}WE_{01} - E_{10}WE_{10} + E_{00}WE_{00})B_{01}^{-1}
\]

\[
+ \left(\frac{\theta - 1}{\theta}\right)^{-1}B_{10}^{-1}(E_{10}WE_{10} - E_{00}WE_{00})(B_{01}^{-1} - B_{00}^{-1}) \right] w_0.
\]

We make use of the fact that \(E_{11} - E_{01} = E_{10} - E_{00}\) in the last equality. Every element of \(B_{11}^{-1} - B_{10}^{-1} = B_{11}^{-1}(B_{10} - B_{11})B_{10}^{-1}\) in the first and second terms in the last expression is nonnegative by (A.11). Likewise, every element of \(B_{01}^{-1} - B_{00}^{-1}\) in the last term is nonnegative. Every element of \(E_{11}WE_{11} - E_{01}WE_{01}\) in the second term and \(E_{10}WE_{10} - E_{00}WE_{00}\) in the last term
is nonnegative by (A.11). As $E_{11} - E_{01} = E_{10} - E_{00}$, the contents of the parentheses in the third term can be rearranged as follows:

$$E_{11}WE_{11} - E_{01}WE_{01} - E_{10}WE_{10} + E_{00}WE_{00}$$

$$= (E_{11} - E_{01})W(E_{11} - E_{10}) + (E_{01} - E_{00})W(E_{11} - E_{01}).$$

Every element of the last expression is nonnegative. Thus, the second term of the expanded (A.24) is nonnegative. These observations establish that the expression (A.24) is nonnegative if $1 < \theta \leq 2$.

Lastly, the term (A.25) is expanded as follows.

$$\frac{R}{\theta} (p_{c11}^{\theta-2} - p_{c01}^{\theta-2}) P_{\theta o1}^{-1} E_{o1} - \frac{R}{\theta} (p_{c10}^{\theta-2} - p_{c00}^{\theta-2}) P_{\theta o0}^{-1} E_{o0}$$

$$= \frac{R}{\theta} (p_{c11}^{\theta-2} - p_{c01}^{\theta-2} - p_{c10}^{\theta-2} + p_{c00}^{\theta-2}) P_{\theta o1}^{-1} E_{o1}$$

(A.26)

$$+ \frac{R}{\theta} (p_{c10}^{\theta-2} - p_{c00}^{\theta-2}) P_{\theta o0}^{-1} E_{o0}$$

(A.27)

$$+ \frac{R}{\theta} (p_{c11}^{\theta-2} - p_{c01}^{\theta-2}) P_{\theta o0}^{-1} (A_{o1} - A_{o0}) E_{o1}$$

(A.28)

$$+ \frac{R}{\theta} (p_{c10}^{\theta-2} - p_{c00}^{\theta-2}) P_{\theta o0}^{-1} (E_{o1} - E_{o0}).$$

(A.29)

Every element of the second (A.27), third (A.28) and fourth (A.29) terms is nonnegative if $1 < \theta \leq 2$ by Lemmas 1, 2, and (A.15). (Note $A_{o1}^{-1} - A_{o0}^{-1} = A_{o1}^{-1}(A_{o0} - A_{o1})A_{o0}^{-1}$).

To see the sign of the first term (A.26), we start by comparing $p_{c}^{1-\theta}$ in each case. From the definition of $p_{c}$ (A.9) we have:

$$p_{c11}^{1-\theta} - p_{c01}^{1-\theta} = [1'(E_{11} - E_{01})B_{11}^{-1}E_{11} + 1'E_{01}B_{11}^{-1}(B_{01} - B_{11})B_{01}^{-1}E_{11} + 1'E_{01}B_{01}^{-1}(E_{11} - E_{01})]w_{0}.$$  

$$p_{c10}^{1-\theta} - p_{c00}^{1-\theta} = [1'(E_{10} - E_{00})B_{10}^{-1}E_{10} + 1'E_{00}B_{10}^{-1}(B_{00} - B_{10})B_{00}^{-1}E_{10} + 1'E_{00}B_{00}^{-1}(E_{10} - E_{00})]w_{0}.$$  

By using the fact that $E_{11} - E_{01} = E_{10} - E_{00}$,

$$p_{c11}^{1-\theta} - p_{c01}^{1-\theta} - p_{c10}^{1-\theta} + p_{c00}^{1-\theta}$$

$$= 1'(E_{11} - E_{01})(B_{11}^{-1}E_{11} - B_{10}^{-1}E_{10})w_{0},$$

(A.30)

$$+ \{1'E_{01}B_{11}^{-1}(B_{01} - B_{11})B_{01}^{-1}E_{11} - 1'E_{00}B_{10}^{-1}(B_{00} - B_{10})B_{00}^{-1}E_{10}\}w_{0},$$

(A.31)

$$+ 1'(E_{01}B_{01}^{-1} - E_{00}B_{00}^{-1})(E_{11} - E_{01})w_{0}.$$  

(A.32)

The first term (A.30) is:

$$1'(E_{11} - E_{01})\{B_{11}^{-1}(B_{10} - B_{11})B_{10}^{-1}E_{11} + B_{10}^{-1}(E_{11} - E_{10})\}w_{0} \geq 0.$$
The last inequality is from (A.11).

The contents of the parentheses of the second term (A.31) can be expanded into the following expression.

\[ 1'(E_{01} - E_{00})B_{11}^{-1}(B_{01} - B_{11})B_{01}^{-1}E_{11} + 1'E_{00}B_{11}^{-1}(B_{10} - B_{11})B_{01}^{-1}(B_{01} - B_{11})B_{01}^{-1}E_{11} \]
\[ + 1'E_{00}B_{10}^{-1}(B_{01} - B_{11} - B_{00} + B_{10})B_{01}^{-1}E_{10} \]
\[ + E_{00}B_{10}^{-1}(B_{00} - B_{10})B_{01}^{-1}(B_{00} - B_{01})B_{00}^{-1}E_{10} + 1'E_{00}B_{10}^{-1}(B_{00} - B_{10})B_{00}^{-1}(E_{11} - E_{10}). \]

Every element in the first and third lines in the above expression is nonnegative by (A.11). The contents of the parentheses in the second line are transformed into the following expression by using the fact that \( E_{11} - E_{01} = E_{10} - E_{00}, \)

\[ \left( \frac{\theta - 1}{\theta} \right)^{\theta - 1} (E_{11} - E_{01} + E_{01} - E_{00})W(E_{11} - E_{10}). \]

All elements of this expression are nonnegative. Thus, the second term (A.31) is nonnegative.

The third term (A.32) is expanded into the following expression.

\[ 1'(E_{01} - E_{00})B_{01}^{-1} + E_{00}B_{01}^{-1}(B_{00} - B_{01})B_{00}^{-1}) \} (E_{11} - E_{01})w_0 \geq 0. \]

The last inequality comes from (A.11).

As all the three lines (A.30), (A.31), and (A.32) are positive or nonnegative we have:

\[ p_{c1}^{1-\theta} - p_{c0}^{1-\theta} > p_{c1}^{1-\theta} - p_{c0}^{1-\theta}. \]

We obtain the following expression by multiplying both sides by: \((p_{c10} + p_{c00})(p_{c11} + p_{c01}).\)

\[ (p_{c1}^{2-\theta} - p_{c0}^{2-\theta})(p_{c10} + p_{c00}) > (p_{c1}^{2-\theta} - p_{c0}^{2-\theta})(p_{c1} + p_{c01}). \]

We can rearrange this inequality into the following form by using Lemma 2.

\[ \frac{p_{c1}^{2-\theta} - p_{c0}^{2-\theta}}{p_{c10}^{2-\theta} - p_{c00}^{2-\theta}} < \frac{p_{c11} + p_{c01}}{p_{c10} + p_{c00}} < 1. \]

Thus,

\[ p_{c1}^{2-\theta} - p_{c0}^{2-\theta} < p_{c10}^{2-\theta} - p_{c00}^{2-\theta}. \]
Multiplying both sides by \((p_{c1} p_{c01} p_{c10} p_{c00})^{\theta - 2}\) and rearranging gives:

\[
(p_{c11} \theta - 2 - p_{c01} \theta - 2) > (p_{c01} p_{c11}) \left( \frac{p_{c01} p_{c11}}{p_{c00} p_{c10}} \right) (p_{c10} \theta - 2 - p_{c00} \theta - 2).
\]

\(\left( \frac{p_{c01} p_{c11}}{p_{c00} p_{c10}} \right)^{\theta - 2}\) is larger than 1 by Lemma 2 if \(1 < \theta \leq 2\). Therefore,

\[
p_{c11} \theta - 2 - p_{c01} \theta - 2 - p_{c10} \theta - 2 + p_{c00} \theta - 2 > 0.
\]

Thus, the first term (A.26) is positive if \(1 < \theta \leq 2\). These observations establish that the term (A.25) is positive. □

**Proposition 1** If \(1 < \theta \leq 2\), there exists a fixed point for the recursive map (A.20), i.e., a maximum of the monopolistic-bank profit.

**Proof.** The domain and region of the map (A.20) is defined on the complete lattice by Lemma 6. The map is monotonically increasing (order-preserving) by Lemma 7. Therefore, there exists a fixed point \(e^* = H(e^*)\) by Tarski’s fixed point theorem (Theorem 1 in Tarski, 1955). □

### A1.3 Comparative statics with respect to the demand influence coefficient

**Proposition 3** For a given set of operating firms \(S = \{i|e_i = 1, i \neq j\}\), the monopolistic bank is more likely to provide a loan to firm \(z\) with a larger demand influence coefficient \(v_z\) if the bank assumes that its decision does not change product prices \(p_i (i = 1, 2, \cdots, n)\).

**Proof.**

\[
\Pi(e_1) - \Pi(e_0) = R_{\theta} p_{c1}^{\theta - 2} 1' P_{\theta 1} A_1^{-1} e_1 - R_{\theta} p_{c1}^{\theta - 2} 1' P_{\theta 0} A_0^{-1} e_0
\]

\[
= R_{\theta} p_{c1}^{\theta - 2} 1' P_{\theta 1} A_1^{-1} (e_1 - e_0)
\]

(33)

\[
+ R_{\theta} p_{c1}^{\theta - 2} 1' (P_{\theta 1} - P_{\theta 0}) A_0^{-1} e_0
\]

(34)

\[
+ R_{\theta} p_{c1}^{\theta - 2} 1' (P_{\theta 1} - P_{\theta 0}) A_0^{-1} e_0
\]

(35)

\[
+ R_{\theta} (p_{c1}^{\theta - 2} - p_{c0}^{\theta - 2}) 1' P_{\theta 0} A_0^{-1} e_0.
\]

(36)

Under the assumptions about \(p_{c1} = p_{c0}\), (A.36) is zero. (A.35) is also zero under the assumption that \(p_{j1} = p_{j0} (j \neq z)\), because the \(z\)th column of \(A_0^{-1} e_0\) is zero.
The second term (A.34) can be transformed as follows.

\[
\frac{R_{p_{cl}}^\theta}{\theta} 1'P_{\theta 1} (A_{1}^{-1} - A_{0}^{-1}) e_0 \\
= \frac{R_{p_{cl}}^\theta}{\theta} 1'P_{\theta 1} (A_{1}^{-1}E_1 - A_{0}^{-1}E_0) e_0 \\
= \frac{R_{p_{cl}}^\theta}{\theta} (1'P_{\theta 1}A_{1}^{-1}E_1 - 1'P_{\theta 0}A_{0}^{-1}E_0) e_0 \\
= \frac{R_{p_{cl}}^\theta}{\theta} (v_{1}'P_{\theta 1} - v_{0}'P_{\theta 0}) e_0 \\
= \frac{R_{p_{cl}}^\theta}{\theta} (v_{1}' - v_{0}') P_{\theta 0} e_0 \\
= \frac{R_{p_{cl}}^\theta}{\theta} \sum_{j \neq z} (v_{j1} - v_{j0}) p_{j0}^{1-\theta}.
\]

We use the property that \( E_1 e_0 = E_0 e_0 = e_0 \) in deriving the second line, and the property \( P_{\theta 1} e_0 = P_{\theta 0} e_0 \) for the fifth line. The first term (A.33) can be transformed into the following expression.

\[
\frac{R_{p_{cl}}^\theta}{\theta} v_{z1} p_{z1}^{1-\theta}.
\]

Thus, under the assumptions about prices,

\[
\Pi(e_1) - \Pi(e_0) = \frac{R_{p_{cl}}^\theta}{\theta} \left\{ v_{z1} p_{z1}^{1-\theta} + \sum_{j \neq z} (v_{j1} - v_{j0}) p_{j0}^{1-\theta} \right\}.
\]

For a given \( v_{j1}, v_{j0} \) for \( j \neq z \), and \( p_{j1}, p_{j0} \) for all \( j \), the marginal increase of the monopolistic-bank profit is increasing in \( v_z \). Thus, the condition for the forbearance lending (A.19) is more likely to hold when \( v_z \) is higher for a given \( (1 + \rho)F_z \).

The first term (A.33) in the profit difference represents the propagation effect, or the positive externality of firm \( z \)'s operation. The second term (A.34) is the effect of the change in \( v_z p_{z1}^{1-\theta} \) \( (i \neq z) \). This part is nonnegative as shown in Lemma 3. The third term (A.35) captures the effect of the change in \( p_{i}^{1-\theta} \). As the variety of inputs increases, the product price decreases because of the love-of-variety assumption in the production function. This has a positive effect on firm profits. The fourth term (A.36) captures the effect of the change in CPI. This increases the relative price of each product, while it also increases the real income of households. The former decreases the demand, while the latter increases it. If \( \theta \) is sufficiently small, the latter effect exceeds the former, and so it increases firm profits.
A1.4 Equilibrium in the decentralized financial market of many infinitesimal investors

We assume that each infinitesimal investor in the decentralized financial market invests in firm $i$ if and only if:

$$\pi_i(e) - (1 + \rho)F_i \geq 0 \quad (A.37)$$

for a given $e$. Let us consider the following recursive map, such that:

$$\bar{e} = \mathcal{G}(e), \quad (A.38)$$

where the $i$th element of $\bar{e}$ ($n \times 1$) is given by

$$\bar{e}_i = \mathbb{1}[\pi_i(e) - (1 + \rho)F_i \geq 0]. \quad (A.39)$$

**Proposition 4** If $1 < \theta \leq 2$, there exists a fixed point $e^\dagger = \mathcal{G}(e^\dagger)$ for the recursive map (A.38), i.e., a competitive equilibrium in the decentralized financial market.

**Proof.** The domain and region of the map (A.38) is defined on the complete lattice by Lemma 6. The map is monotonically increasing (order-preserving) by Lemma 4. Therefore, there exists a fixed point $e^\dagger = \mathcal{G}(e^\dagger)$ by Tarski’s fixed point theorem (Theorem 1 in Tarski, 1955). □
Online Appendix 2: Numerical example

We present a numerical example to illustrate Proposition 2. In the example, there are four firms that supply inputs to each other in a network similar to Figure 1.

The first example illustrates network-motivated forbearance lending. We set

\[
W = \begin{pmatrix}
0 & .3 & .3 & .3 \\
.1 & 0 & .1 & 0 \\
.1 & 0 & 0 & .1 \\
.1 & .1 & 0 & 0
\end{pmatrix}, \quad w_0 = \begin{pmatrix}
.1 \\
.8 \\
.8 \\
.8
\end{pmatrix}, \quad F = \begin{pmatrix}
14 \\
10 \\
10 \\
10
\end{pmatrix},
\]

where \( F \) is the vector of the required amount of refinancing, \( \theta = 2, \rho = 0.1, p_0 = 1, \) and \( R = 100. \) Firm 1 depends on the inputs from the other firms more than the others, and its required refinancing is larger than that of the others. The other firms are symmetric. When all firms are refinanced, the demand influence coefficient of Firm 1 is 1.417 and those of the others are 1.046. The monopolistic bank can achieve the profit of 37.07 by refinancing all these firms and making them operate despite the loan to Firm 1 incurring a loss of 2.12.

In the case of infinitesimal investors in the decentralized financial market, the loss-making Firm 1 cannot be refinanced, whereas the others make profits and are refinanced. In this case, the demand influence coefficient of each operating firm is 1.0256 and the profit of each firm is 10.59. Thus, the total profit of the network is 31.78. This is smaller than the monopolistic-bank case in the previous paragraph, which is 37.07.

The marginal profit from forbearing loans to firm 1 is 37.07 − 31.78 = 5.29. The marginal cost is 2.12. The bank can benefit by forbearing the loss-making firm 1.
Online Appendix 3: The bias of the OLS estimator

We provide a justification for the use of the OLS estimator to estimate the spatial autoregressive models. We derive the formula for the bias of the OLS estimator and evaluate the bias in our sample. As a result, we find that the bias is likely to be small.\(^1\)

Recall that the model is:

\[ \Delta \mathbf{s} = \beta \mathbf{G} \Delta \mathbf{s} + \gamma_1 \mathbf{I}_{\text{Ind}} + \gamma_p \mathbf{Pre} + \epsilon, \]

where \( \beta = \gamma_0 I + \gamma_1 I_{\text{mfg}} + \gamma_2 I_{\text{ws}} \). We assume that \( \mathbf{Ind} \) and \( \mathbf{Pre} \) are exogenously given variables, so that they are uncorrelated with \( \epsilon \). In this section, we assume that \( \epsilon \) is homoskedastic. However, note that the standard errors reported in the main text are robust to heteroskedasticity.

The bias of the OLS estimator of \( \gamma = (\gamma_0, \gamma_1, \gamma_2)' \) can be derived in the following way. Let \( \tilde{\mathbf{X}} \) be the matrix of the residuals from the regression of \( \mathbf{G} \Delta \mathbf{s}, I_{\text{mfg}} \mathbf{G} \Delta \mathbf{s} \) and \( I_{\text{ws}} \mathbf{G} \Delta \mathbf{s} \) on \( \mathbf{Ind} \) and \( \mathbf{Pre} \). Thus, \( \tilde{\mathbf{X}} \) is an \( n \times 3 \) matrix. The OLS estimator of \( \gamma \) is:

\[ \hat{\gamma} = (\tilde{\mathbf{X}}' \tilde{\mathbf{X}})^{-1} \tilde{\mathbf{X}}' \Delta \mathbf{s} = \gamma + (\tilde{\mathbf{X}}' \tilde{\mathbf{X}})^{-1} \tilde{\mathbf{X}}' \Delta \epsilon. \]

As \( \mathbf{Ind} \) and \( \mathbf{Pre} \) are assumed to be exogenous, \( E(\tilde{\mathbf{X}}' \epsilon) \approx E((\mathbf{G} \Delta \mathbf{s}, I_{\text{mfg}} \mathbf{G} \Delta \mathbf{s}, I_{\text{ws}} \mathbf{G} \Delta \mathbf{s})' \epsilon) \). The reduced-form equation for \( \Delta \mathbf{s} \) is:

\[ \Delta \mathbf{s} = (I_n - \beta \mathbf{G})^{-1}(\gamma_1 \mathbf{I}_{\text{Ind}} + \gamma_p \mathbf{Pre} + \epsilon). \]

As all terms appearing on the right-hand side except \( \epsilon \) are assumed to be exogenous, we have:

\[ E((\mathbf{G} \Delta \mathbf{s})' \epsilon) = E(\epsilon'(\mathbf{G})'(I_n - \beta \mathbf{G})^{-1} \epsilon) = \sum_{k=0}^{\infty} E\left(\epsilon'(\mathbf{G})'(\beta \mathbf{G})^k \epsilon\right). \]

Similarly, we have:

\[ E((I_{\text{mfg}} \mathbf{G} \Delta \mathbf{s})' \epsilon) = \sum_{k=0}^{\infty} E\left(\epsilon'(I_{\text{mfg}} \mathbf{G})'(\beta \mathbf{G})^k \epsilon\right), \]

\[ E((I_{\text{ws}} \mathbf{G} \Delta \mathbf{s})' \epsilon) = \sum_{k=0}^{\infty} E\left(\epsilon'(I_{\text{ws}} \mathbf{G})'(\beta \mathbf{G})^k \epsilon\right). \]

\(^1\)Note that Lee (2002) also examines the conditions under which the OLS estimator is consistent. However, their argument is not applicable to our model.
Let

$$
B = \left( \begin{array}{c}
\sum_{k=0}^{\infty} E \left( \epsilon' (GM)'(\beta GM)^k \epsilon \right) \\
\sum_{k=0}^{\infty} E \left( \epsilon' (I_{mfg}GM)'(\beta GM)^k \epsilon \right) \\
\sum_{k=0}^{\infty} E \left( \epsilon' (I_{ws}GM)'(\beta GM)^k \epsilon \right)
\end{array} \right)
$$

The asymptotic bias of the OLS estimator is:

$$
\left( \frac{1}{N} \lim_{N \to \infty} \bar{X}' \bar{X} \right)^{-1} \frac{1}{N} \lim_{N \to \infty} \frac{1}{N} B.
$$

The bias of the OLS estimator may be numerically evaluated. The value of $\bar{X}' \bar{X}$ can be computed from the data. Let $g_{0ii}(k)$ be the $i$-th element on the main diagonal of $(GM)'(\beta GM)^k$. Similarly, let $g_{1ii}(k)$ and $g_{2ii}(k)$ be the $i$-th elements on the main diagonals of $(I_{mfg}GM)'(\beta GM)^k$ and $(I_{ws}GM)'(\beta GM)^k$, respectively. Assume that $\epsilon$ is homoskedastic with variance $\sigma^2$. Noting also that the elements of $\epsilon$ are uncorrelated across $i$, we have:

$$
\sum_{k=0}^{\infty} E \left( \epsilon' (GM)'(\beta GM)^k \epsilon \right) = \sigma^2 \sum_{k=0}^{\infty} \sum_{i=1}^{n} g_{0ii}(k),
$$

$$
\sum_{k=0}^{\infty} E \left( \epsilon' (I_{mfg}GM)'(\beta GM)^k \epsilon \right) = \sigma^2 \sum_{k=0}^{\infty} \sum_{i=1}^{n} g_{1ii}(k),
$$

$$
\sum_{k=0}^{\infty} E \left( \epsilon' (I_{ws}GM)'(\beta GM)^k \epsilon \right) = \sigma^2 \sum_{k=0}^{\infty} \sum_{i=1}^{n} g_{2ii}(k).
$$

As $g_{0ii}(1) = 0$ for $l = 0, 1, 2$ by the definition of $G$ and $M$ being a diagonal matrix, the value at $k = 0$ does not contribute to the sum. Note that $g_{0ii}(k)$ can be computed from the data once we have the values of $\sigma^2$ and $\gamma$. We then compute the bias using the formula:

$$
\left( \bar{X}' \bar{X} \right)^{-1} \sigma^2 \sum_{k=1}^{\infty} \sum_{i=1}^{n} \begin{pmatrix} g_{0ii}(k) \\ g_{1ii}(k) \\ g_{2ii}(k) \end{pmatrix}.
$$

We compute an approximated value of the bias in the data and find that the bias is likely to be small. To do so, we use the OLS estimator of $\beta$ and the estimate $\sigma^2$ from the OLS estimation to evaluate: $\sigma^2 \sum_{k=0}^{\infty} \sum_{i=1}^{n} g_{lii}(k)$ for $l = 0, 1, 2$. The infinite sum is truncated at $k = 2$. This truncation is justified because the value of $\beta$ is small (recall that the OLS estimate is at most 0.0103 for a seller in the wholesale sector, and $M$ is a diagonal matrix with elements less than one).

The bias is listed in Table A.1. The ratios of the bias over the estimated coefficients are less than 1% in for 2005 and 2013. The ratios are somewhat higher for 2010, but they are still at most 2.5%. Thus, the bias is negligible and so we can rely on the OLS estimator. Here, we use the
Table A.1: Bias of the spatial autoregressive model

<table>
<thead>
<tr>
<th></th>
<th>2005 series</th>
<th>2010 series</th>
<th>2013 series</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_0$</td>
<td>9.52E-06</td>
<td>6.42E-05</td>
<td>1.82E-05</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>-6.59E-06</td>
<td>-5.34E-05</td>
<td>-1.53E-05</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>4.07E-05</td>
<td>1.58E-04</td>
<td>4.19E-05</td>
</tr>
</tbody>
</table>

values of $\sigma^2$ and $\beta$ from the OLS estimation. The computed bias would be small even if different (reasonable) values of $\sigma^2$ and $\beta$ were used.
Online Appendix 4: Standard errors when regressors are generated

In this appendix, we provide the formula for the standard errors for the forbearance lending probability regression, taking into account the fact that the key regressor (demand influence coefficient) is estimated. We first briefly review the effect of a generated regressor in general linear regression frameworks. We then provide the formula for the specific case in which the generated regressor is a demand influence coefficient.

General results

We first examine the effect of a generated regressor in linear regression models. In particular, we explain how to modify the asymptotic variance estimator for the OLS estimator. Note that this is a rather standard exercise in econometrics.

We consider the following linear regression model:

\[ y_i = x_i' \beta + u_i. \]

However, \( x_i \) is not directly observed. We know that \( x_i \) can be written as \( x_i = x_i(\gamma_0) \), where the function \( x_i(\cdot) \) is known and \( \gamma_0 \) is estimable. Let \( \hat{\gamma} \) be an estimate of \( \gamma_0 \). We thus use a generated regressor \( \hat{x}_i = x_i(\hat{\gamma}) \) instead of \( x_i \). The OLS estimator of \( \beta \) with \( \hat{x}_i \) is:

\[ \hat{\beta} = \left( \sum_{i=1}^{N} \hat{x}_i \hat{x}_i \right)^{-1} \sum_{i=1}^{N} \hat{x}_i y_i. \]

We make the following assumptions about the generated regressor. We assume that \( x_i(\cdot) \) is differentiable and use the mean value theorem so that:

\[ x_i - \hat{x}_i = -\frac{\partial x_i(\bar{\gamma})}{\partial \gamma}(\hat{\gamma} - \gamma_0), \]

where \( \bar{\gamma} \) is between \( \hat{\gamma} \) and \( \gamma_0 \). We also assume that \( \hat{\gamma} \) is asymptotically linear:

\[ \hat{\gamma} - \gamma = \frac{1}{N^*} \sum_{j=1}^{N^*} \phi_j, \]

where \( \phi_j \) has mean zero and finite variance. We note that here we allow \( \gamma \) to be estimated from a different sample than the sample used in the estimation of \( \beta \). These samples are allowed to be
overlapping or disjointed. Let \( N^* \) denote the sample size of the sample used in the estimation of \( \gamma \); \( N^* \) may be different from \( N \). We assume that \( \lim_{N,N^* \to \infty} (N/N^*) = \kappa < \infty \).

The asymptotic distribution of the OLS estimator \( \hat{\beta} \) depends on the estimation error in \( \hat{\gamma} \). We observe that:

\[
y_i = \hat{x}_i' \beta + (x_i - \hat{x}_i)' \beta + u_i.
\]

We therefore have the following expansion of \( \hat{\beta} \):

\[
\hat{\beta} = \beta + \left( \sum_{i=1}^{N} \hat{x}_i \hat{x}_i \right)^{-1} \left( \sum_{i=1}^{N} \hat{x}_i u_i - \sum_{i=1}^{N} \hat{x}_i \beta' \frac{\partial x_i}{\partial \gamma} (\gamma) \sum_{j=1}^{N^*} \phi_j \right).
\]

The second term on the right-hand side yields the usual asymptotic distribution of the OLS estimator. The third term depends on the estimation error in \( \hat{\gamma} \). Using the assumption on \( \hat{x}_i \), we have:

\[
\hat{\beta} - \beta = \left( \frac{1}{N} \sum_{i=1}^{N} \hat{x}_i \hat{x}_i \right)^{-1} \left( \frac{1}{N} \sum_{i=1}^{N} \hat{x}_i u_i - \frac{1}{N} \sum_{i=1}^{N} \hat{x}_i \beta' \frac{\partial x_i}{\partial \gamma} (\gamma) \sum_{j=1}^{N^*} \phi_j \right).
\]

Let

\[
A = E \left( x_i \beta' \frac{\partial x_i}{\partial \gamma} (\gamma_0) \right),
\]

and

\[
B = \operatorname{plim}_{N \to \infty} \frac{1}{N} \sum_i x_i u_i \phi_i',
\]

where \( \sum_i \) in \( B \) is taken over the set of observations that appear in both the sample used for the estimation \( \gamma \) and that for \( \beta \). From the expansion of \( \hat{\beta} \), it is easy to derive the asymptotic distribution of \( \hat{\beta} \), which is:

\[
\sqrt{N}(\hat{\beta} - \beta) \to_d N(0, V),
\]

where

\[
V = (E(x_i x_i'))^{-1} \left( E(u_i^2 x_i x_i') - B A' - A B' + A \kappa E(\phi_j \phi_j') A' \right) (E(x_i x_i'))^{-1}.
\]

We note that when \( \kappa = 0 \) (i.e., \( N^* \) is much larger than \( N \)), the estimation error in the generated regressor does not affect the asymptotic distribution of \( \hat{\beta} \).
The asymptotic variance can be estimated by:

\[
\hat{V} = \left( \frac{1}{N} \sum_{i=1}^{N} \hat{x}_i \hat{x}_i \right)^{-1} \left( \frac{1}{N} \sum_{i=1}^{N} \hat{u}_i^2 \hat{x}_i \hat{x}_i' - \hat{B} \hat{A}' - \hat{A} \hat{B}' + \hat{A} \frac{N}{N^*} \frac{1}{N^*} \sum_{j=1}^{N^*} \hat{\phi}_j \hat{\phi}_j' \hat{A}' \right) \left( \frac{1}{N} \sum_{i=1}^{N} \hat{x}_i \hat{x}_i \right)^{-1},
\]

where

\[
\hat{A} = \frac{1}{N} \sum_{i=1}^{N} \hat{x}_i \hat{\beta}' \frac{\partial x_i}{\partial \gamma}(\hat{\gamma}),
\]

and

\[
\hat{B} = \frac{1}{N} \sum_{i} \hat{x}_i \hat{u}_i \hat{\phi}_i'.
\]

In this, \( \hat{\phi}_i \) is an estimate of \( \phi_i \).

**When the demand influence coefficient is generated**

In our application, the generated regressor is the value of the demand influence coefficient. This subsection provides the formula for \( \phi_j \) and \( x_i(\cdot) \) in our case.

Recall that the demand influence coefficient is computed from the OLS estimate of the model:

\[
\Delta s = \beta GM \Delta s + \gamma'_I \text{Ind} + \gamma'_P \text{Pre} + \epsilon,
\]

where \( \beta \equiv \gamma_0 I + \gamma_1 I_{\text{mfg}} + \gamma_2 I_{\text{ws}} \). Thus, in our setting, \( \gamma = (\gamma_0, \gamma_1, \gamma_2)' \). As \( x_i \) is the logarithm of the demand influence coefficient, we have \( x_i(\gamma) = \log((\sum_{k=0}^{\infty} (\beta GM)^k)_i) \), where \( (a)_i \) denotes the element of \( a \) corresponding to the \( i \)-th firm.

The formula for \( \phi_j \) and \( \partial x_i/(\partial \gamma) \) can be derived easily in our setting. Let \( \bar{X} \) be the matrix of the residuals from the regression of \( GM \Delta s, I_{\text{mfg}}GM \Delta s \) and \( I_{\text{ws}}GM \Delta s \) on \( \text{Ind} \) and \( \text{Pre} \). The OLS estimator of \( \gamma \) is:

\[
\hat{\gamma} = \left( \bar{X}' \bar{X} \right)^{-1} \bar{X}' \Delta s = \gamma + \left( \bar{X}' \bar{X} \right)^{-1} \bar{X}' \Delta \epsilon.
\]

Therefore, the formula for \( \phi_j \) is:

\[
\phi_j = \left( \frac{1}{N} \bar{X}' \bar{X} \right)^{-1} \bar{x}_j \epsilon_j,
\]

where \( \bar{x}_j \) is the \( j \)-th row of \( \bar{X} \). The formula for \( \partial x_i/(\partial \gamma) \) can be computed directly by taking the derivative. This gives:

\[
\frac{\partial x_i}{\partial \gamma}(\gamma) = \frac{1}{(\sum_{k=0}^{\infty} (\beta GM)^k)_i} \sum_{k=1}^{\infty} k \left( \frac{(1' \beta GM)^{k-1} GM_i}{(1' \beta GM)^{k-1} I_{\text{mfg}} GM_i} \right) \left( \frac{(1' \beta GM)^{k-1} I_{\text{ws}} GM_i}{(1' \beta GM)^{k-1} I_{\text{ws}} GM_i} \right).
\]
Online Appendix 5: Additional empirical results

A5.1 Result from the annual cross-section data

We estimate the baseline model (28) with the annual cross-section data of distressed firms in each year as a robustness check. We assign lagged influence coefficients for each annual set of financial data, i.e., $v_{2005}$ for the observations in 2008–10, $v_{2010}$ for those in 2011–13, and $v_{2013}$ for those in 2014–16. The set of control variables is almost the same as in the time-aggregated estimation except that we refrain from including the main-bank fixed effects because of the small sample size. The shortcoming of this method is that the number of observations in each cross-section is much smaller and this makes it difficult to introduce the bank fixed effects.

Table A.2 shows that the demand influence coefficient is highly positive in 2010 and 2011, right after the global financial crisis. The coefficient in 2010 is statistically significant at the 1% level. The estimated coefficient implies that a 0.4 percentage point increase from one in the demand influence coefficient increases the probability of obtaining forbearance by 2.4 percentage points in 2010 and 2.1 percentage points in 2011. This is economically significant because the average forbearance probability is 12.7%. The estimated coefficient of the demand influence coefficient in the period from 2008 to 2013 is positive and its value is relatively high. However, many of the coefficients of the demand influence coefficients are not statistically significant because of the high standard errors, probably because of the small sample size. The estimated coefficient becomes smaller and insignificant in the years after 2014. We assume that this is due to QQE and the economic recovery since April 2013 as mentioned before.

A5.2 Sample selection model

So far, we have used the dataset comprising distressed firms only to test our hypothesis that a distressed firm with a higher demand influence coefficient has a higher probability of obtaining a forbearance loan. A potential problem is sample selection bias due to dropping firms that are not in financial distress. To address this potential problem, we apply the Heckit procedure (Heckman, 1979).

In the first stage, we estimate the probability of $Distress_{it} = 1$ using a probit model. We include all the independent variables used in the baseline estimation for the forbearance loan probability
Table A.2: Result from the annual cross-section data
(Notes) Estimated by OLS. The dependent variable is forbear (0,1). Each row is the cross-section regression for a particular year. The estimated coefficients of the control variables, industry dummies, regional dummies, and the constant term are omitted from the report. The cross-section data for the accounting year ending in each period are used. The sample includes firms that have been distressed (distress = 1) in each year. v for 2005 is in rows 1–3, for 2010 is in rows 4–6 and for 2013 is in rows 7–9, respectively. The set of controls includes those in Table 9. The set of industry dummies is based on the large classification listed in Table 4 (seven dummies). The region dummies are the same as those in Table 9. Main-bank-clustered standard errors of the estimated coefficient of ln(v) in parentheses. *** p<0.01, ** p<0.05, * p<0.1 (two-tail t-test).

<table>
<thead>
<tr>
<th>Year</th>
<th>Coef. of ln(v)</th>
<th>(s.e.)</th>
<th>N</th>
<th>Adj. $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008</td>
<td>3.508</td>
<td>(3.562)</td>
<td>749</td>
<td>0.023</td>
</tr>
<tr>
<td>2009</td>
<td>2.005</td>
<td>(3.725)</td>
<td>881</td>
<td>0.035</td>
</tr>
<tr>
<td>2010</td>
<td>6.008***</td>
<td>(2.211)</td>
<td>1,115</td>
<td>0.046</td>
</tr>
<tr>
<td>2011</td>
<td>5.347</td>
<td>(3.937)</td>
<td>925</td>
<td>0.030</td>
</tr>
<tr>
<td>2012</td>
<td>3.536</td>
<td>(4.117)</td>
<td>808</td>
<td>0.007</td>
</tr>
<tr>
<td>2013</td>
<td>4.364</td>
<td>(3.885)</td>
<td>818</td>
<td>0.012</td>
</tr>
<tr>
<td>2014</td>
<td>0.215</td>
<td>(1.614)</td>
<td>705</td>
<td>0.008</td>
</tr>
<tr>
<td>2015</td>
<td>-0.111</td>
<td>(1.163)</td>
<td>572</td>
<td>0.044</td>
</tr>
<tr>
<td>2016</td>
<td>1.097</td>
<td>(1.840)</td>
<td>431</td>
<td>0.024</td>
</tr>
</tbody>
</table>

in the previous section. In the second stage, we regress the forbearance loan dummy on the set of the independent variables that we used in the baseline estimation and the inverse Mills ratio, which is calculated by the predicted value in the first stage. We can estimate the model including main-bank fixed effect in the second stage using the procedure proposed by Wooldridge (1995).

The estimated coefficients with the time-aggregated dataset are listed in Table A.3. The estimated coefficients from the first stage probit for the distress probability are listed in panel (a). The demand influence coefficients are positive in all periods. This suggests the possibility of moral hazard because of the anticipated network-motivated forbearance lending. However, they are not statistically significant. The firms with higher interest coverage ratio $sale/R^*$, higher sales growth $\Delta real sale$, larger tangible assets $collateral$, and larger total assets $asset$ are less likely to face financial distress. Older firms and firms with larger numbers of employees are more likely to be distressed. The negative and significant coefficient of the number of lenders $#banks$ suggests that the variable works as a proxy for firm size or creditworthiness.

Panel (b) shows the estimated coefficients in the second stage. The coefficient of the inverse
Mills ratio is positive and significant. This implies that the sample selection bias does matter in our baseline estimation. However, the estimated coefficient and its statistical significance with respect to the demand influence coefficient are not affected by it.

Table A.4 presents the estimated coefficients of important variables from the sample selection model with the year-by-year repeated cross-section data. The coefficient of the inverse Mills ratio is positive and statistically significant in all years after 2011. However, the estimated coefficients of the demand influence coefficient in the second stage regression are only marginally different. The influence coefficient in the first stage is positive in most years, but statistically significant only in 2008 and 2015.

Thus, the estimations by the sample selection model support the baseline results consistently. In addition, they provide evidence suggestive of possible moral hazard by firms who may expect network-motivated forbearance lending.
Table A.3: Sample selection model with time-aggregated data
(Notes) Estimated by the sample selection model with the main-bank fixed effect. The first stage is the probit model for the distress probability. The dependent variable in the second stage is forbear (0,1). Time-aggregated data for each period are used. The control variables are those corresponding to the first year of each period. Column 1 uses \( v \) for 2005, column 2 for 2010, and column 3 for 2013. The year dummy is a dummy indicating the first year of distress (two dummies). The industry dummy is based on the detailed classification listed in Table 4 (seven dummies) in the first stage, whereas it is based on the two-digit classification in the second stage. Region dummies are the same as in Table 9. The estimates of the constant term and the coefficients of these dummies are not reported. The contents of the parentheses are standard errors of the estimated coefficients, which are clustered in the second stage by main bank. *** \( p<0.01 \), ** \( p<0.05 \), * \( p<0.1 \) (two-tail t-test).

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1) ( v=v_{2005} ) Period: 2008-10</th>
<th>(2) ( v=v_{2010} ) 2011-13</th>
<th>(3) ( v=v_{2013} ) 2014-16</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln(( v ))</td>
<td>2.279 (1.600)</td>
<td>3.598 (2.671)</td>
<td>2.473 (1.752)</td>
</tr>
<tr>
<td>ln(sale/( R^* ))</td>
<td>-0.418*** (0.024)</td>
<td>-0.473*** (0.029)</td>
<td>-0.428*** (0.035)</td>
</tr>
<tr>
<td>ln(1+( \Delta ) real sale)</td>
<td>-1.317*** (0.102)</td>
<td>-0.923*** (0.113)</td>
<td>-1.214*** (0.163)</td>
</tr>
<tr>
<td>ln(1 + collateral(t-1))</td>
<td>-0.525*** (0.070)</td>
<td>-0.589*** (0.085)</td>
<td>-0.269*** (0.098)</td>
</tr>
<tr>
<td>ln(age)</td>
<td>0.111** (0.050)</td>
<td>0.221*** (0.058)</td>
<td>0.254*** (0.069)</td>
</tr>
<tr>
<td>ln(#employee(t-1))</td>
<td>0.074** (0.030)</td>
<td>0.098*** (0.035)</td>
<td>0.146*** (0.044)</td>
</tr>
<tr>
<td>ln(asset(t-1))</td>
<td>-0.270*** (0.025)</td>
<td>-0.373*** (0.030)</td>
<td>-0.340*** (0.038)</td>
</tr>
<tr>
<td>leverage(t-1)</td>
<td>3.777*** (0.145)</td>
<td>4.280*** (0.161)</td>
<td>4.999*** (0.204)</td>
</tr>
<tr>
<td>switch</td>
<td>-0.073 (0.118)</td>
<td>0.036 (0.161)</td>
<td>-0.034 (0.203)</td>
</tr>
<tr>
<td>#bank</td>
<td>-0.039*** (0.010)</td>
<td>-0.053*** (0.012)</td>
<td>-0.053*** (0.014)</td>
</tr>
<tr>
<td>share</td>
<td>-0.224 (0.187)</td>
<td>-0.024 (0.216)</td>
<td>-0.104 (0.262)</td>
</tr>
<tr>
<td>hi</td>
<td>-0.155 (0.252)</td>
<td>-0.278 (0.277)</td>
<td>0.172 (0.355)</td>
</tr>
<tr>
<td>ln(MB asset)</td>
<td>0.024* (0.014)</td>
<td>0.009 (0.015)</td>
<td>0.004 (0.018)</td>
</tr>
<tr>
<td>MB cap ratio</td>
<td>-1.193 (1.183)</td>
<td>-0.179 (1.793)</td>
<td>-1.135 (2.206)</td>
</tr>
</tbody>
</table>

| Industry dummy                        | yes                                | yes                            | yes                            |
| Year dummy                            | yes                                | yes                            | yes                            |
| Region dummy                          | yes                                | yes                            | yes                            |
Table A.3: (cont.)

(b) Second stage: forbearance lending probability

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$v = v_{2005}$</td>
<td>$v_{2010}$</td>
<td>$v_{2013}$</td>
</tr>
<tr>
<td>ln($v$)</td>
<td>3.476</td>
<td>8.370***</td>
<td>0.777</td>
</tr>
<tr>
<td></td>
<td>(2.897)</td>
<td>(2.856)</td>
<td>(1.590)</td>
</tr>
<tr>
<td>ln(sale/$R^*$)</td>
<td>-0.029**</td>
<td>-0.061***</td>
<td>-0.027</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.016)</td>
<td>(0.021)</td>
</tr>
<tr>
<td>ln(1+Δ real sale)</td>
<td>-0.065</td>
<td>0.026</td>
<td>-0.056</td>
</tr>
<tr>
<td></td>
<td>(0.057)</td>
<td>(0.055)</td>
<td>(0.073)</td>
</tr>
<tr>
<td>ln(1 + collateral(t-1))</td>
<td>-0.039</td>
<td>-0.093*</td>
<td>0.037</td>
</tr>
<tr>
<td></td>
<td>(0.040)</td>
<td>(0.053)</td>
<td>(0.068)</td>
</tr>
<tr>
<td>ln(age)</td>
<td>-0.014</td>
<td>0.017</td>
<td>0.019</td>
</tr>
<tr>
<td></td>
<td>(0.024)</td>
<td>(0.025)</td>
<td>(0.037)</td>
</tr>
<tr>
<td>ln(#employee(t-1))</td>
<td>-0.021</td>
<td>0.007</td>
<td>0.022</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td>(0.020)</td>
<td>(0.028)</td>
</tr>
<tr>
<td>ln(asset(t-1))</td>
<td>-0.006</td>
<td>-0.043***</td>
<td>-0.051**</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.015)</td>
<td>(0.023)</td>
</tr>
<tr>
<td>leverage(t-1)</td>
<td>0.091***</td>
<td>0.036*</td>
<td>0.030</td>
</tr>
<tr>
<td></td>
<td>(0.031)</td>
<td>(0.019)</td>
<td>(0.021)</td>
</tr>
<tr>
<td>switch</td>
<td>-0.019</td>
<td>0.163*</td>
<td>0.081</td>
</tr>
<tr>
<td></td>
<td>(0.040)</td>
<td>(0.093)</td>
<td>(0.123)</td>
</tr>
<tr>
<td>#bank</td>
<td>-0.009</td>
<td>-0.017**</td>
<td>-0.016**</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.007)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>share</td>
<td>-0.068</td>
<td>-0.327*</td>
<td>-0.371*</td>
</tr>
<tr>
<td></td>
<td>(0.133)</td>
<td>(0.187)</td>
<td>(0.213)</td>
</tr>
<tr>
<td>hi</td>
<td>-0.050</td>
<td>-0.098</td>
<td>0.112</td>
</tr>
<tr>
<td></td>
<td>(0.173)</td>
<td>(0.154)</td>
<td>(0.184)</td>
</tr>
<tr>
<td>ln(MB asset)</td>
<td>0.380</td>
<td>0.575**</td>
<td>-2.284*</td>
</tr>
<tr>
<td></td>
<td>(0.567)</td>
<td>(0.288)</td>
<td>(1.283)</td>
</tr>
<tr>
<td>MB cap ratio</td>
<td>-0.786</td>
<td>7.426</td>
<td>-1.702</td>
</tr>
<tr>
<td></td>
<td>(1.247)</td>
<td>(5.032)</td>
<td>(16.355)</td>
</tr>
<tr>
<td>Inverse Mills Ratio</td>
<td>0.069***</td>
<td>0.103***</td>
<td>0.096***</td>
</tr>
<tr>
<td></td>
<td>(0.030)</td>
<td>(0.022)</td>
<td>(0.026)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>N</th>
<th>10,638</th>
<th>9,050</th>
<th>8,584</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adj. $R^2$ in 2nd stage</td>
<td>0.083</td>
<td>0.066</td>
<td>0.066</td>
</tr>
<tr>
<td>Bank FE</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Industry (2 digit) dummy</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Year dummy</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Region dummy</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>#group(banks)</td>
<td>253</td>
<td>243</td>
<td>208</td>
</tr>
</tbody>
</table>
Table A.4: Sample selection model with year-by-year data
(Notes) Estimated by the sample selection model. The first stage is the probit model for the distress probability. The dependent variable in the second stage is forbear (0,1). Each row represents a separate year. The estimated coefficients of the control variables, industry dummies, regional dummies, and the constant term are omitted from the report. Rows 1–3 represent v for 2005, rows 4–6 for 2010, and rows 7–9 for 2013. The set of independent variables is the same as in those in Table A.3 except that the industry dummies in the second stage are based on the classification in Table 4 (seven dummies). Main-bank-clustered standard errors are reported for the second stage regression. *** p<0.01, ** p<0.05, * p<0.1 (two-tail t-test).

<table>
<thead>
<tr>
<th>Year</th>
<th>Coef. of ln(v) for forbearance prob. (clustered s.e.)</th>
<th>Coef. of inverse-Mills ratio (clustered s.e.)</th>
<th>Coef. of ln(v) for distress prob. (s.e.)</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008</td>
<td>3.541</td>
<td>0.011</td>
<td>5.836**</td>
<td>7,823</td>
</tr>
<tr>
<td></td>
<td>(2.483)</td>
<td>(0.028)</td>
<td>(2.705)</td>
<td></td>
</tr>
<tr>
<td>2009</td>
<td>2.413</td>
<td>0.024</td>
<td>-0.156</td>
<td>7,806</td>
</tr>
<tr>
<td></td>
<td>(4.414)</td>
<td>(0.028)</td>
<td>(5.122)</td>
<td></td>
</tr>
<tr>
<td>2010</td>
<td>5.964**</td>
<td>0.041</td>
<td>1.961</td>
<td>8,436</td>
</tr>
<tr>
<td></td>
<td>(2.376)</td>
<td>(0.033)</td>
<td>(1.762)</td>
<td></td>
</tr>
<tr>
<td>2011</td>
<td>5.366</td>
<td>0.071***</td>
<td>3.607</td>
<td>7,418</td>
</tr>
<tr>
<td></td>
<td>(3.578)</td>
<td>(0.023)</td>
<td>(3.444)</td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>3.881</td>
<td>0.095***</td>
<td>4.531</td>
<td>7,299</td>
</tr>
<tr>
<td></td>
<td>(3.832)</td>
<td>(0.023)</td>
<td>(2.881)</td>
<td></td>
</tr>
<tr>
<td>2013</td>
<td>4.843</td>
<td>0.067***</td>
<td>3.744</td>
<td>7,398</td>
</tr>
<tr>
<td></td>
<td>(3.885)</td>
<td>(0.025)</td>
<td>(3.679)</td>
<td></td>
</tr>
<tr>
<td>2014</td>
<td>0.820</td>
<td>0.079***</td>
<td>2.229</td>
<td>7,676</td>
</tr>
<tr>
<td></td>
<td>(2.396)</td>
<td>(0.024)</td>
<td>(2.973)</td>
<td></td>
</tr>
<tr>
<td>2015</td>
<td>-0.045</td>
<td>0.087***</td>
<td>3.683*</td>
<td>7,200</td>
</tr>
<tr>
<td></td>
<td>(1.494)</td>
<td>(0.022)</td>
<td>(2.091)</td>
<td></td>
</tr>
<tr>
<td>2016</td>
<td>1.155</td>
<td>0.051**</td>
<td>3.005</td>
<td>6,188</td>
</tr>
<tr>
<td></td>
<td>(2.160)</td>
<td>(0.022)</td>
<td>(2.892)</td>
<td></td>
</tr>
</tbody>
</table>
References


