Specific Human Capital Investments in Weak Skills under Uncertain Terms of Trade

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Abstract

Specialization and division of labor are the sources of high productivity in the modern society. Workers may face the choice between general and specific human capital investments. Given that the individual agents face uncertainty in the terms of trade, what are the optimal strategy for heterogeneous individual agents in human capital investments? In the absense of insurance markets, general investments give an option value for the changes in the environment. When workers are born heteregoneous in two-dimension skills used in different sectors, do they invest in what they were good at when they were born? Do they invest in general human capital? Or do they invest in what they were not originally good at? I find that some workers choose to invest in their weakest skills via specific human capital investments, provided that the size of risk is big and that the parameter of relative risk aversion is larger than one.

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1 Introduction

In the previous paper by the author, I explored the problem of occupational choice by individual agents with multiple skills. In that paper, skills or talents of individuals are assumed to be innately given. Given the fixed level of skills, the choice of agents is in which sector they work given the environment. This assumption of fixed skill levels was made because of analytical convenience and of the nature of the problems at hand, namely, the analysis on the occupational choice of each agent, gains or losses for each agent from trade liberalization and the exploration of the reasons for difficulty of compensation.

Of course this assumption was somewhat restrictive because, in the real world, people try to change the level of their skills over time. In fact, it is widely known that human capital investment activities such as education and on-the-job training are prevalent[1]. The purpose of this paper is to endogenize the skills of individual agents and to analyze the two period model of occupational choice with human capital investments. I will investigate the individuals' incentives to invest in their multi-dimensional skills. In particular, we are interested in whether the heterogeneous individuals want to invest in their specific skills or in general skills in the presence of uncertainty caused by possible trade liberalization.

The importance of specialization has been widely recognized from long time ago. (See, for example, the very first chapter in *Wealth of Nations* by Adam Smith 1776.) The division of labor and specialization are the key sources of higher productivity of the modern world. The division of labor may be the result of endowed differences in individual comparative advantages as in Rosen (1978). Specialization, however, may also be a result of specialized human capital investments as in Rosen (1983).

There is a paper by Findlay and Kierzkowski (1983) that looked at the problem of human capital investments with international trade. However, the paper dealt with one-dimensional skills. So there was no question about specialization versus generalization in skill investments. In this paper, I will try to work on the dichotomy of specialized skill investments versus general skill investments within the set up of open economy. When there is no uncertainty about which sector individuals are employed, individual agents have incentives to specialize. (Murphy, 1986, p.37)

When the world is uncertain, ex-ante specific human capital investments may be a risky strategy because specialization makes individual agents inflexible as factors of production ex post. To borrow the expressions from Gene M. Grossman and Carl Shapiro (1982), the investment toward specialization is like reducing "the degree of intersectoral mobility" in later period. A general (as opposed to specific) human capital investment is a form of self-insurance if the insurance markets are absent. Murphy (1986, section 5.2) also looked at the case where there is uncertainty about which sector will become favorable. He also concluded that less specialization exposes workers to less risk. Both Grossman

[1] See Becker (1993).

and Shapiro (1982) and Murphy (1986) examined the individuals' incentives to general versus specific human capital investment where every agent is *identical* before he conducts an investment in human capital skill(s). So, what happens to these results if we start from the situation where agents are heterogeneous in their innate comparative advantage before they conduct human capital investments? In this paper, I would like to analyze incentives of individual agents to invest in human capital skills when the role of endowed differences is important.

By looking at the directional incentives to invest in human capital skills for the model of occupational choice, I could address the following set of research questions: What is the role of endowed differences of individuals in human capital investments? Do people with heterogeneous skills specialize or generalize their innate skills when the world is uncertain? Do people invest in their innately strong skills? Do people specialize in what they were good at when they were born? If so, under what conditions? Do agents invest their time (and money) to enhance their skills in the socially inefficient way? Do they go to schools in which they will learn something they are innately poor at? Will there be the same amount of job-switching individuals if we allow dynamic development of human capitals? All these questions were unanswered by the previous works in this field since they assumed the identical agents before the decision of human capital investments. Although there is a large amount of literature on human capital investments to date, to my knowledge there does not exist previous works that start from multi-dimensional heterogeneity. I will introduce such a model in the next section.

I find that, in general, many workers tend to specialize in their strong skills even under uncertainty. This is true especially for the workers who have very strong comparative advantage in one sector. For example, if you are born to be a good singer and you did not do well in high school general study classes, then you will choose to go to the school of music to enhance your strong innate skills in terms of comparative advantage. For the workers who are born to be good in both sectors, then the problem becomes very difficult. Some workers try to invest in general skills so that they can choose their jobs in a flexible manner once the uncertainty resolves.

In the case where workers are very risk averse, I find that some workers decide to conduct specialized investments in their weak skills rather than the strong ones. This happens to those who have skills whose levels are similar enough so that their degree of comparative advantage is not very strong. For example, consider a person A who has high talent in music but also with good IQ, he might choose to give up music to go to college even if his talent in music is actually the stronger one of his skills. On the other hand, suppose there is another person, B, who has similar classes of skills as the person A. Suppose also that B's music skills is not as good as A's and that B's IQ is slightly higher than A's. Therefore, if there is no uncertainty, then A chooses to be a musician and B gives up to be a musician and focuses on study. However, given the setup of the model with uncertainty, A goes to college by giving up becoming a musician and B goes to the college of music to be a musician! So the reversal of training occurs for those who are in certain classes of skills in this model. The result is quite new and innovative. So let's look at the model in the next section.

2 The Model

We consider a simple two-period and two-sector model of a small open economy that faces exogenously given international output prices. In period 0, no production takes place, agents are endowed with multi-dimensional heterogeneous skills and they invest in their human capital to enhance their innate skills. Uncertainty about the terms of trade will be realized in period 1 and agents choose their occupations and engage in production. Multi-dimensional skills are embodied in an individual agent and can only be sold as a package. Therefore, we assume that an individual agent can take only one job at a time.

Output markets for sectors X and Y are assumed to be competitive, both internationally and domestically. In making the investment decision in period 0, each agent is assumed to have rational expectations concerning the prices that will prevail in period 1.

The economy consists of a continuum of self-employed agents $j \in J$, each of whom is endowed with an individual-specific occupational skill vector (θ^j, τ^j) jointly distributed over a unit square $[0,1] \times [0,1] \subset \mathbb{R}^2$ in period 0. Let Θ_t and $F_t(\theta, \tau)$ denote the space and the joint distribution function of human capital skills for each period $t = 0, 1$. Thus, we know that $\Theta_0 = [0,1]^2$. Let $f_t(\theta, \tau) > 0$ denote the joint density function for $F_t(\theta, \tau)$, and assume that f_t is integrable over any partition of the human capital skill space Θ_t for $t = 0, 1$. Agents are price takers in the output markets. Each component of the skill vector (θ^j, τ^j) represents an sector-specific human capital skill; their magnitudes measure the capabilities of the agent j in the production of X and Y in efficiency units.

Once the terms of trade become known in period 1, each agent decides either to produce X using θ, or Y using τ. Each individual undertakes only one occupation at a time because human capital skills are assumed to be embodied in human being. (Murphy, 1986, Sec.II) Each component of the skill vector (θ^j, τ^j) is indivisible and non-transferable. The size of the components of individual skill vector in each period $(\theta^j, \tau^j) \in \Theta_t$ is private information for j, but its aggregate distribution F_t is publicly known.

To summarize, the timing of the model is as follows:

1. In period 0, the type of each individual $(\theta^j, \tau^j) \in [0,1]^2 = \Theta_0$ is given.

2. An individual choose to invest in human capital skills for either sector X or Y or both. This individual decision will create the new skill space Θ_1 and new distribution of skills $F_1(\theta, \tau)$ for period 1.

3. In period 1, a new relative price $P_1 = (P_X, P_Y)$ is realized.

4. An individual chooses occupation and produce.

Let us now state the assumptions about the human capital investments and uncertainty.

2.1 Key Assumptions of the Model

Let us now state the assumptions about how individual agents can invest in their skills in period 0. There are two kinds of human capital investment: (1) General Human Capital Investment and (2) Sector Specific Human Capital Investment. Because there are two directions in sector specific investments, we can categorize possible invetments into 3 investment types.

A1: [**Human Capital Investment**] An individual with the skill vector (θ, τ) can choose among the 3 types of human capital investments: $I^{HC} \in \{S_X, G, S_Y\}$ which are listed as:

1. S_X = Specific Investment in Sector X: $(\theta, \tau) \Longrightarrow (\beta\theta, \tau)$
2. G = General Investment in both sectors: $(\theta, \tau) \Longrightarrow (\alpha\theta, \alpha\tau)$
3. S_Y = Specific Investment in Sector Y: $(\theta, \tau) \Longrightarrow (\theta, \beta\tau)$

where α and β represent parameters such that

$$\beta > \alpha > 1. \tag{1}$$

The fact that both parameters are larger than 1 implies that human capital investments, regardless of specificity or generality, are effective. The size of investment efficiency for specific investment β is assumed to be larger than the size of α. Otherwise, every one will invest in general skills alone because specific skill investment will be dominated by general investment in either direction.

It is assumed that the specific investments in two sectors share the same parameter β. It means that the effect of the specific investment is symmetric between sector X and Y. It is also assumed that the only one type of general investments can occur. General investment enhances skills in both sector in a symmetric manner. Both components will be multiplied by the same α. These symmetry assumptions are made in favor of simplicity of the analysis.

Another peculiarity about investment technology is that the effect of investments is proportional to the individual's original strength (innate capabilities) in each sector. If your innate skill θ in sector X production is very large, then your post investment skill in the same sector $\beta\theta$ will be proportionally large. This assumption is appropriate if all the agents in the economy are considered to be young. For example, young Michael Jordan can be trained to be a superstar because of his exceptional innate talent as a basketball player. The effect of training Jordan is much larger than the effect of training a mediocre player. The assumption of proportional effectiveness of human capital skill investments will create a theoretical problem if we start thinking the economy with matured (and old) agents who already have invested in their skills. However, let us focus on this particular case of proportionality in this paper.

Compared to more general investment frontier depicted in Murphy (1986) and Grossman and Shapiro (1982), this paper's specification of human capital investment given by the above assumption is somewhat restrictive by not allowing varieties of intermediate cases in general investments. This restrictive assumptions are made in favor of simplicity. The fact that β is larger than α means an implicit assumption about some form of increasing returns to specialization in human capital investments. Because β is larger than α, we can conclude that the type space in period 1 is $\Theta_1 \subset [0, \beta]^2$.

Now let us assume all the consumers have identical and homothetic preferences that can be represented by a utility function $u(x, y)$ for positive consumption amounts for each product: $x > 0$ and $y > 0$. When we denote income of the individuals as $M > 0$ and a vector of output prices as $P \equiv (P_X, P_Y)$, then the indirect utility function can be written as the following separable function:

$$V(P, M) = C(P) \cdot v(M) \tag{2}$$

We know, by Deaton and Muellbauer (1980), that the price-index function $C(P)$ is linear homogeneous and concave in P. The part $v(M)$ is an increasing function of income M of an individual. If the original utility function $u(x, y)$ is homogeneous of degree one, then $v(M) = M$ should hold. Therefore, if the agents are risk neutral, then the indirect utility can be written as

$$V(P, M) = C(P) \cdot M \tag{3}$$

and if the agents are risk averse, then $v(M)$ in the equation (2) is strictly concave in M.(Hirshleifer and Riley, 1992, p.32) Thus, we could write $v' > 0$ and $v'' < 0$. For now, let us leave the second one as $v'' \leq 0$, in order to include both cases of risk neutrality and risk aversion.

Now let us introduce an uncertainty in period 1. Since we assume that we live in a small open economy, every individual agent takes terms of trade as given. Suppose there are two states of nature in the following manner.

A2. **[Uncertainty]** Uncertainty about the terms of trade in period 1 takes the following symmetric form:

$$P_1 = \begin{cases} (p, 1) & \text{with probability} \quad \frac{1}{2} \\ (1, p) & \text{with probability} \quad \frac{1}{2} \end{cases} \tag{4}$$

where p is a positive parameter larger than 1.

We concentrate on the case with same probability for the two states of nature. When the states of nature is such that $P_1 = (p, 1)$, producers in sector X will be benefitted in period 1 because P_X will be more expensive relative to P_Y. When $P_1 = (1, p)$ occurs, producers in sector Y will be benefitted because P_Y will be more expensive relative to P_X. As we have seen in the previous paper, the realization of a particular terms of trade may induce some workers to take different jobs once the uncertainty is resolved. Thus, some agents will choose to work in a favorable sector while others may stay in the sector at which they were innately good.

Let us further assume that the demand condition is symmetric in sectors X and Y. In the equation of indirect utility (2), $C(P)$ represents a consumer price index (CPI).

A3. [Equal CPI] The consumer price index (CPI) in period 1 is the same for the different states of the world, namely

$$C(p,1) = C(1,p) \tag{5}$$

holds in the neighborhood.

This assumption will allow us to compare welfare by directly looking at $v(M)$ in (2), the utility part only from income in period 1 without worrying about substitution effects in consumption. (See Appendix for an explanation.) This way, regardless of the outcome of uncertain terms of trade, we can compare economic welfare of individual agents only by looking at $v(M)$.

Assume also that no income-insurance market exists. (See Grossman and Shapiro [1982] for the comparison between self-insurance versus insurance markets.)

A4. [No Insurance] The market for income insurance is absent.

This may be because there is no market for skills per se. Or this may be because information asymmetry[2] prevents insurance firms from operating profitably. In any case, we focus on the case without insurance.

In period 1, given the investment made in period 0 and given the resolved terms of trade, individual agents choose their occupations and start producing either X or Y. How do individual agents decide which sector to work? In order to see it, let's first introduce a constant-returns-to-scale production function for each sector:

$$\begin{cases} x &= N_X \\ y &= N_Y \end{cases} \tag{6}$$

where N_X and N_Y are the total sum of (effective) skill levels employed in each sector[3]. Given the realization of terms of trade, P, the skill space in period 1, Θ_1, can be partitioned into two: (1) $\Theta_X(P)$ $\cdots$ group of producers of X and (2) $\Theta_Y(P)$ $\cdots$ group of producers of Y. Then, N_X and N_Y can be written as follows:

$$\begin{cases} N_X &= \int_{\Theta_X(P)} \theta dF_1(\theta,\tau) \\ N_Y &= \int_{\Theta_Y(P)} \tau dF_1(\theta,\tau) \end{cases} \tag{7}$$

All individual agents earn wages that are equal to their value of marginal product. Therefore, when the terms of trade is given as $P = (P_X, P_Y)$, and if an individual with a vector of skills (θ,τ) engages in the production in sector X would earn the wage of $P_X\theta$ and the same individual would earn $P_Y\tau$ if he engages in the production in sector Y. So whether an individual (θ,τ) works in sector X or Y depends on the direction of inequality in

$$P_X\theta \gtrless P_Y\tau. \tag{8}$$

[2] Skill levels are private information of the individual agents and the insurance company cannot know them.

[3] We could add coefficients in the production functions such as $x = \gamma N_X$, but we omit them for the sake of simplicity.

Thus, an individual who conducted human capital investment $I^{HC} \in \{S_X, G, S_Y\}$ in period 0 faces terms of trade $P \equiv (P_X, P_Y) \in \{(p,1),(1,p)\}$ in period 1 will earn the income $M(I^{HC}, P)$. The possible combinations can be written as follows:

$$\begin{cases} M(S_X,(p,1)) &= \max \; [\; p\beta\theta \;,\; \tau \;] \\ M(G,(p,1)) &= \max \; [\; p\alpha\theta \;,\; \alpha\tau \;] \\ M(S_Y,(p,1)) &= \max \; [\; p\theta \;,\; \beta\tau \;] \\ M(S_X,(1,p)) &= \max \; [\; \beta\theta \;,\; p\tau \;] \\ M(G,(1,p)) &= \max \; [\; \alpha\theta \;,\; p\alpha\tau \;] \\ M(S_Y,(1,p)) &= \max \; [\; \theta \;,\; p\beta\tau \;] \end{cases} \tag{9}$$

Given this information about possible income in period 1, we can now consider expected utility of an individual (θ, τ) who conducted human capital investment $I^{HC} \in \{S_X, G, S_Y\}$ in period 0. Since the lottery about terms of trade in period 1 is given by (4), expected utility $U(I^{HC})$ for each investment pattern is written as

$$\begin{cases} U(S_X) &= \{\frac{1}{2}v[M(S_X,(p,1))] + \frac{1}{2}v[M(S_X,(1,p))]\} \cdot C(P_1) \\ U(G) &= \{\frac{1}{2}v[M(G,(p,1))] + \frac{1}{2}v[M(G,(1,p))]\} \cdot C(P_1) \\ U(S_Y) &= \{\frac{1}{2}v[M(S_Y,(p,1))] + \frac{1}{2}v[M(S_Y,(1,p))]\} \cdot C(P_1) \end{cases} \tag{10}$$

where $C(P_1) \equiv C(p,1) = C(1,p)$ is defined as such from (5). Thus, in order to compare the sizes among (10), we only need to compare the inside of the curly brackets $\{\}$ because we can think of $C(P_1)$ as an exogenous parameter. We will come back to this point later.

Now, in order to save our space for analysis, I would like to make additional innocuous assumption about the size of risk. Let us consider the case with large risk in this paper.

A5. [**Risk Size**] We focus on our analysis for the case of $p > \beta$. The size of price parameter is larger than the size of a parameter for specific human capital investment.

The reason for this assumption is a matter of simplicity. By assuming otherwise, the analysis is very similar except for the combination of parameters and cases. For those who are interested in the other case of $p < \beta$, please consult my Ph.D. Dissertation, (Ichida, 2004, Chapter 4), for the case of risk-neutral agents.

2.2 Incentives for Each Individual Agent

Given the assumptions given by A1.-A5. above, let us now look into the incentives for each individual agent. What type of human capital investments do agents choose? What jobs do agents take once the uncertainty is resolved?

If all agents were identical before investment decision as in Grossman and Shapiro (1982) and in Murphy (1986), then in the equilibrium all the investment choice must give the same expected utility. Otherwise, every agent will pick the best alternative that gives the highest expected utility level. The income of all agents in period 1 must also be equalized across sectors for identical-agents-case.

That is how the allocation of jobs would be done. However, this shall not be the case for the model in this paper because agents are born to be different. In fact, agents are heterogeneous in both absolute and relative skill levels in both periods. Depending on the relative sizes of skills in the vector (θ, τ), the largest size of income that is chosen among those in (9) may differ.

Therefore, let us break down the initial type space $\Theta_0 = [0,1]^2$ (in period 0) into the following 8 partitions by the possible occupational choice with different investment choices and with different outcomes of risk.

$$\begin{cases} K_1 : (\theta,\tau) \in [0,1]^2 & \cap & \{ & 0 & < & \tau & < & \frac{1}{p\beta}\theta & \} \\ K_2 : (\theta,\tau) \in [0,1]^2 & \cap & \{ & \frac{1}{p\beta}\theta & < & \tau & < & \frac{1}{p}\theta & \} \\ K_3 : (\theta,\tau) \in [0,1]^2 & \cap & \{ & \frac{1}{p}\theta & < & \tau & < & \frac{\beta}{p}\theta & \} \\ K_4 : (\theta,\tau) \in [0,1]^2 & \cap & \{ & \frac{\beta}{p}\theta & < & \tau & < & \theta & \} \\ K_5 : (\theta,\tau) \in [0,1]^2 & \cap & \{ & \theta & < & \tau & < & \frac{p}{\beta}\theta & \} \\ K_6 : (\theta,\tau) \in [0,1]^2 & \cap & \{ & \frac{p}{\beta}\theta & < & \tau & < & p\theta & \} \\ K_7 : (\theta,\tau) \in [0,1]^2 & \cap & \{ & p\theta & < & \tau & < & p\beta\theta & \} \\ K_8 : (\theta,\tau) \in [0,1]^2 & \cap & \{ & p\beta\theta & < & \tau & < & \infty & \} \end{cases} \tag{11}$$

I will explain the meaning of these partitions in words.

1. Think about the case where an individual conducted S_X (specific investment in sector X skill) in period 0. We think of the case of $M(S_X, (p,1))$ or $M(S_X, (1,p))$. If the terms of trade turn out to be favorable to sector X, i.e. $(p,1)$ is realized[4], then agents in $\bigcup_{i=1}^{7} K_i$ will produce in sector X while those in K_8 will produce in sector Y. If the terms of trade turn out to be favorable to sector Y, i.e. $(1,p)$ is realized, then agents in $\bigcup_{i=1}^{3} K_i$ will produce in sector X while those in $\bigcup_{i=4}^{8} K_i$ will produce in sector Y.

2. Think about the case where an individual conducted G (general investment) in period 0. We think of the case of $M(G, (p,1))$ or $M(G, (1,p))$. If the terms of trade turn out to be favorable to sector X, i.e. $(p,1)$ is realized, then agents in $\bigcup_{i=1}^{6} K_i$ will produce in sector X while those in $\bigcup_{i=7}^{8} K_i$ will produce in sector Y. If the terms of trade turn out to be favorable to sector Y, i.e. $(1,p)$ is realized, then agents in $\bigcup_{i=1}^{2} K_i$ will produce in sector X while those in $\bigcup_{i=3}^{8} K_i$ will produce in sector Y.

3. Think about the case where an individual conducted S_Y (specific investment in sector Y skill) in period 0. We think of the case of $M(S_Y, (p,1))$ or $M(S_Y, (1,p))$. If the terms of trade turn out to be favorable to sector X, i.e. $(p,1)$ is realized[5], then agents in $\bigcup_{i=1}^{5} K_i$ will produce in sector X while those in $\bigcup_{i=6}^{8} K_i$ will produce in sector Y. If the terms of trade turn out to be favorable to sector Y, i.e. $(1,p)$ is realized, then agents in K_1 will produce in sector X while those in $\bigcup_{i=2}^{8} K_i$ will produce in sector Y.

[4] This also means that the individual guessed correctly.
[5] This also means that the individual guessed wrong.

Given the above examination of individual decisions about which sector agents work in period 1, we want to think about the investment incentives of individuals in each partition by using backward induction. However, what kind of investments each individual chooses depends on the agent's attituted toward risk.

2.3 Expected Utility and Attitude toward Risk

Now, because expected utility function can represent the same preference up to the monotonic linear transformation[6], we can define the expected utility for the agents in partition K_i and who conducted investment I^{HC} as follows: for all $i \in \{1, ..., 8\}$, a new von-Neumann Morgenstern expected utility function $V(K_i, I^{HC})$ is defined by

$$V(K_i, I^{HC}) \equiv \left. \frac{U(I^{HC})}{C(P_1)} \right|_{(\theta,\tau)\in K_i}. \tag{12}$$

This can be done because $C(P_1) \equiv C(p, 1) = C(1, p)$ is an exogenous parameter as we said before. For the agents in different partitions in (11), we are able to calculate von-Neumann Morgenstern expected utility function $V(K_i, I^{HC})$ by combining the equations of income (9) and the equations of expected utility (10). $v(M)$ part of the equations (10) can be thought of Bernoulli utility function. When $v'' = 0$, we can say agents are risk neutral and when $v'' < 0$, we can say agents are strictly risk averse.

Here I would like to introduce specific functional forms for Bernoulli utility function. Three cases of different attitude toward risk are analyzed: (1) RN = risk neutral case, (2) RA1 = risk aversion with coefficient of relative risk aversion is 1, and (3) RA2 = risk aversion with coefficient of relative risk aversion is 2. Let us start with the risk neutral case.

RN When the agents are risk neutral, Bernoulli utility function takes the linear form: $v(M) = M$.

The case for risk neutrality (RN) is straight forward. We adopt simply the linear function without coefficients for the Bernoulli utility.

The case for risk averse is more complicated because there are many ways to be risk averse. For the sake of ease of getting the analytical results, let us focus on the following two cases of risk aversion.

RA1 One type of risk averse agents have Bernoulli utility function in the form of $v(M) = \ln M$. The parameter for relative risk aversion for this log form is constant at 1.

The first type of risk aversion RA1 utilizes log utility form. This utility has a constant elasticity for any income level.

[6] See Prop.6B1 in page 173 of Mas-Colell et al. (1995).

RA2 Another type of risk averse agents have Bernoulli utility function in the form of $v(M) = -\frac{1}{M}$. The parameter for relative risk aversion for this form is 2.

The second type of risk aversion RA2 utilizes negative inverse form of utility. The parameter for relative risk aversion is larger than 1. Therefore, the degree of risk aversion is the strongest among these three cases.

2.4 The results for individual agents

Let us state the main result for the RN case as Theorem 1.

Theorem 1 *When the the assumptions given by A1.-A5. hold in the model with the RN Bernoulli utility function $v(M) = M$, the following three situations will occur depending on the size of the parameter value of α.*

(i) All the agents in the economy will conduct specific investments in the direction of their innate comparative advantage; agents $(\theta, \tau) \in \Theta_0$ with $\tau < \theta$ will conduct S_X (Specific investment in sector X) and agents $(\theta, \tau) \in \Theta_0$ with $\tau > \theta$ will conduct S_Y (Specific investment in sector Y), if the parameter of general investment is smaller than a particular threshold value, or

$$\alpha < \frac{\beta + 1}{2}.$$

(ii) There will be some agents who will conduct general investments G. In particular, agents $(\theta, \tau) \in \Theta_0$ with

$$\frac{\beta - \alpha}{\alpha - 1} \cdot \theta < \tau < \frac{\alpha - 1}{\beta - \alpha} \cdot \theta$$

will conduct G (General investment in both sectors), agents $(\theta, \tau) \in \Theta_0$ with

$$0 < \tau < \frac{\beta - \alpha}{\alpha - 1} \cdot \theta$$

will conduct S_X, and agents $(\theta, \tau) \in \Theta_0$ with

$$\frac{\alpha - 1}{\beta - \alpha} \cdot \theta < \tau < \infty$$

will conduct S_Y, if the parameter of general investment is within a particular threshold values, or

$$\frac{\beta + 1}{2} < \alpha < \frac{(p+1)\beta}{p + \beta}.$$

(iii) There will be some agents who will conduct general investments G. In particular, agents $(\theta, \tau) \in \Theta_0$ with

$$\frac{p\beta + \beta - p\alpha}{p\alpha} \cdot \theta < \tau < \frac{p\alpha}{p\beta + \beta - p\alpha} \cdot \theta$$

will conduct G (General investment in both sectors), agents $(\theta, \tau) \in \Theta_0$ with

$$0 < \tau < \frac{p\beta + \beta - p\alpha}{p\alpha} \cdot \theta$$

will conduct S_X, and agents $(\theta, \tau) \in \Theta_0$ with

$$\frac{p\alpha}{p\beta + \beta - p\alpha} \cdot \theta < \tau < \infty$$

will conduct S_Y, if the parameter of general investment is larger than a particular threshold value, or

$$\frac{(p+1)\beta}{p+\beta} < \alpha.$$

The proof is in the later section. In the case of risk neutrality, specific investments are conducted only to strengthen the agent's comparative advatage. When the parameter of general investment is smaller than a particular threshold value, no general investment occurs. If the parameter is larger, then the general investment occurs for the agents with intermediate comparative advantage.

Let us state the main result for the RA1 case with log utility.

Theorem 2 *When the the assumptions given by A1.-A5. hold in the model with the RA1 Bernoulli utility function $v(M) = \ln M$, the following two situations will occur depending on the size of the parameter value of α.*

(i) All the agents in the economy will conduct specific investments, but the direction of investments vary depending on the strength of their innate comparative advantage; agents $(\theta, \tau) \in \Theta_0$ with

$$\begin{cases} 0 < \tau < \frac{\beta^2}{p\alpha}\theta & \iff & \textit{conduct } S_X \\ \frac{\beta^2}{p\alpha}\theta < \tau < \frac{p\alpha}{\beta^2}\theta & \iff & \textit{indifferent between } S_X \textit{ and } S_Y \\ \frac{p\alpha}{\beta^2}\theta < \tau < \infty & \iff & \textit{conduct } S_Y \end{cases}$$

if the parameter of general investment is smaller than a particular threshold value, or

$$\alpha < \sqrt{\beta}.$$

(ii) There will be some agents who will conduct general investments G. In particular, agents $(\theta, \tau) \in \Theta_0$ with

$$\frac{\beta}{p}\theta < \tau < \frac{p}{\beta}\theta$$

will conduct G (General investment in both sectors), agents $(\theta, \tau) \in \Theta_0$ with

$$0 < \tau < \frac{\beta}{p}\theta$$

will conduct S_X, and agents $(\theta, \tau) \in \Theta_0$ with

$$\frac{p}{\beta}\theta < \tau < \infty$$

will conduct S_Y, if the parameter of general investment is larger than a particular threshold value, or

$$\sqrt{\beta} < \alpha.$$

Similarly to the risk neutral case, when the parameter of general investment is larger than a particular threshold value, then the general investment occurs for the agents with intermediate comparative advantage. When the parameter of general investment is smaller, however, the direction of specific investments is quite different. For those who have very strong innate comparative advantage, they conduct specific investments in their strong skills. For those with intermediate comparative advantage, they are indifferent in conducting specific investments in sector X and sector Y.

Let us state the main result for the RA2 case.

Theorem 3 *When the the assumptions given by A1.-A5. hold in the model with the RA2 Bernoulli utility function $v(M) = -\frac{1}{M}$, the following two situations will occur depending on the size of the parameter value of α.*

(i) All the agents in the economy will conduct specific investments, but the direction of investments vary depending on the strength of their innate comparative advantage; agents $(\theta, \tau) \in \Theta_0$ with

$$\begin{cases} 0 < \tau < \frac{1}{1+p-\beta}\theta & \Longleftrightarrow \quad \textit{conduct } S_X \\ \frac{1}{1+p-\beta}\theta < \tau < \theta & \Longleftrightarrow \quad \textit{conduct } S_Y \\ \theta < \tau < (1+p-\beta)\theta & \Longleftrightarrow \quad \textit{conduct } S_X \\ (1+p-\beta)\theta < \tau < \infty & \Longleftrightarrow \quad \textit{conduct } S_Y \end{cases}$$

if the parameter of general investment is smaller than a particular threshold value, or

$$\alpha < \frac{2\beta}{\beta+1}.$$

(ii) There will be some agents who will conduct general investments G. In particular, agents $(\theta, \tau) \in \Theta_0$ with

$$\begin{cases} 0 < \tau < \frac{1}{1+p-\beta}\theta & \Longleftrightarrow \quad \textit{conduct } S_X \\ \frac{1}{1+p-\beta}\theta < \tau < \frac{\beta-\alpha}{\beta(\alpha-1)}\theta & \Longleftrightarrow \quad \textit{conduct } S_Y \\ \frac{\beta-\alpha}{\beta(\alpha-1)}\theta < \tau < \frac{\beta(\alpha-1)}{\beta-\alpha}\theta & \Longleftrightarrow \quad \textit{conduct } G \\ \frac{\beta(\alpha-1)}{\beta-\alpha}\theta < \tau < (1+p-\beta)\theta & \Longleftrightarrow \quad \textit{conduct } S_X \\ (1+p-\beta)\theta < \tau < \infty & \Longleftrightarrow \quad \textit{conduct } S_Y \end{cases}$$

if the parameter of general investment is larger than a particular threshold value, or

$$\frac{2\beta}{\beta+1} < \alpha.$$

The proofs are given below. When the risk aversion is very strong, we observe a very interesting phenomenon. Some agents specialize in their weak skills.

2.5 Analysis for Each Partition

Let us start analyzing the incentives to invest in human capital skills for agents in each partition. Depending on the realized outcome of uncertain terms of trade, the choice of occupation differs among different partitions. Therefore, the realized income may differ. Based on expected choice of occupation and expected income level, the individuals choose to decide to invest in specific or general human capital investments.

2.5.1 Partition K_1 and K_8

Consider agents in partition K_1. All agents in this partition will work in sector X in period 1 regardless of the realized terms of trade. Their expected utility can be written as follows:

$$\left\{\begin{array}{lcl} V(K_1,S_X) & = & \frac{1}{2}v(p\beta\theta)+\frac{1}{2}v(\beta\theta) \\ V(K_1,G) & = & \frac{1}{2}v(p\alpha\theta)+\frac{1}{2}v(\alpha\theta) \\ V(K_1,S_Y) & = & \frac{1}{2}v(p\theta)+\frac{1}{2}v(\theta) \end{array}\right. . \tag{13}$$

In this case, it is straight forward to show

$$V(K_1,S_X) > V(K_1,G) > V(K_1,S_Y) \tag{14}$$

regardless of the shapes of Bernoulli utility function. Regardless of the attitude toward risks, i.e. RN, RA1, or RA2, the Bernoulli utility functions are increasing monotonic functions, i.e. $v' > 0$. We know that $p\beta\theta > p\alpha\theta > p\theta$ and $\beta\theta > \alpha\theta > \theta$ for $p > \beta > \alpha > 1$ by assumptions of the model. (14) shows that all agents in partition K_1 will conduct specific investment in sector X. The same logic hold true for the agents in partition K_8 and they conduct S_Y.

2.5.2 Partition K_2 and K_7

Next, consider agents in partition K_2. They work in sector X except for the case when they have invested in S_Y **and** the terms of trade turn out to be unfavorable to sector X, i.e. $P_1 = (1,p)$. Thus, their expected utility can be written as

$$\left\{\begin{array}{lcl} V(K_2,S_X) & = & \frac{1}{2}v(p\beta\theta)+\frac{1}{2}v(\beta\theta) \\ V(K_2,G) & = & \frac{1}{2}v(p\alpha\theta)+\frac{1}{2}v(\alpha\theta) \\ V(K_2,S_Y) & = & \frac{1}{2}v(p\theta)+\frac{1}{2}v(p\beta\tau) \end{array}\right. . \tag{15}$$

From the analysis for K_1, we know that

$$V(K_2, S_X) > V(K_2, G) \tag{16}$$

holds always true since $V(K_2, S_X) = V(K_1, S_X)$ and $V(K_2, G) = V(K_1, G)$ can be seen in (15). So we are left to compare the size between $V(K_2, S_X)$ and $V(K_2, S_Y)$. We should note that the partition K_2 belongs to the region

$$\frac{1}{p\beta}\theta < \tau < \frac{1}{p}\theta. \tag{17}$$

We first look at the RN case: $v(M) = M$. By looking at

$$2\left\{V(K_2, S_X) - V(K_2, S_Y)\right\} = (p\beta + \beta - p)\theta - p\beta\tau \tag{18}$$

we can conclude that the expression (18) is positive because

$$\frac{p\beta + \beta - p}{p\beta} = \frac{1}{p} + \left(1 - \frac{1}{\beta}\right) > \frac{1}{p}$$

together with (17).

Second, look at the case for RA1: $v(M) = \ln M$. The expression

$$2\left\{V(K_2, S_X) - V(K_2, S_Y)\right\} = \ln\theta - \ln\tau + \ln\beta - \ln p \tag{19}$$

is positive because $\tau < \frac{\beta}{p}\theta$ always hold for agents in (17).

Third, look at the RA2 case: $v(M) = -\frac{1}{M}$. Let's look at the following expression:

$$2\left\{V(K_2, S_X) - V(K_2, S_Y)\right\} = \frac{\theta - (p + 1 - \beta)\tau}{p\beta\theta\tau} \tag{20}$$

Because it is easy to verify that

$$\frac{1}{p} < \frac{1}{p + 1 - \beta} \tag{21}$$

and, by noting the inequality (17), we can say that the expression (20) is always positive for agents in K_2.

So we can conclude that

$$V(K_2, S_X) > V(K_2, S_Y) \tag{22}$$

for all three cases. Therefore, we can conclude that in the partition K_2, the specific investment for sector X, S_X, is always chosen for all RN, RA1 and RA2 cases. The same logic hold true for the agents in partition K_7 and they conduct S_Y.

2.5.3 Partition K_3 and K_6

Now consider agents in partition K_3 who are in the region:

$$\frac{1}{p}\theta < \tau < \frac{\beta}{p}\theta. \tag{23}$$

They work in sector Y when the terms of trade is $(1, p)$ only if they had invested either G or S_Y. Otherwise, they work in sector X. Therefore, their expected utility can be written as

$$\left\{ \begin{array}{lcl} V(K_3, S_X) & = & \frac{1}{2}v(p\beta\theta) + \frac{1}{2}v(\beta\theta) \\ V(K_3, G) & = & \frac{1}{2}v(p\alpha\theta) + \frac{1}{2}v(p\alpha\tau) \\ V(K_3, S_Y) & = & \frac{1}{2}v(p\theta) + \frac{1}{2}v(p\beta\tau) \end{array} \right. . \tag{24}$$

Note that $V(K_3, S_X) = V(K_2, S_X)$ and $V(K_3, S_Y) = V(K_2, S_Y)$, but $V(K_3, G) \neq V(K_2, G)$.

To censerve the space, Let $\succ$ denote preference over the human capital investment $I^{HC} \in \{S_X, G, S_Y\}$ for a particular partition. If it is clear from the context, assume that the following notation can be used for any $i \in \{1, ..., 8\}$:

$$I^{HC} \succ I^{HC\prime} \quad \text{for agents in } K_i \iff V(K_i, I^{HC}) > V(K_i, I^{HC\prime}).$$

By using the notation $\succ$, we look at the three cases for agents in K_3 and K_6.

RN case: K_3 and K_6 Let us first look at the RN case: $v(M) = M$. By looking at

$$2\left\{V(K_3, S_X) - V(K_3, S_Y)\right\} = (p\beta + \beta - p)\theta - p\beta\tau \tag{25}$$

we know this is the same as (18). By looking at the following expresssion:

$$\frac{p\beta + \beta - p}{p\beta} - \frac{\beta}{p} = \frac{(p - \beta)(\beta - 1)}{p\beta},$$

we can conclude that

$$\frac{\beta}{p} < \frac{p\beta + \beta - p}{p\beta} \tag{26}$$

holds true since $\beta > 1$ and $p > \beta$. Then we can say that $S_X \succ S_Y$ for RN agents in K_3. Now we have to compare S_X with G. Look at

$$2\left\{V(K_3, S_X) - V(K_3, G)\right\} = (p\beta - p\alpha + \beta)\theta - p\alpha\tau \tag{27}$$

The sign of (27) depends on the direction of the following inequality:

$$\tau \gtrless \frac{p\beta + \beta - p\alpha}{p\alpha}\theta$$

Considering the condition for partition (23),

$$\tau < \frac{p\beta + \beta - p\alpha}{p\alpha}\theta$$

always hold if

$$\frac{\beta}{p} < \frac{p\beta + \beta - p\alpha}{p\alpha} \Leftrightarrow \alpha < \frac{\beta(p+1)}{p+\beta}$$

holds or

$$\alpha < \frac{\beta(p+1)}{p+\beta} \Leftrightarrow S_X \succ G. \tag{28}$$

When the parameter α is larger, then there exist regions which the reversal may occur.

$$\alpha > \frac{\beta(p+1)}{p+\beta} \Leftrightarrow \begin{cases} S_X \succ G & \text{for} \quad \frac{1}{p}\theta < \tau < \frac{p\beta - p\alpha + \beta}{p\alpha}\theta \\ G \succ S_X & \text{for} \quad \frac{p\beta - p\alpha + \beta}{p\alpha}\theta < \tau < \frac{\beta}{p}\theta \end{cases} \tag{29}$$

The same logic can explain symmetrically about the agents in K_6.

RA1 case: K_3 **and** K_6 Look at the case for RA1: $v(M) = \ln M$. We can conclude that $S_X \succ S_Y$ for RA1 agents in K_3 because

$$2\{V(K_3, S_X) - V(K_3, S_Y)\} = \ln\beta + \ln\theta - \ln p - \ln\tau \tag{30}$$

is positive for $\tau < \frac{\beta}{p}\theta$. If we now compare S_X and G, there are two cases depending on the parameter values for α and β. Look at

$$2\{V(K_3, S_X) - V(K_3, G)\} = 2\ln\beta + \ln\theta - 2\ln\alpha - \ln p - \ln\tau \tag{31}$$

and we find that the sign of this depends on

$$\tau \lesseqgtr \frac{\beta^2}{p\alpha^2}\theta$$

hence, the direction of inequality in $\beta \lesseqgtr \alpha^2$. We can easily see that

$$\beta > \alpha^2 \Longrightarrow S_X \succ G \tag{32}$$

for the left side of the arrow in (32) implies

$$\frac{\beta}{p} < \frac{\beta^2}{p\alpha^2}.$$

Otherwise, the region in K_3 is divided into two in the following manner:

$$\beta < \alpha^2 \Leftrightarrow \begin{cases} S_X \succ G & \text{for} \quad \frac{1}{p}\theta < \tau < \frac{\beta^2}{p\alpha^2}\theta \\ G \succ S_X & \text{for} \quad \frac{\beta^2}{p\alpha^2}\theta < \tau < \frac{\beta}{p}\theta \end{cases} \tag{33}$$

Some workers in K_3 choose to conduct specific investment in X while others make general investments. The same logic can explain symmetrically about the agents in K_6.

RA2 case: K_3 and K_6 Now let us look at the case for RA2: $v(M) = -\frac{1}{M}$. In order to provide the integrated analysis, we postpone the analysis for the RA2 case for agents in K_3 and K_6 to the section under the title: RA2 case: K_4 and K_5.

2.5.4 Partition K_4 and K_5

Now consider agents in partition K_4 who are in the region:

$$\frac{\beta}{p}\theta < \tau < \theta. \tag{34}$$

They are the most flexible workers. So all of them work in sector X (respectively, Y) when the terms of trade is $(p, 1)$ (respectively, $(1, p)$) regardless of their investment decisions. Therefore, their expected utility can be written as

$$\begin{cases} V(K_4, S_X) &= \frac{1}{2}v(p\beta\theta) + \frac{1}{2}v(p\tau) \\ V(K_4, G) &= \frac{1}{2}v(p\alpha\theta) + \frac{1}{2}v(p\alpha\tau) \\ V(K_4, S_Y) &= \frac{1}{2}v(p\theta) + \frac{1}{2}v(p\beta\tau) \end{cases}. \tag{35}$$

Note that $V(K_4, G) = V(K_3, G)$ and $V(K_4, S_Y) = V(K_3, S_Y)$, but $V(K_4, S_X) \neq V(K_3, S_X)$.

RN case: K_4 and K_5 Let us first look at the RN case: $v(M) = M$. By looking at

$$2\{V(K_4, S_X) - V(K_4, S_Y)\} = p(\beta - 1)(\theta - \tau)$$

which is positive because of (34) and $p > 0$ and $\beta > 1$, we can conclude that

$$S_X \succ S_Y$$

for agents in K_4. Now we have to compare S_X with G. Look at

$$2\{V(K_4, S_X) - V(K_4, G)\} = p[(\beta - \alpha)\theta - (\alpha - 1)\tau] \tag{36}$$

The sign of (36) depends on the direction of the following inequality:

$$\tau \gtrless \frac{\beta - \alpha}{\alpha - 1}\theta.$$

There are 3 possible parameter spaces:

1. All agents in K_4 choose S_X if

$$1 < \frac{\beta - \alpha}{\alpha - 1} \Leftrightarrow \alpha < \frac{\beta + 1}{2} \tag{37}$$

because (34) and (37) imply

$$\tau > \frac{\beta - \alpha}{\alpha - 1}\theta.$$

2. Some agents choose S_X and others choose G depending on the location if

$$\frac{\beta}{p} < \frac{\beta - \alpha}{\alpha - 1} < 1 \Leftrightarrow \frac{\beta + 1}{2} < \alpha < \frac{(p+1)\beta}{p + \beta}$$

holds. Threfore,

$$\begin{cases} 0 < \tau < \frac{\beta-\alpha}{\alpha-1}\theta & \Longleftrightarrow \quad \text{conduct } S_X \\ \frac{\beta-\alpha}{\alpha-1}\theta < \tau < \theta & \Longleftrightarrow \quad \text{conduct } G \end{cases}$$

should hold.

3. All agents in K_4 choose S_X if

$$\frac{\beta - \alpha}{\alpha - 1} < \frac{\beta}{p} \Leftrightarrow \frac{(p+1)\beta}{p + \beta} < \alpha \tag{38}$$

because (34) and (38) imply

$$\tau > \frac{\beta - \alpha}{\alpha - 1}\theta$$

for all agents in K_4.

The last case must combine with the analysis on K_3. The same logic can explain symmetrically about the agents in K_5.

RA1 case: K_4 and K_5 Let us first look at the RA1 case: $v(M) = \ln M$. By looking at

$$2\left\{V(K_4, S_X) - V(K_4, S_Y)\right\} = 0,$$

we must conclude that $S_X \sim S_Y$ for all agents in K_4. In order to compare with general investments, look at

$$2\left\{V(K_4, S_X) - V(K_4, G)\right\} = \ln\beta - 2\ln\alpha$$

whose sign depends on the direction of inequality in

$$\beta \gtrless \alpha^2.$$

If $\beta > \alpha^2$ holds, then we should know that $S_X \sim S_Y \succ G$ for all agents in K_4. Therefore, we can say that

$$S_X \sim S_Y \Leftrightarrow \frac{\beta}{p}\theta < \tau < \theta.$$

But if $\beta < \alpha^2$ holds, then we should know that

$$\frac{\beta^2}{p\alpha^2} < \frac{\beta}{p}$$

holds. Therefore, we know that $G \succ S_X \sim S_Y$ for all agents in K_4. This last case must combine with the analysis on K_3. The same logic can explain symmetrically about the agents in K_5.

RA2 case: K_4 and K_5 Let us look at the RA2 case: $v(M) = -\frac{1}{M}$. Here we want to analyze partitions K_3 and K_4 together. The case for partitions K_5 and K_6 is similar, so we omit the analysis.

We first make it clear that agents belong to the regions:

$$\begin{cases} K_3: & \frac{1}{p}\theta < \tau < \frac{\beta}{p}\theta \quad \text{and} \\ K_4: & \frac{\beta}{p}\theta < \tau < \theta \quad . \end{cases}$$

We also list here the conditions for preference of agents for invetment types. Because $V(K_4, G) = V(K_3, G)$ and $V(K_4, S_Y) = V(K_3, S_Y)$ holds, the relationship between G and S_Y is the same between K_3 and K_4. By checking the sign of the expression

$$2\{V(K_i, S_Y) - V(K_i, G)\} = \frac{(\beta - \alpha)\theta - \beta(\alpha - 1)\tau}{p\alpha\beta\theta\tau} \text{ for } i = 3, 4,$$

we can conclude that

$$\begin{cases} \tau > \frac{\beta-\alpha}{\beta(\alpha-1)}\theta & \Leftrightarrow \quad G \succ S_Y \quad \text{for} \quad K_3 \text{ and } K_4 \\ \tau < \frac{\beta-\alpha}{\beta(\alpha-1)}\theta & \Leftrightarrow \quad S_Y \succ G \quad \text{for} \quad K_3 \text{ and } K_4 \end{cases} \tag{39}$$

Now start looking at the partition K_3, to compare the two specific investments, we must look at the sign of the following expression:

$$2\{V(K_3, S_X) - V(K_3, S_Y)\} = \frac{\theta - (1 + p - \beta)\tau}{p\beta\theta\tau} \tag{40}$$

and we can conclude that

$$\begin{cases} \tau > \frac{1}{1+p-\beta}\theta & \Leftrightarrow \quad S_Y \succ S_X \quad \text{for} \quad K_3 \\ \tau < \frac{1}{1+p-\beta}\theta & \Leftrightarrow \quad S_X \succ S_Y \quad \text{for} \quad K_3 \end{cases} . \tag{41}$$

If we compare S_X and G, we must check the sign of the following:

$$2\{V(K_3, S_X) - V(K_3, G)\} = \frac{\beta\theta - (\alpha + p\alpha - \beta)\tau}{p\alpha\beta\theta\tau} \tag{42}$$

and we can conclude that

$$\begin{cases} \tau > \frac{\beta}{\alpha+p\alpha-\beta}\theta & \Leftrightarrow \quad G \succ S_Y \quad \text{for} \quad K_3 \\ \tau < \frac{\beta}{\alpha+p\alpha-\beta}\theta & \Leftrightarrow \quad S_Y \succ G \quad \text{for} \quad K_3 \end{cases} . \tag{43}$$

Look at the partition K_4. Let us first compare the two specific investments. Look at

$$2\{V(K_4, S_X) - V(K_4, S_Y)\} = \frac{\alpha(\beta - 1)(\tau - \theta)}{p\alpha\beta\theta\tau}$$

which is always positive because $\alpha > 0$, $\beta > 1$ and $\tau > \theta$ for all agents in K_4. Therefore, we can conclude that

$$S_Y \succ S_X \quad \text{for all agents in} \quad K_4 \ . \tag{44}$$

If we compare S_X and G for the partition K_4, we must check the sign of the following:

$$2\{V(K_4,S_X)-V(K_4,G)\}=\frac{(\beta-\alpha)\tau-\beta(\alpha-1)\theta}{p\alpha\beta\theta\tau}$$

and we can conclude that

$$\begin{cases} \tau>\frac{\beta(\alpha-1)}{\beta-\alpha}\theta & \Leftrightarrow & S_X\succ G & \text{for} & K_4 \\ \tau<\frac{\beta(\alpha-1)}{\beta-\alpha}\theta & \Leftrightarrow & G\succ S_X & \text{for} & K_4 \end{cases}. \tag{45}$$

In order to prepare the further analysis, we now claim the following two results.

Lemma 1 *The following relationship holds true.*

$$\frac{1}{p}<\frac{1}{p-(\beta-1)}<\frac{\beta}{p} \tag{46}$$

Proof. The left side of (46) is obvious because $\beta>1$. To prove the right side,

$$\frac{1}{p-(\beta-1)}<\frac{\beta}{p}\Leftrightarrow\beta(\beta-1)<p(\beta-1)$$

which is true for $\beta<p$ by assumption. ■

Second result is given here.

Lemma 2 *The following relationship holds true.*

$$\frac{2\beta}{p+1}<\frac{p+\beta}{p+1}<\frac{2\beta}{\beta+1}<\frac{\beta(\beta+p)}{\beta^2+p} \tag{47}$$

Proof. From the leftside, because $p>\beta$, it is obvious that

$$\frac{\beta+\beta}{p+1}<\frac{p+\beta}{p+1}.$$

Next, look at the mid inequality,

$$\frac{p+\beta}{p+1}<\frac{2\beta}{\beta+1}\Leftrightarrow(\beta-1)(\beta-p)<0$$

which is true because $\beta>1$ and $p>\beta$. From the right side,

$$\frac{2\beta}{\beta+1}<\frac{\beta(\beta+p)}{\beta^2+p}\Leftrightarrow(\beta-1)(\beta-p)<0$$

which is also true. This concludes the proof. ■

Now, given the relative size in (47), we can analyze 5 cases depending on the size of the parameter α.

Case 1: $\alpha < \frac{2\beta}{p+1}$

Case 2: $\frac{2\beta}{p+1} < \alpha < \frac{p+\beta}{p+1}$

Case 3: $\frac{p+\beta}{p+1} < \alpha < \frac{2\beta}{\beta+1}$

Case 4: $\frac{2\beta}{\beta+1} < \alpha < \frac{\beta(\beta+p)}{\beta^2+p}$

Case 5: $\frac{\beta(\beta+p)}{\beta^2+p} < \alpha$

2.5.5 Case 1: $\alpha < \frac{2\beta}{p+1}$

When $\alpha < \frac{2\beta}{p+1}$ holds, then the following relationship holds true:

$$\frac{1}{p} < \frac{1}{1+p-\beta} < \frac{\beta}{p} < 1 \tag{48}$$

and

$$\begin{cases} \frac{\beta(\alpha-1)}{\beta-\alpha} < \frac{\beta}{p} \\ 1 < \frac{\beta-\alpha}{\beta(\alpha-1)} \\ 1 < \frac{\beta}{\alpha+p\alpha-\beta} \end{cases} . \tag{49}$$

This is because $\alpha < \frac{2\beta}{p+1}$ automatically imply $\alpha < \frac{p+\beta}{p+1}$ and $\alpha < \frac{2\beta}{\beta+1}$, and we can derive a few results:

$$\begin{cases} \alpha < \frac{2\beta}{p+1} & \Leftrightarrow & 1 < \frac{\beta}{\alpha+p\alpha-\beta} \\ \alpha < \frac{p+\beta}{p+1} & \Leftrightarrow & \frac{\beta(\alpha-1)}{\beta-\alpha} < \frac{\beta}{p} < \frac{\beta}{\alpha+p\alpha-\beta} \\ \alpha < \frac{2\beta}{\beta+1} & \Leftrightarrow & \frac{\beta(\alpha-1)}{\beta-\alpha} < 1 < \frac{\beta-\alpha}{\beta(\alpha-1)} \end{cases}$$

Together with (46), (48) and (49) can be shown. If we summarize all the conditions (39)-(45), we can state the following results for the case $\alpha < \frac{2\beta}{p+1}$.

$$\begin{cases} \frac{1}{p}\theta < \tau < \frac{1}{1+p-\beta}\theta & \Leftrightarrow & S_X \succ S_Y \succ G \\ \frac{1}{1+p-\beta}\theta < \tau < \frac{\beta}{p}\theta & \Leftrightarrow & S_Y \succ S_X \succ G \\ \frac{\beta}{p}\theta < \tau < \theta & \Leftrightarrow & S_Y \succ S_X \succ G \end{cases}$$

2.5.6 Case 2: $\frac{2\beta}{p+1} < \alpha < \frac{p+\beta}{p+1}$

When $\alpha < \frac{p+\beta}{p+1}$ holds, then the following relationship holds true:

$$\frac{1}{p} < \frac{1}{1+p-\beta} < \frac{\beta}{p} < \frac{\beta}{\alpha+p\alpha-\beta} < 1 \tag{50}$$

and

$$\begin{cases} \frac{\beta(\alpha-1)}{\beta-\alpha} < \frac{\beta}{p} \\ 1 < \frac{\beta-\alpha}{\beta(\alpha-1)} \end{cases} .$$

Because $\alpha < \frac{p+\beta}{p+1}$ automatically imply $\alpha < \frac{2\beta}{\beta+1}$, we can derive a few results:

$$\begin{cases} \frac{2\beta}{p+1} < \alpha & \Leftrightarrow & \frac{\beta}{\alpha+p\alpha-\beta} < 1 \\ \alpha < \frac{p+\beta}{p+1} & \Leftrightarrow & \frac{\beta(\alpha-1)}{\beta-\alpha} < \frac{\beta}{p} < \frac{\beta}{\alpha+p\alpha-\beta} \\ \alpha < \frac{2\beta}{\beta+1} & \Leftrightarrow & \frac{\beta(\alpha-1)}{\beta-\alpha} < 1 < \frac{\beta-\alpha}{\beta(\alpha-1)} \end{cases}$$

Together with (46), (50) can be shown. If we summarize all the conditions (39)-(45), we can state the following results for the case $\frac{2\beta}{p+1} < \alpha < \frac{p+\beta}{p+1}$.

$$\begin{cases} \frac{1}{p}\theta < \tau < \frac{1}{1+p-\beta}\theta & \Leftrightarrow & S_X \succ S_Y \succ G \\ \frac{1}{1+p-\beta}\theta < \tau < \frac{\beta}{p}\theta & \Leftrightarrow & S_Y \succ S_X \succ G \\ \frac{\beta}{p}\theta < \tau < \theta & \Leftrightarrow & S_Y \succ S_X \succ G \end{cases}$$

2.5.7 Case 3: $\frac{p+\beta}{p+1} < \alpha < \frac{2\beta}{\beta+1}$

When $\frac{p+\beta}{p+1} < \alpha < \frac{2\beta}{\beta+1}$ holds, then the following relationship holds true.

$$\frac{1}{p} < \frac{1}{1+p-\beta} < \frac{\beta}{\alpha+p\alpha-\beta} < \frac{\beta}{p} < \frac{\beta(\alpha-1)}{\beta-\alpha} < 1 < \frac{\beta-\alpha}{\beta(\alpha-1)} \tag{51}$$

We can derive a few results:

$$\begin{cases} \frac{p+\beta}{p+1} < \alpha & \Leftrightarrow & \frac{\beta}{\alpha+p\alpha-\beta} < \frac{\beta}{p} < \frac{\beta(\alpha-1)}{\beta-\alpha} \\ \alpha < \frac{2\beta}{\beta+1} & \Leftrightarrow & \frac{\beta(\alpha-1)}{\beta-\alpha} < 1 < \frac{\beta-\alpha}{\beta(\alpha-1)} \end{cases}$$

Also we have to check if

$$\frac{\beta}{\alpha+p\alpha-\beta} \leq \frac{1}{1+p-\beta} \tag{52}$$

cannot occur. Suppose it does. Then we will encounter contradiction because there must exist the agents within

$$\frac{\beta}{\alpha+p\alpha-\beta}\theta \leq \tau \leq \frac{1}{p-(\beta-1)}\theta \tag{53}$$

whose preference can be represented by the following three:

$$\begin{array}{ccc} S_Y & \succ & G \\ S_X & \succeq & S_Y \\ S_X & \succeq & G \end{array} \tag{54}$$

which violates transitivity of the preference.

First, $S_Y \succ G$ because all agents in (53) must satisfy

$$\tau < \frac{\beta-\alpha}{\beta(\alpha-1)}\theta$$

since (51) and (52). $S_X \succeq S_Y$ because of the rightside in (53) and whose preference can also be represented by $S_X \succeq G$ because of the leftside of (53). This leads to (54).

If we summarize all the conditions (39)-(45), we can state the following results for the case 3.

$$\begin{cases} \frac{1}{p}\theta < \tau < \frac{1}{1+p-\beta}\theta & \Leftrightarrow \quad S_X \succ S_Y \succ G \\ \frac{1}{1+p-\beta}\theta < \tau < \frac{\beta}{\alpha+p\alpha-\beta}\theta & \Leftrightarrow \quad S_Y \succ S_X \succ G \\ \frac{\beta}{\alpha+p\alpha-\beta}\theta < \tau < \frac{\beta(\alpha-1)}{\beta-\alpha}\theta & \Leftrightarrow \quad S_Y \succ G \succ S_X \\ \frac{\beta(\alpha-1)}{\beta-\alpha}\theta < \tau < \theta & \Leftrightarrow \quad S_Y \succ S_X \succ G \end{cases}$$

2.5.8 Case 4: $\frac{2\beta}{\beta+1} < \alpha < \frac{\beta(\beta+p)}{\beta^2+p}$

When $\frac{2\beta}{\beta+1} < \alpha < \frac{\beta(\beta+p)}{\beta^2+p}$ holds, then the following relationship holds true.

$$\frac{1}{p} < \frac{1}{1+p-\beta} < \frac{\beta}{\alpha+p\alpha-\beta} < \frac{\beta}{p} < \frac{\beta-\alpha}{\beta(\alpha-1)} < 1 < \frac{\beta(\alpha-1)}{\beta-\alpha} \tag{55}$$

Because $\frac{2\beta}{\beta+1} < \alpha$ automatically imply $\frac{p+\beta}{p+1} < \alpha$, we can derive a few results:

$$\begin{cases} \frac{p+\beta}{p+1} < \alpha & \Leftrightarrow \quad \frac{\beta}{\alpha+p\alpha-\beta} < \frac{\beta}{p} < \frac{\beta(\alpha-1)}{\beta-\alpha} \\ \frac{2\beta}{\beta+1} < \alpha & \Leftrightarrow \quad \frac{\beta-\alpha}{\beta(\alpha-1)} < 1 < \frac{\beta(\alpha-1)}{\beta-\alpha} \\ \alpha < \frac{\beta(\beta+p)}{\beta^2+p} & \Leftrightarrow \quad \frac{\beta}{p} < \frac{\beta-\alpha}{\beta(\alpha-1)} \end{cases} \tag{56}$$

In addition to the above, we must check if

$$\frac{\beta}{\alpha+p\alpha-\beta} \leq \frac{1}{1+p-\beta} \tag{57}$$

cannot occur. Suppose it does. Then we will encounter contradiction because there must exist the agents within

$$\frac{\beta}{\alpha+p\alpha-\beta}\theta \leq \tau \leq \frac{1}{p-(\beta-1)}\theta \tag{58}$$

whose preference can be represented by the following three:

$$\begin{array}{rcl} S_Y & \succ & G \\ S_X & \succeq & S_Y \\ S_X & \succeq & G \end{array} \tag{59}$$

which violates transitivity of the preference.

First, $S_Y \succ G$ because all agents in (58) must satisfy

$$\tau < \frac{\beta-\alpha}{\beta(\alpha-1)}\theta$$

since (55). $S_X \succeq S_Y$ because of the rightside in (58) and whose preference can also be represented by $S_X \succeq G$ because of the leftside of (58). This leads to (59).

If we summarize all the conditions (39)-(45), we can state the following results for the case 4.

$$\begin{cases} \frac{1}{p}\theta < \tau < \frac{1}{1+p-\beta}\theta & \Leftrightarrow \quad S_X \succ S_Y \succ G \\ \frac{1}{1+p-\beta}\theta < \tau < \frac{\beta}{\alpha+p\alpha-\beta}\theta & \Leftrightarrow \quad S_Y \succ S_X \succ G \\ \frac{\beta}{\alpha+p\alpha-\beta}\theta < \tau < \frac{\beta-\alpha}{\beta(\alpha-1)}\theta & \Leftrightarrow \quad S_Y \succ G \succ S_X \\ \frac{\beta-\alpha}{\beta(\alpha-1)}\theta < \tau < \theta & \Leftrightarrow \quad G \succ S_Y \succ S_X \end{cases}$$

2.5.9 Case 5: $\frac{\beta(\beta+p)}{\beta^2+p} < \alpha$

When $\frac{\beta(\beta+p)}{\beta^2+p} < \alpha$ holds, then the following relationship holds true.

$$\frac{1}{p} < \frac{1}{1+p-\beta} < \frac{\beta}{\alpha+p\alpha-\beta} < \frac{\beta-\alpha}{\beta(\alpha-1)} < \frac{\beta}{p} < 1 < \frac{\beta(\alpha-1)}{\beta-\alpha} \tag{60}$$

Because $\frac{\beta(\beta+p)}{\beta^2+p} < \alpha$ automatically imply $\frac{p+\beta}{p+1} < \alpha$ and $\frac{2\beta}{\beta+1} < \alpha$, we can derive a few results:

$$\begin{cases} \frac{p+\beta}{p+1} < \alpha & \Leftrightarrow \quad \frac{\beta}{\alpha+p\alpha-\beta} < \frac{\beta}{p} < \frac{\beta(\alpha-1)}{\beta-\alpha} \\ \frac{2\beta}{\beta+1} < \alpha & \Leftrightarrow \quad \frac{\beta-\alpha}{\beta(\alpha-1)} < 1 < \frac{\beta(\alpha-1)}{\beta-\alpha} \\ \frac{\beta(\beta+p)}{\beta^2+p} < \alpha & \Leftrightarrow \quad \frac{\beta-\alpha}{\beta(\alpha-1)} < \frac{\beta}{p} \end{cases} \tag{61}$$

In addition to the above, we must check if

$$\frac{\beta-\alpha}{\beta(\alpha-1)} \leq \frac{\beta}{\alpha+p\alpha-\beta} \tag{62}$$

cannot occur. Suppose it does. Then we will encounter contradiction because there must exist the agents within

$$\frac{\beta-\alpha}{\beta(\alpha-1)}\theta \leq \tau \leq \frac{\beta}{\alpha+p\alpha-\beta}\theta \tag{63}$$

whose preference can be represented by the following three:

$$\begin{array}{ccc} G & \succeq & S_Y \\ S_Y & \succ & S_X \\ S_X & \succeq & G \end{array} \tag{64}$$

which violates transitivity of the preference.

First, $S_Y \succ S_X$ because all agents in (63) must satisfy

$$\tau > \frac{1}{1+p-\beta}\theta$$

since (60). $G \succeq S_Y$ because of the leftside in (63) and whose preference can also be represented by $S_X \succeq G$ because of the rightside of (63). This leads to (64).

If we summarize all the conditions (39)-(45), we can state the following results for the case 5.

$$\begin{cases} \frac{1}{p}\theta < \tau < \frac{1}{1+p-\beta}\theta & \Leftrightarrow \quad S_X \succ S_Y \succ G \\ \frac{1}{1+p-\beta}\theta < \tau < \frac{\beta}{\alpha+p\alpha-\beta}\theta & \Leftrightarrow \quad S_Y \succ S_X \succ G \\ \frac{\beta}{\alpha+p\alpha-\beta}\theta < \tau < \frac{\beta-\alpha}{\beta(\alpha-1)}\theta & \Leftrightarrow \quad S_Y \succ G \succ S_X \\ \frac{\beta-\alpha}{\beta(\alpha-1)}\theta < \tau < \theta & \Leftrightarrow \quad G \succ S_Y \succ S_X \end{cases}$$

2.5.10 Summary of Cases

Case 1-3 can be summarized to be

$$\begin{cases} \frac{1}{p}\theta < \tau < \frac{1}{1+p-\beta}\theta & \Longleftrightarrow \quad \text{conduct } S_X \\ \frac{1}{1+p-\beta}\theta < \tau < \theta & \Longleftrightarrow \quad \text{conduct } S_Y \end{cases}$$

for the agents in K_3 and K_4 with $\alpha < \frac{2\beta}{\beta+1}$.

Case 4-5 can be summarized to be

$$\begin{cases} \frac{1}{p}\theta < \tau < \frac{1}{1+p-\beta}\theta & \Longleftrightarrow \quad \text{conduct } S_X \\ \frac{1}{1+p-\beta}\theta < \tau < \frac{\beta-\alpha}{\beta(\alpha-1)}\theta & \Longleftrightarrow \quad \text{conduct } S_Y \\ \frac{\beta-\alpha}{\beta(\alpha-1)}\theta < \tau < \theta & \Longleftrightarrow \quad \text{conduct } G \end{cases}$$

for the agents in K_3 and K_4 with $\frac{2\beta}{\beta+1} < \alpha$.

The same logic can explain symmetrically about the agents in K_5 and K_6. And together with the analysis for K_1 and K_2 (K_7 and K_8), this proves the result in Theorem 3.

3 Conclusion

In this paper, I extended the model of occupational choice to a dynamic case. I now allow human capital skills investment by individual agents and will examine the incentives by individuals when they start out from different innate capabilities. In general, specialization enhances productive capabilities of the society through division of labor. Investing into specific skills, however, may be a risky strategy if there is no insurance market under uncertainty. Human capital investment in general skills are usually considered to be a form of self-insurance. Previous works in this field deal with the case where every individual is identical before his human capital investment. The model introduced in this paper analyzed the investment decision problem when individual agents are heterogeneous in the sense of both absolute and comparative advantages in different sectors. The paper analyzed the case in which we encounter the terms of trade uncertainty.

I find the conditions about when agents will invest in general skills and when they specialize their skills in a particular sector. Depending on parameter values:

α, β, and p, it is quite possible for all individuals to invest in their innately strong skills even if there is no insurance market. For the agents with smaller degree of comparative advantage (those who are close to the 45 degree line in the unit square), general investment may occur when the parameter α is stronger. I did not find anyone who invested in human capital skills in the direction of their innate comparative disadvantage.

Our results are favored toward specific investment in the skills which agents have innate comparative advantages probably because of the following reasons. First, the agents are assumed to be risk neutral rather than risk averse. Risk neutral agents simply try to maximize expected income rather than expected utility. Therefore, given the set up of the model in this paper, agents find it worthwhile to enhance their strong skills rather than compensating their weaker skills. Second, investment technology is assumed to be the one of constant returns to scale. The investment coefficients α and β are multiplied proportionately by θ and τ, agents' skill levels prior to investment. Thus, agents who had already high θ (before investment) will have a better effect of training either from general or specific investment than the agents with lower value of θ. Agents with stronger comparative advantage in one sector will find themselves better off when they invest in the skills they can get most out of the investments. Third, the structure of uncertainty in this paper is limited to a very specific case of equal probabilities over two states of nature. The choice of this uncertainty structure is done in favor of simplicity, but it is true that individual incentives will change if we change the probability distribution. Future research can address these extensions.

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A The Relationship between CPI and Welfare for Homothetic Demand

When the utility function $u(x, y)$ is homothetic and strictly monotone, then we can write its indirect utility function as separable in prices and income I.

$$V(P_X, P_Y, I) = v(P_X, P_Y) \cdot w(I) \tag{65}$$

where $w(\cdot)$ is an strictly increasing function. We also know that when $u(x, y)$ is a linear homogeneous, then $w(I) = I$. (This case corresponds to the case of risk neutrality.)

Claim 1 $\frac{1}{v(P_X,P_Y)}$ *is a measure of consumer price index.*

When we want to compare welfare across different times, the change in relative prices is always a nuisance. But if we assume that $v(P_X, P_Y)$ is symmetric in P_X and P_Y, then we can conclude that

$$v(p, 1) = v(1, p) \tag{66}$$

Thus, when equation (66) holds, then the comparison of income function $w(I)$ is one to one relationship with the comparison of welfare.