News Credibility and the Quest for Clicks*

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Abstract

We examine a model of dynamic communication by a media outlet. In each period, the uninformed public can consult the outlet's report at a cost. The outlet, which is primarily driven by profit maximization, has an incentive to induce uncertainty in order to encourage future consultation and thereby generate revenue. In an intermediate cost range, the public and the outlet may be worse off with a cheaper cost of access since it leads the outlet to distort information more, by making the public's future consultation decision more responsive to the current report.

Keywords: cheap talk, strategic information transmission, media, Markovian processes.

JEL classification: D81, D83.

1 Introduction

In contemporary liberal societies, citizens obtain their information to a large extent via news media. Four stylized facts of media markets stand out. First, news providers generate revenue from readers (or viewers) whether directly by selling access to news or through advertising fees (Doyle, 2013; Noam, 2013). Second, the average cost of accessing news contents has decreased significantly over the years, in consequence of the rise of online news (Newman et. al., 2016).[1] Third,

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[1]While technology does allow news outlets to implement paywalls, most news media adopt either no paywall or a "soft" paywall on which key articles are still freely available. See Ingram, M. "The Guardian, Paywalls and the Death of Print Newspapers," Fortune, 27 February 2016. See also Arrese (2016) for the development of payment systems for online media.

the revenue (advertising and circulation combined) of the newspaper industry has decreased significantly.[2] Fourth, news accuracy is widely perceived to have declined in recent decades.[3]

Katherine Viner, Editor in Chief of the British newspaper the *Guardian*, summarizes the overall situation as follows: "*(T)he trouble is that the business model of most digital news organisations is based around clicks. News media around the world has reached a fever-pitch of frenzied binge-publishing, in order to scrape up digital advertising's pennies and cents. (...) The impact on journalism of the crisis in the business model is that, in chasing down cheap clicks at the expense of accuracy and veracity, news organisations undermine the very reason they exist: to find things out and tell readers the truth – to report, report, report.*"[4]

We study the relationship between the informativeness of media reports and the (exogenous) cost of access to news in a simple dynamic game of information transmission. An informed sender (a media firm) faces a representative reader, who seeks information to improve his decision-making. The state of the world follows a Markov process, and each current state entails a different (low or high) level of uncertainty about the state tomorrow. When today's state implies low uncertainty about tomorrow, the media firm has an incentive to lie about the current state in order to induce the rational receiver to acquire another report tomorrow. We find that in an intermediate cost range, cheaper access to news lowers the accuracy of the report in equilibrium since the receiver's decision whether to read a news report becomes more "manipulable", which increases the sender's incentive to lie. This negative effect on the accuracy of news may be strong enough to outweigh the benefit of cheaper access for both the sender and the receiver. As a result, they may be worse off with cheaper access to news.

In our model, an informed sender (S) who perfectly observes the state produces a report in each period. An uninformed receiver (R) makes two decisions in each period, namely i) whether to access the report by incurring costs, including opportunity costs; and ii) which action to take in order to match the state of the world. While S's primary source of payoffs is visits/accesses by R, he also derives a positive payoff from truth-telling, but the latter is assumed small relative to the revenue generated from visits.

We find that if the expected cost of visit is in an intermediate range, equilibrium communication is partially but not fully informative.[5] S occasionally sends a false report that the current state

[2]"Newspapers Fact Sheet", Pew Research Center, 9 July 2019. Cagé (2016) points to the recent decline in the number of journalists per outlet and in the amount of space for news in newspapers.

[3]"Decline in Credibility Ratings for Most News Organizations", Pew Research Center, 16 August 2012; "U.S. Adults under 30 Now Trust Information from Social Media Almost As Much As from National News Outlets", Pew Research Center, 27 October 2022.

[4]Viner, K. "How Technology Disrupted the Truth," Guardian, 12 July 2016. See also Murtha, J. "What It's Like to be Paid for Clicks," Columbia Journalism Review, 13 July 2015. See also Sherman, J. "Daily Mail 'Too Unreliable' for Wikipedia," The Times, 10 February 2017. See Rusbridger (2018) for a comprehensive description of fundamental changes that have occurred in the print media industry during the last decades. See also Kilgo et al. (2018) for the link between social media recommendations and sensationalism in online news. See also Arbaoui et a. (2020), which provides evidence that commercial media funded through advertising are more prone to sensationalism than publicly funded media.

[5]The cost of access being in an intermediate range has a natural interpretation. If the cost is very low or near zero, R will acquire a news report in the next period no matter what the report in the current period is and how informative the report may be.

is the one that involves higher future uncertainty, with the aim of inducing a future visit. Reports are informative on average, which is why R is willing to acquire them even if it is costly to do so. Equilibrium communication thus features a combination of uncertainty resolution and uncertainty generation. Our second main finding is that in the intermediate cost range, the accuracy of S's reports (i.e. his tendency to lie) is *decreasing* in the expected cost of access since R's decision to access a future report becomes more responsive to the contents of the current report, which in turn gives S a stronger incentive to misreport. Our third main finding is that this adverse effect may be strong enough that both S and R can be worse off with cheaper access to news.

The analysis of our simple model is followed by extensions. The first extension considers the case of multiple news outlets, which allows R to cross-check reports. We find that if outlets can with some probability coordinate their reports, the accuracy of individual reports can decrease as the visiting cost decreases, as in the single outlet model. In particular, we show that an increased level of independence in the production of reports (, which could be interpreted as more competition) does not necessarily increase the accuracy of reports especially when the cost of access is small and the reports in equilibrium are less accurate. In other words, cross-checking may not be effective in inducing more accurate reports when the readers' behaviour is highly manipulable.

The second extension reconsiders our single and multiple firms models with the added feature that access to an outlet's reports requires a subscription whose price is endogenously set. We find that in the single outlet case, a subscriptions allows the outlet to appropriate some of R's surplus only in the monopoly case, while Bertrand competition over subscription leads the outlets to set zero price in the case of multiple outlets.[6]

On the face of it, our results echo some findings in the literature on planned obsolescence, in that reducing the quality of the current product leads to higher profits over periods (Bulow, 1986; Waldman, 1993, 1995). In particular, Bulow (1986) showed that a durable goods monopolist may inefficiently reduce the durability of the product in the current period in order to mitigate his tendency to overproduce later (due to the lack of commitment). However, the economic incentives we examine in this paper are very different from those in the literature on planned obsolescence. In Bulow (1986) for example, lower durability/quality does *not* increase the future demand, while in our model the purpose of reducing the quality of a report is to increase the demand in the future.[7]

The literature on strategic media has studied factors that determine the characteristics of media reports in various environments.[8] A key distinction with regards to media firms' incentives to misreport in the literature is between partisan motives and profit motives. While the partisan approach has provided important insights (e.g. Duggan and Martinelli, 2011), our model is more

[6]In Appendix B, we provide a third extension, which is an infinite horizon version of our baseline two periods setup, which we find relevant to both technical aspects and realism. While the state in the two period model can be interpreted as the development of a specific event that has a predetermined end date, the infinite-horizon model better represents reporting on an economic or political state of affairs in perpetual evolution. We find that our main results and intuitions carry over from the two period setup.

[7]In Bulow (1986), the future demand function is unaffected but the future (resale) price is negatively affected by durability through the future total supply. Moreover, in our model the media firm's incentive to misreport comes from the reader's (or buyer's) lack of commitment to a future consultation strategy, while in Bulow (1986) the driving force of low durability is time inconsistency on the seller's side.

[8]See e.g. Gentzkow, Shapiro and Stone (2015) and Puglisi and Snyder (2015) for surveys.

closely related to the second strand of the literature. In Mullainathan and Shleifer (2005) and Bernhardt, Krasa, and Polborn (2008), a profit seeking media firm biases news reports to target a subset of behavioural readers who, besides seeking information, value confirmatory news more.[9] In contrast to these papers, readers in our model care only about being informed, are fully rational, and have no behavioural preferences such as positive payoff from confirmatory news.[10]

The effects of media competition on newspaper reports have been studied extensively. Gentzkow, Shapiro and Sinkinson (2014) document that competitive forces in the US newspaper market made newspapers ideologically differentiate themselves from competitors and focus on serving specific tastes of consumers. Angelucci, Cagé and Sinkinson (2022) show that newspapers in the US reduced the amount of local news items in response to the entry of national television news. Cagé (2020) finds that an increased number of newspapers is associated with fewer articles and less hard news provision. Anand, Di Tella, and Galetovic (2007) study theoretically how competition induces media bias as product differentiation, and Perego and Yuksel (2022) analyze how the differentiation of news induced by competition affects voting behaviour of the audience. Gentzkow, Shapiro and Stone (2015) observe that in theory, competition generally reduces distortion since competition tends to align outcomes better with the demand of consumers, especially when they are unbiased. In our model the audience are also unbiased and readers wish purely to be better informed, but the effect of competition (parameterized as the degree to which reports are independently produced) on the informativeness of reports is ambiguous when the cost of access is low.

The economic incentives behind the media firm's misreporting may also be related to credence goods, for which the buyer cannot observe the quality of treatment even after purchase.[11] The closest paper in the literature to ours is Fong (2005), where the buyer can choose whether to purchase a treatment after a "recommendation" from the seller. However, the "recommendation" in Fong (2005) is a take-it-or-leave-it offer specifying the price and type of the treatment the expert commits to, while in our model a media firm's report is information about the state and does not affect the receiver's action space in any way.

The incentive to create future uncertainty in our paper is reminiscent of "suspense" and "surprise" in Ely, Frankel and Kamenica (2015), which refer to the expected and current changes in the receiver's belief. While the receiver in their model enjoys positive payoffs directly from uncertainty as represented by "suspense" and "surprise", the receiver in our model is strictly worse off with higher uncertainty since his action is less likely to match the state. Moreover, unlike our model, in Ely, Frankel and Kamenica (2015) the sender commits to a disclosure policy on the state that remains unchanged over periods. Hörner and Skrzypacz (2016) also develop a model of

[9] Empirical studies on the sources of media's ideological biases include Gentzkow and Shapiro (2006, 2010), Martin and Yurukoglu (2017), Qin, Strömberg, and Wu (2018), Levy (2021).

[10] The empirical study of Ahern and Sosyura (2015) examines financial media reporting on speculated corporate mergers. They find that less accurate stories are more likely to feature well-known firms with broader readership appeal. This suggests that the primary source of inaccuracy is the incentive to attract a broader readership, as we argue in this paper. Angelucci and Cagé (2019) demonstrate theoretically and empirically that a reduction in advertising revenue for media firms leads to a reduction in the subscription price and the amount of journalistic contents.

[11] See Dulleck and Kerschbamer (2006) for a survey.

dynamic information revelation by an agent of verifiable information concerning his ability level ("gradual persuasion").

Renault, Solan, and Vieille (2013) and Golosov et al. (2014) are among the recent papers that study repeated cheap talk games. In their models, the incentives to misreport stem from the misalignment of preferences with regards to the action taken by the receiver given the information provided by the sender. In our paper, the sender's payoff is primarily affected by whether the receiver obtains a report, but not the receiver's action informed by the report. The incentive to misreport stems from the drive to induce future visits, which to our knowledge has not been studied previously. A recent paper by Che, Kim and Mierendorff (2022) studies dynamic persuasion with a listening cost for the receiver as well as an information provision cost for the sender, where the receiver can stop listening and take the final action at any point.

We proceed as follows. Section 2 studies a single firm model with two periods. Section 3 extends the model to the case of two firms, and Section 4 discusses subscription pricing. Section 5 concludes. Most proofs are relegated to Appendix A. Appendix B presents the analysis of an infinite horizon version of the single firm model in Section 2.

2 Single Media Firm

There are two periods $t = 1,2$. At each t, the state ω_t is drawn from state space $\{A,B\}$. The prior distribution of the state in period 1 is given by $P(\omega_1 = A) = \theta$, where $\theta \in [\frac{1}{2},1)$. The state is governed by a Markovian process with transition probabilities $P(\omega_{t+1} = A \mid \omega_t = A) = \gamma \in \left(\frac{1}{2},1\right)$ and $P(\omega_{t+1} = B | \omega_t = B) = \frac{1}{2}$. State B thus entails higher uncertainty concerning the state in the next period. There are two players, namely a sender S and a receiver R. S observes the state ω_t at the beginning of period t and chooses a message (or "report") $m_t \in \{A,B\}$ for the period. In period t, R observes neither ω_t nor ω_{t-1}.

R's decision making in each period is divided into three stages. In the first stage, R has the option to consult (or "visit") S at a cost and thereby observe m_t. The consultation cost is $w+v_t$, where $w \in (0,\frac{1}{2}]$ is a fixed cost constant across both periods, and v_t is a random cost privately observed by R that is drawn anew at each t from a uniform distribution on $(0,\frac{1}{2}]$.[12] The consultation cost includes not only various material costs borne when acquiring the report but also the opportunity cost of time and cognitive effort to read the report. R observes v_t before deciding whether to consult in period t. Let d_t be a variable that takes value 1 if R consults in period t and 0 otherwise. In the second stage of period t, R chooses an action $a_t \in \{A,B\}$. The choice of a_t is based on R's posterior given m_t if it has been acquired, and otherwise on R's prior at the beginning of period t. In the last stage of period t, R observes m_t even if he chose not to visit S at t. This captures the notion that news reports are only excludable for a short time, as information spreads fast through various channels such as word of mouth.

The payoff of R for period t depends on both his consultation choice and whether his action matches the state. Besides the consultation cost already described, R obtains an action-payoff of

[12] Our results qualitatively do not depend on the specific upper bound of the support.

1 in period t if $a_t = \omega_t$, and otherwise 0.

S's payoff in period t depends on two aspects, visits and truth-telling. S receives a per visit revenue f if R makes a visit. Reporting $m_t = \omega_t$ yields a truth-telling benefit $z = xf$, where $x \in (0,1)$ is small so that $x < (1-\theta)(2\gamma-1)$, while reporting $.m_t \neq \omega_t$ yields no truth-telling reward.[13] The truth-telling benefit z captures S's intrinsic preference for reporting the truth, or unmodelled reputational concerns, which we will discuss shortly. By construction $z < f$ so that S's primary concern is to induce consultation. We assume no discounting for simplicity.

A strategy of S for the whole game is given by the combination of communication strategies for both periods. A communication strategy for period t is informative if $P(m_t = A \mid \omega_t = A) \neq P(m_t = A \mid \omega_t = B)$ and it is otherwise uninformative. We define a *simple* communication strategy for period t as one that has the following two features. First, the probability of sending $m_t \in \{A, B\}$ is only a function of the current state ω_t. Denote by τ_J^t the probability that message $m_t = J$ is sent at t when $\omega_t = J$. The second feature is $\tau_B^t = 1$, which means that S reports truthfully whenever the state is B. Intuitively, since state B implies higher uncertainty about the future than state A, there is no reason for S to misreport if $\omega_t = B$. Thus if misreporting is to occur, it must be when $\omega_t = A$. Given $\tau_B^t = 1$, a simple communication strategy for period t is *partially informative* if $\tau_A^t \in (0,1)$, *fully informative* if $\tau_A^t = 1$ and *uninformative* if $\tau_A^t = 0$. Thus τ_A^t measures the informativeness of the S's report in period t. A simple communication strategy for the whole game is such that S uses simple communication strategies in both periods. S is free to use any strategy, i.e. we allow S to use strategies that are not simple communication strategies.

A strategy of R for the whole game is given by the combination of strategies for both periods. R's strategy for each period specifies i) whether he visits (i.e. acquires m_t before choosing a_t) and ii) which action a_t he chooses, conditional on the information R has.

A strategy profile together with a set of equilibrium beliefs constitutes a perfect Bayesian equilibrium if each player's strategy is sequentially rational given their beliefs and the other's strategy, while the beliefs are derived from Bayes' rule whenever possible. We thus consider the game where neither S nor R can commit to a decision rule. That is, at any point in time where a player is called upon to make a choice (e.g. choosing a message in S's case), their choice maximizes their expected utility at that point. Our analysis thus is in line with those of standard cheap talk models such as Crawford and Sobel (1982) or Renault, Solan, and Vieille (2013).[14]

Note that we do *not* restrict our equilibrium analysis to the equilibria that features a simple communication strategy described above. We shall however establish that for a large range of parameter values there is a unique equilibrium, and that such an equilibrium has the property that the S uses a simple communication strategy.

Let us conclude the presentation of our model with a discussion of some of its assumptions.

[13] In principle, *truthful* communication (or truth-telling) and *fully informative* communication are conceptually different. S communicates truthfully if $m_t = \omega_t$ regardless of the actual meaning of the message in equilibrium, as pinned down by Bayes' rule. In contrast, equilibrium communication is fully informative if R can always infer the state perfectly based on the message observed, using Bayes' rule and S's communication strategy. We will see later that in our model, fully informative equilibrium communication has to come in the form of truth-telling.

[14] An alternative, which we do not explore, would have been to let S commit ex ante to a communication rule with the aim of maximizing his ex ante expected payoff, as in Kamenica and Gentzkow (2011).

The assumption that R exogenously observes m_1 at the end of period 1 captures information diffusion through free media, social networks (off- or online), or word of mouth. Technically speaking, the assumption simplifies the equilibrium analysis since R's belief at the end of $t = 1$ is independent of his consultation choice in period 1.

We interpret f as advertising revenue generated by a visit of R to S's website (a click), rather than as a direct transfer from R to S. Advertising revenue is typically a main source of income for many news websites that do not charge for access. This interpretation also implies that even with a subscription contract, S may well have an incentive to induce clicks since they still may generate revenue through advertisements or the sales of access data. We will discuss subscription contracts in Section 4.

The truth-telling reward $z = fx$ captures journalistic commitment to truthful reporting, unmodelled reputational concerns, or potential legal costs associated with inaccurate reporting.[15] It appears natural to assume that the truth-telling reward is proportional to the size of the market and/or revenue. Note that our formalization is equivalent to simply assuming $z = r$, where r is some positive constant. We will see later that key equilibrium quantities (in particular the truth-telling probabilities) are affected by z and f exclusively through the ratio $\frac{z}{f}$, so that the assumption $z = fx$ greatly simplifies the exposition throughout. All of our results, except those directly related to the effect of exogenous changes in f, are qualitatively unaffected even if f does not enter the truth-telling reward.

Finally, for expositional convenience we assume that there is only one receiver up until our discussion of subscription pricing in Section 4. However, throughout the paper our representative receiver R can be reinterpreted as featuring a continuum of receivers with mass one. In this alternative interpretation, the distribution of v_t is i.i.d. across all receivers and periods.

In what follows, we present our equilibrium analysis according to three adjacent ranges of w; low, middle and high. We demonstrate that the accuracy of the report in period 1 is weakly increasing in w over the three ranges, but strictly increasing in the middle range. Specifically we will see that there is no information transmission in the low range, noisy transmission in the intermediate range where the message becomes less noisy as w increases, and full transmission in the high range. When w is very low, the temptation to misreport in order to induce a future visit is too high to allow information transmission. The main focus of our analysis is on the intermediate range, which is in our view the most relevant to the credibility of online news.

2.1 Low visiting costs

We first consider the case w is low, specifically

$$w \in \left[0, \frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}\right].$$

[15] Let us recall that the model features no uncertainty about S's preference or behavioural type, which is often associated with models of reputational concerns.

Here, the low fixed cost of consultation implies that R's consultation decision at period 2 is very sensitive to his posterior at the end of period 1 and thus highly manipulable from the viewpoint of S, which in turn affects S's choice of report in period 1. S would like R to believe that $\omega_1 = B$ and therefore that the future state ω_2 is highly uncertain.

It is useful to note that the report in period 2 must be truthful (i.e. $m_2 = \omega_2$) in equilibrium. Since period 2 is the last period, there is no incentive to induce a future visit by misreporting. Moreover, because of the presence of the truth-telling benefit z, and S strictly prefers to report truthfully.

Proposition 1. *If* $w \in \left[0, \frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}\right]$*, there is a unique equilibrium. It features uninformative communication where* $m_1 = B$ *regardless of the state in period 1, and truth-telling in period 2.*

A very low consultation cost thus makes it impossible to achieve any informative communication by S in period 1, since the incentive to manipulate R's future consultation decision is too strong.

Notice that even if w is 0, R still bears a random cost of visiting $v_t \in [0, \frac{1}{2}]$. Therefore, as w tends to 0, the expected cost of consultation does not tend to 0 but to $\frac{1}{4}$. If there was no random cost v_t, as w tends to 0 R's cost of access becomes completely free and visiting decision in period 2 would unresponsive to m_1, since he would visit for sure regardless of m_1, which in turn would generate an equilibrium with truth-telling in period 1. However, in the context of online news we are concerned with in this paper, no readers would have unlimited capacity to read any online articles without any material or opportunity costs, and therefore our assumption that R has some non-negligible cost of access would be valid.

2.2 Intermediate visiting costs

Let us now assume that the fixed cost of consultation is an intermediate range, i.e.

$$w \in \left(\frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}, \frac{1-x}{2}\right). \tag{1}$$

In this range of w, R's decision whether to visit in period 2 is affected by m_1 but its influence is attenuated.

2.2.1 Equilibrium

As in the case of low w examined in (2.1), the report in period 2 must be truthful in equilibrium because there is no incentive to induce a future visit by misreporting in the final period. Also, there is no incentive to misreport when the state is B in period 1 (i.e. $\tau_B^1 = 1$) because reporting truthfully maximizes R's uncertainty about ω_2 and thus R's probability of visit in period 2. We obtain the following equilibrium characterization:

Proposition 2. *Let* $w \in \left(\frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}, \frac{1-x}{2}\right)$.

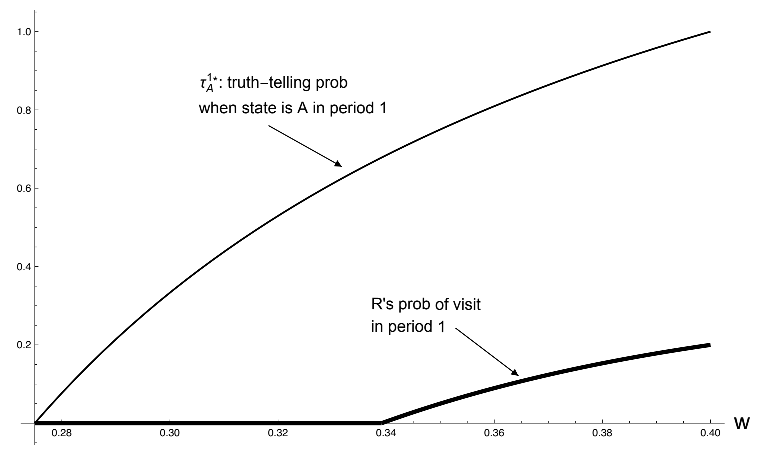


Figure 1: Equilibrium behaviour with respect to w, where $\theta = 1/2, \gamma = 3/4$, and $x = 1/5$

a) *There exists a unique equilibrium. S reports truthfully in period 2 and uses a partially informative simple communication strategy in period 1, where*

$$\tau_A^{1*} = \frac{2w + \theta(2\gamma - 1) + x - 1}{\theta(2\gamma + 2w + x - 2)} \in (0, 1). \tag{2}$$

R consults with probability x in period 2 if $m_1 = B$*, and with probability zero if* $m_1 = A$*. In periods* $t = 1, 2$*, R chooses* $a_t = m_t$ *if he consults and* $a_t = A$ *if he does not consult.*

b) τ_A^{1*} *is constant in f, increasing in w,* γ *and x.*

Recall that we allow S to use any communication strategy, and thus we do not restrict our analysis to equilibria that involve a simple communication strategy. Proposition 2 however establishes that there is a unique equilibrium, and that a property of this equilibrium is that S uses a simple communication strategy. Communication in period 1 is thus partially informative, as S randomizes between $m_1 = A$ and $m_1 = B$ when $\omega_1 = A$. Note that τ_A^{1*} is the only aspect of S's communication strategy that varies with exogenous parameters and affects the informativeness of m_1.

A key finding Part b) of Proposition 2 is that τ_A^{1*} increases in w, as illustrated in Figure 1. This is consistent with the discussion in the Introduction, where we suggest that online news, which are less costly to access relative to traditional paper-based media, have also become less reliable.

Technically, the fact that τ_A^{1*} is increasing in w is an immediate implication of the following indifference condition (3) that holds when S is indifferent between sending messages A and B in period 1 when $\omega_1 = A$:

$$fx = fP(d_2 = 1 \mid m_1 = B). \tag{3}$$

The LHS corresponds to the truth-telling reward in period 1 when sending $m_1 = A$. Recall that $m_1 = A$ induces no visit in period 2. The RHS of (3) is the expected revenue from R's visit in period 2 induced by sending $m_1 = B$ (thereby misreporting), where $P(d_2 = 1 \mid m_1 = B)$ denotes R's conditional probability of visit in period 2 given $m_1 = B$.[16] Clearly, f cancels out on both sides of (3), so that the indifference condition reduces to a requirement that the probability of visit

[16]Recall that d_t is a variable that takes value 1 if R visits in period t and 0 otherwise.

in period 2 conditional on $m_1 = B$ equals x, the parameter for the size of the truth-telling reward.

It is easy to check that $P(d_2 = 1 \mid m_1 = B)$ is strictly increasing in τ_A^{1*}. Intuitively, a higher τ_A^{1*} means higher uncertainty about ω_2 conditional on $m_1 = B$, since it implies $m_1 = B$ provides stronger evidence that $\omega_1 = B$ (i.e. $m_1 = B$ is "discounted" less). Note also that as the expected cost of consultation increases, R's willingness to consult in period 2 after $m_1 = B$ decreases, holding fixed the informativeness of the report in period 1. Therefore, (3) implies that as the expected cost of consultation increases, the truth-telling probability τ_A^{1*} must increase in order to ensure that R's visiting probability after $m_1 = B$ remains unchanged at x.

The relationship between the cost of consultation and the informativeness of the report in period 1 is a reflection of the commitment problem that R faces. A small reduction in the cost of consultation makes R's decision to visit more manipulable, in the sense that his probability of visit in period 2 after $m_1 = B$ increases (while the probability of visit following $m_1 = A$ remains zero). The increase in R's responsiveness to $m_1 = B$ gives rise to a larger temptation to misreport when $\omega_1 = A$ and, as a result, to lower informativeness of the equilibrium report in period 1. A higher cost of consultation thus de facto acts as an imperfect but effective commitment device for R not to overreact to a message that indicates higher future uncertainty.

Proposition 2 does not describe R's visiting probability in period 1, but it is shown in Figure 1.[17] We see that the probability of visit is zero when w is low in the intermediate range because the low credibility of m_1 cannot justify the cost of visit. As w increases, the improvement in the informativeness of m_1 outweighs the increase in the cost of visit, and the probability of visit in period 1 may be increasing.

The intuition behind the positive effect of an increase in γ shown in Proposition 2 is similar to the intuition for the effect of w. For a given informativeness of m_1, an increase in γ reduces R's responsiveness to message $m_1 = B$ by reducing his uncertainty in period 2 conditional on $m_1 = B$, which benefits R.

2.2.2 Welfare

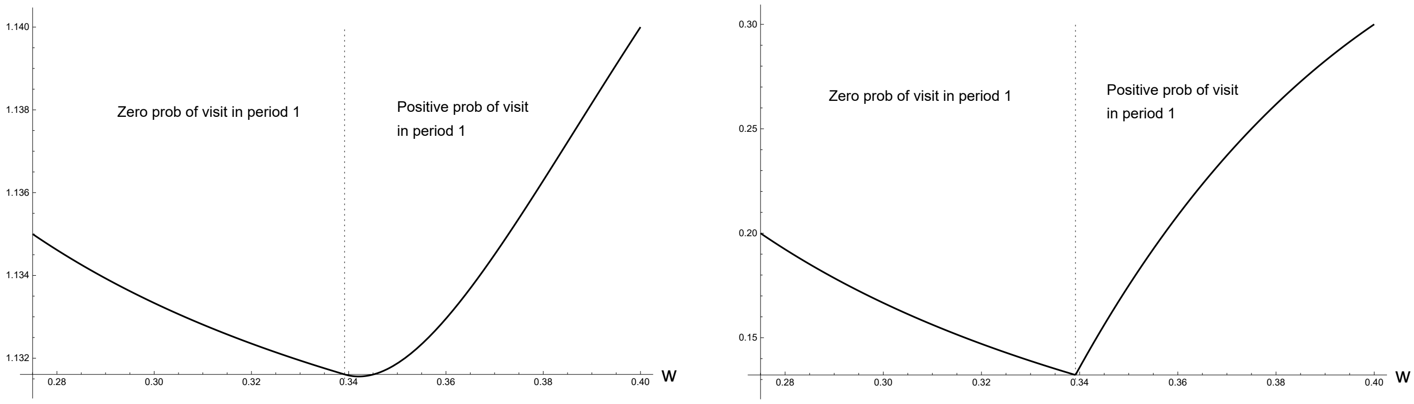
We now consider equilibrium payoffs. In order to obtain clear results regarding payoffs, for the rest of our analysis of the intermediate range of w we assume maximal uncertainty regarding the state in period 1, i.e. $\theta = 1/2$. This maximizes R's incentive to consult before choosing the action in period 1, all else equal.

Proposition 3.

1. *For w sufficiently close to but below $\frac{1-x}{2}$, a) R consults with strictly positive probability in period 1 and his consultation probability is increasing in w; and b) R's expected payoff in period 1 is increasing in w.*

2. *S's total expected payoff is weakly increasing in w.*

[17] Since S chooses m_1 to influence R's future visiting decision and R always observes m_1 at the end of period 1, S's equilibrium strategy in period 1 can be derived without calculating R's visiting probability period 1.



(a) R's total expected payoff

(b) S's total expected *accounting* profit

Figure 2: Equilibrium payoff with respect to w, where $\theta = 1/2, \gamma = 3/4$, and $x = 1/5$

Consider first Part 1 of the Proposition, which considers R's expected payoff. R's total expected payoff across the two periods has two components, namely the expected cost of consultation and the expected payoff from whether his action matches ω_t in each period. An increase in w, which shifts the average consultation upwards, generates a trade-off between an increased informativeness of m_1 and an increased average cost of consultation in periods 1 and 2. As we saw earlier, the informativeness of m_2 is independent of w since S always reports truthfully. However, an increase in w, by improving the informativeness of m_1, helps R to better decide whether to visit in period 2. The effect of w on R's payoff in period 2 is thus ambiguous. Meanwhile, the effect of w on R's payoff in period 1 is simpler, involving a pure trade-off between an increased informativeness of m_1 and an increased cost of consulting in period 1. Part 1-a) identifies a sufficient (not necessary) condition guaranteeing a positive probability of visit in period 1 as well as a positive effect of w on the period-1 visiting probability. Part 1-b) describes the comparative statics of R's payoff in period 1 with respect to w. The benefit of the increase in the informativeness of m_1 due to higher w outweighs the adverse increase in the expected visiting cost.

The effect of a change in w on R's total expected payoff cannot be obtained analytically, but we derive R's total expected payoff over the two periods explicitly in Appendix A and obtain insights through numerical examples in Figure 2a. The Figure shows that the benefit of a more informative report m_1 may outweigh the increase in the expected visiting cost when w is relatively high, which leads to the upward sloping curve. In other words, R may be worse off with a lower cost of access ("cheap clicks") to reports, because the reduced cost may not be sufficient to compensate for the simultaneous decline in the informativeness of the report.

Part 2 of Proposition 3 considers the expected payoff of S and provides a clear comparative statics result. The effect of an increase in w occurs through three effects. First, the higher probability of truth-telling in period 1 leads to higher truth-telling payoffs. Second, the expected advertising revenue in period 1 increases if the probability of visit in the period is positive, since in this case the probability of visit in period 1 increases as τ_A^{1*} increases. Third, the expected revenue in period 2 decreases as the probability of visit in period 2 decreases since the accuracy of m_2 remains unchanged, $m_1 = B$ becomes less likely (because τ_A^{1*} increases) and the cost of visit increases.

We may be interested not only in the expected payoff of S but also in the expected *accounting profit* of S since it captures the financial performance of a media firm, leaving out the less tangible truth-telling benefits. We are unable to obtain an analytical result for the expected accounting profit, but Figure 2b shows that it can also be increasing in w. The Figure shows that, when the probability of visit in period 1 is positive, S's accounting profit may be increasing in w. As we noted earlier, an increase in w has two opposing effects on accounting profits. Figure 2b indicates that the increase in the revenue in period 1 outweighs the decline the revenue in period 2. Since R faces very high uncertainty in period 1 ($\theta = 1/2$), his decision to purchase m_1 is very sensitive to the informativeness of m_1. On the other hand, $\gamma > 1/2$ implies that there is less overall uncertainty in period 2 than in period 1, and thus the demand for the report in period 2 is relatively low and less sensitive to an increase in w.

2.3 High visiting costs

Finally, let us consider the case of high w, namely

$$w \in \left[\frac{1-x}{2}, \frac{1}{2}\right).$$

When w is high, R's visiting decision in period 2 becomes insensitive to the report in period 1, and this naturally reduces the incentive to misreport.

Proposition 4. *If $w \in \left[\frac{1-x}{2}, \frac{1}{2}\right)$, there exists an equilibrium featuring truth-telling in both periods and a strictly positive probability of consultation in period 2.*

Proposition 4 shows that for a high range of fixed cost of consultation, there is an equilibrium where S reports truthfully and the probability of consultation in period 2 is positive. In this equilibrium, from S's viewpoint the benefit of revealing truthfully outweighs the benefit of inducing a higher probability of visit by misreporting (i.e. sending $m_1 = B$ when $\omega_1 = A$) since the latter probability remains very low due to the high expected cost of visit.

Note that $w < 1/2$ ensures that the probability of visit in period 2 is strictly positive under truthful reporting (when $\omega_1 = m_1 = B$), while the probability of visit is zero when $\omega_1 = m_1 = A$. Thus here, truth-telling is induced not by the complete absence of response to the message in period 1, but by the muted response to m_1 due to high w.

3 Multiple Media Firms

So far we have studied the informativeness of reports assuming that there is one media firm that generates a report for each period. We now extend our single firm model to the case of two media firms denoted by S_1 and S_2. In the model R observes the reports from both firms at the end of period 1 that can be used to better infer ω_1, and we will see that this "cross-checking" effect should increase the informativeness of the reports in equilibrium relative to the case with one firm. The aim of our analysis in this section is not to compare the single firm model and the two-firm model, but to demonstrate that in the two-firm model the fixed cost of access w may still have a positive effect on the informativeness of the reports from the firms, if their choices of reports are not entirely independent. As in Section (2) we assume away pricing, which shall be discussed later in Section (4).

In what follows, the assumptions of the model that we do not explicitly describe below are assumed to be the same as those in the single-firm model (e.g. the process governing the state, the sources of payoffs, etc) in Section 2. For simplicity, let the prior distribution of the state be uniform, that is, $\theta = 1/2$. Each firm S_i observes the state $\omega_t \in \{A, B\}$ perfectly at the beginning of t before choosing $m_t^{S_i}$. The truth-telling benefit for i of sending $m_t^{S_i} = \omega_r$ is still $z = xf$. In each period, reports are sent simultaneously with probability β, and instead sequentially with probability $1-\beta$. In the latter case, each firm is equally likely to be the first mover. The firms do not directly observe which scenario (simultaneous or sequential) has materialized. However, if communication is sequential *and* a firm is the second mover, the firm observes the first mover's message before choosing its own and correctly learns that communication is sequential. Unless a firm is the second mover under sequential communication, the firm is uncertain as to whether communication is simultaneous or sequential (i.e. the firm only knows that it is not the second mover). We interpret β as the degree of independence of firms in choosing their reports. In reality, we observe that reports from various outlets are often remarkably similar, which can be attributed to the fact that journalists occasionally observe other outlets' coverage of a given topic prior to producing their own report.

In each period t, after the two firms send their reports but before R chooses the action a_t for the period, R decides to acquire $m_t^{S_1}$, $m_t^{S_2}$, or neither them. We assume R cannot consult both. Acquiring a report in period t comes at cost $w + v_t$. If R is to consult in period t, he chooses the more informative of the two outlets. If these are equally informative, he chooses each with equal probability. At the end of period 1, after choosing a_1, R observes both messages $\left\{m_1^{S_1}, m_1^{S_2}\right\}$ regardless of whether R has acquired one of the reports. Therefore, at the end of the period, R has the opportunity to cross-check messages. Finally, so as to keep our exposition clear, we assume state A is sufficiently stable relative to the fixed cost of visiting so that $w > 1-\gamma$, which ensures that R never visits in period 2 if R knows for certain that $\omega_1 = A$.

Let us study the equilibrium and welfare of the two-firm model. We derive an equilibrium similar to the one we saw in the single firm model, in which misreporting occurs with a positive probability only in period 1 when $\omega_1 = A$. Note first that as in the single firm model, S_1 and S_2 report truthfully in period 2 as there is no benefit from lying in this final period. Also, in period 1 any incentive to misreport is present only when $\omega_1 = A$, since when $\omega_1 = B$ the probability of visit is maximized by reporting truthfully. We therefore focus on communication in period 1 and derive an equilibrium where the two firms use simple symmetric communication strategies represented by the parameter τ^C, the probability of truthful reporting when the state is A.[18]

[18]While we derive the equilibrium in simple symmetric communication strategies, the firms are allowed to adopt

Specifically, in an equilibrium in simple symmetric communication strategies, if firm i is the second mover under sequential communication, the firm aligns its report with the report from firm j, so that $m_1^{S_i} = m_1^{S_j}$.[19] Meanwhile, if firm i is not the second mover under sequential communication, without knowing the report from firm j (or whether the communication is simultaneous or sequential), firm i chooses $m_1^{S_i} = B$ with probability 1 if $\omega_1 = B$, and $m_1^{S_i} = A$ with probability τ^C when $\omega_1 = A$.

Let us consider the beliefs and incentives of R in such an equilibrium. Note that R correctly infers that $\omega_1 = A$ whenever at least one report indicates A, in which case R never visits any of the firms in period 2. Thus R is uncertain regarding ω_1 only when $m_1^{S_1} = m_1^{S_2} = B$. Note that the conditional probability that $m_1^{S_1} = m_1^{S_2} = B$ given $\omega_1 = B$ is one, while the conditional probability of this same message profile given $\omega_1 = A$ is

$$\beta(1-\tau^C)^2 + (1-\beta)(1-\tau^C). \tag{4}$$

The first term in (4) corresponds to the case where senders communicate simultaneously, whereas the second term corresponds to the case of sequential communication. Using (4), R's posterior given $m_1^{S_1} = m_1^{S_2} = B$ is written

$$P(\omega_1 = A \mid m_1^{S_1} = m_1^{S_2} = B) = \left(\frac{\frac{1}{2}\beta(1-\tau^C)^2 + \frac{1}{2}(1-\beta)(1-\tau^C)}{\frac{1}{2} + \frac{1}{2}\beta(1-\tau^C)^2 + \frac{1}{2}(1-\beta)(1-\tau^C)} \right). \tag{5}$$

We have $P(\omega_1 = A \mid m_1^{S_1} = m_1^{S_2} = B) \in (0,1)$ if $\tau^C < 1$. Accordingly, the conditional probability of visit in period 2 given $m_1^{S_1} = m_1^{S_2} = B$ is

$$\begin{aligned} &P(d_2 = 1 \mid m_1^{S_1} = m_1^{S_2} = B) \\ &= \frac{1 - P(\omega_1 = A \mid m_1^{S_1} = m_1^{S_2} = B)\gamma - \left(1 - P(\omega_1 = A \mid m_1^{S_1} = m_1^{S_2} = B)\right)\frac{1}{2} - w}{1/2}. \end{aligned} \tag{6}$$

It is easy to see from (5) that (6) is increasing in β and τ^C, and decreasing in w. This implies that the posterior probability that $\omega_1 = B$ given $m_1^{S_1} = m_1^{S_2} = B$ is higher, when it is more likely that the reports are independently generated, and the reports are more informative. As a result, higher β and τ^C lead to a higher probability of visit in period 2 given $m_1^{S_1} = m_1^{S_2} = B$.

Let us now look at the incentives of the media firms. Note first that complete independence ($\beta = 1$) leads to an equilibrium with truth-telling in period 1 (i.e. $\tau^C = 1$) where the off-the-equilibrium belief is such that any contradiction in the two reports means $\omega_1 = A$ with probability 1, so that if one firm reports truthfully, the other firm has no incentive to misreport since lying is always exposed.[20] In contrast, $\beta = 0$ implies that the strategic situation is equivalent to the case

any strategies for deviation.

[19]As we will show later, contradicting the first mover's report is never beneficial for the second mover. This is because, if the first mover reveals truthfully any misreporting by the second mover is exposed at the end of the period, and if the first mover misreports then the symmetry between the firms implies that reporting truthfully does not make the second mover better off.

[20]This type of equilibrium construction appears in the literature on cheap talk games with multiple senders (e.g.

of a single firm, because the second mover always sends the same report as the first mover's and thus the first mover is effectively the only sender that determines the report to be communicated.

It is interesting to note that β influences the media firms' incentives through two opposing effects, namely a credibility effect and a cross-checking effect. The credibility effect is that for fixed $\tau^C > 0$, an increase in β leads to an increase in $P(\omega_1 = A \mid m_1^{S_1} = m_1^{S_2} = B)$ since the reports are less likely to be a result of coordination between the firms. This effect alone *increases* the benefit of lying when $\omega_1 = A$, since R regards $m_1^{S_1} = m_1^{S_2} = B$ as a stronger indication that $\omega_1 = B$ and thus the probability of consultation in period 2 is higher. The cross-checking effect is that for fixed $\tau^C > 0$, an increase in β makes it more likely that misreporting by firm i when $\omega_1 = A$ will be exposed by the other firm. This effect alone *reduces* the benefit of lying.

Suppose that S_i learns that it is *not* the second mover in period 1. Given $\omega_1 = A$ and $m_1^{S_i} = B$, S_i assigns the following probability to $m_1^{S_j} = B$ (and thus to the event that R observes $m_1^{S_1} = m_1^{S_2} = B$)

$$\frac{\beta}{(1-\beta)\frac{1}{2}+\beta}(1-\tau^C) + \frac{(1-\beta)\frac{1}{2}}{(1-\beta)\frac{1}{2}+\beta}, \tag{7}$$

which denotes the probability that cross-checking *fails*.[21] It is easy to see that (7) is not only decreasing in β but also decreasing in τ^C. In other words, lying by one firm is more likely to be exposed when the report from the opponent becomes more accurate (that is, higher τ^C).

In an equilibrium in simple symmetric communication strategies such that $\tau^C \in (0,1)$, S_i must be indifferent between $m_1^{S_i} = B$ (lying) and $m_1^{S_i} = A$ (truth-telling) when $\omega_1 = A$. As in the single firm model of Section 2, $m_1^{S_i} = B$ triggers a positive probability of visit in period 2 but yields no truth-telling benefit, and $m_1^{S_i} = A$ instead induces zero probability of visit in period 2 but positive truth-telling benefit. The indifference condition given by

$$\begin{aligned} &f\left(\frac{\beta}{(1-\beta)\frac{1}{2}+\beta}(1-\tau^C) + \frac{(1-\beta)\frac{1}{2}}{(1-\beta)\frac{1}{2}+\beta}\right)P(d_2 = 1 \mid m_1^{S_1} = m_1^{S_2} = B)\frac{1}{2} \\ &= fx. \end{aligned} \tag{8}$$

The LHS of (8) is from (6) and (8), and represents the probability from firm i's viewpoint that firm i receives a visit in period 2 given $\omega_1 = A$ and $m_1^{S_i} = B$. The fraction $\frac{1}{2}$ that appears at the end of the expression captures the fact that R visits one of the firms with equal probability since both firms' reports are truthful in period 2. The RHS is simply the truth-telling benefit from sending $m_1^{S_i} = A$ when $\omega_1 = A$. If a given $\tau^C \in (0,1)$ satisfies (8), then it corresponds to a mixed strategy symmetric equilibrium. If for $\tau^C = 1$, the LHS of (8) is smaller than x, then there exists a symmetric truth-telling equilibrium.

We add a final comment on the incentives of the second mover. As mentioned earlier, the

Krishna and Morgan, 2001).

[21] If S_i learns that it is *not* the second mover, then it attributes probability $\frac{\beta}{(1-\beta)\frac{1}{2}+\beta}$ (in the first term) to simultaneous communication and probability $\frac{(1-\beta)\frac{1}{2}}{(1-\beta)\frac{1}{2}+\beta}$ (in the second term) to being the first mover under sequential communication.

second mover never has a strict incentive to contradict the first mover's report. Consider for example an equilibrium with $\tau^C \in (0,1)$. If the first mover i's message is $m_1^{S_i} = A$, then R's belief cannot be influenced by the second mover's message given the (correct) belief that any contradiction in the reports implies $\omega_1 = A$. If the first mover i's message is $m_1^{S_i} = B$, then the second mover is strictly better off sending the same message when $\omega_1 = B$ and indifferent between the two messages when $\omega_1 = A$ since the same indifference condition (8) applies to the second mover.

For our results presented in the next two Propositions, let x be small enough that

$$1-\gamma < \frac{3-2\gamma-4x}{4} < \frac{1}{2} - \frac{x(1+\beta)}{(1-\beta)}.$$

In the following Proposition we provide a sufficient condition for the existence of an equilibrium with truth-telling in period 1.

Proposition 5. *There exists an equilibrium with truth-telling ($\tau^C = 1$) in period 1 if $w \geq \frac{1}{2} - \frac{x(1+\beta)}{(1-\beta)}$, which is equivalent to $\beta \geq \frac{1-2w-2x}{1-2w+2x}$.*

We have already seen that a truth-telling equilibrium exists if $\beta = 1$. The same can also be seen in (8), where the left hand side, which represents the payoff from lying given $\omega_1 = A$, is 0 if we let $\beta = 1$ and $\tau^C = 1$, which confirms that S_i strictly prefers truth-telling since $x > 0$. Proposition 5 extends the result and shows that truth-telling is feasible if β is large enough. The underlying intuition is the same as for the case of $\beta = 1$: misreporting is more likely to be exposed when the reports are generated independently, which in turn reduces the benefit of misreporting and hence leads to truth-telling. In other words, the cross-checking effect dominates when β is high.

We now turn to an equilibrium that features noisy communication in period 1, i.e. an interior value of τ^C that solves the indifference condition (8). Unfortunately the explicit solution to the indifference condition is too complex to lend itself to a complete analysis within this paper, and we rely on numerical examples to obtain economic insights. However, the existence of an equilibrium with noisy communication $\tau^C \in (0,1)$ can be established as follows:

Proposition 6. *Let $w \in \left(\frac{3-2\gamma-4x}{4}, \frac{1}{2} - \frac{x(1+\beta)}{(1-\beta)}\right)$. There exists an equilibrium with noisy communication in period 1 ($\tau^C \in (0,1)$). There is no equilibrium with truth-telling ($\tau^C = 1$) or uninformative communication ($\tau^C = 0$) in period 1.*

As in the single firm model, we have an equilibrium with noisy communication for intermediate values of w is noisy communication, and the informativeness of the reports $\tau^C \in (0,1)$ in equilibrium is characterized by the indifference condition (8). The Proposition follows from a standard intermediate value argument. The interval condition on w in Proposition 6 ensures that the LHS of (8) is strictly larger than x given $\tau^C = 1$ and strictly smaller than x given $\tau^C = 0$. Since the LHS is continuous in τ^C for $\tau^C \in [0,1]$, there must be at least one interior value of τ^C for which the indifference condition is satisfied.

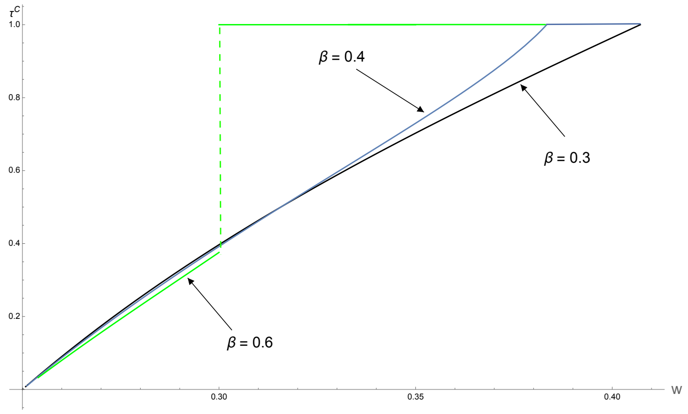


Figure 3: Truth-telling probability in period 1 given $\omega_1 = A$ $(\gamma = 0.9, x = 0.05)$

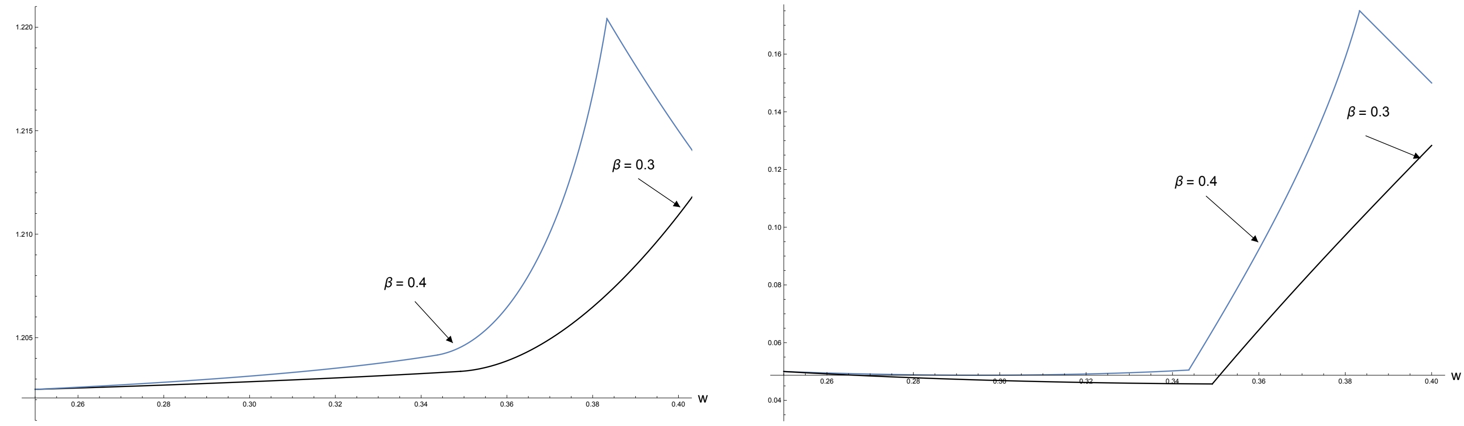
The LHS is a complex function of β and τ^C, and as a result explicit solutions for τ^C are also complex, but we are able to make some qualitative observations. We know that at the highest interior solution for $\tau^C \in (0,1)$, the LHS expression must be crossing fx from below. Accordingly, the fact that the LHS is decreasing in w immediately implies that the highest interior solution forτ^C must be increasing in w. In other words, as in the single firm model, a higher expected cost of visit makes R's decision to visit less attractive and consequently lying less attractive. Finally, the argument we have used to establish the existence of a truth-telling equilibrium and an uninformative equilibrium implies that in the intermediate range of w there exists no equilibrium that features $\tau^C = 1$ or $\tau^C = 0$.

Figure 3 plots τ^C in the most informative equilibrium as a function of w. If no truth-telling equilibrium exists, there is a unique mixed strategy equilibrium such that $\tau^C \in (0,1)$. If instead the truth-telling equilibrium exists, it may coexist with mixed equilibria (see details below). We see that for all β's considered in the example, the equilibrium value of τ^C is increasing in w as in the single firm case in Section 2.

Let us consider how w interacts with β in equilibrium through Figure 3. When w is small, β does not affect τ^C significantly, where all curves are clustered. There, a low τ^C makes cross-checking ineffective as both firms are likely to misreport. In fact, the equilibrium τ^C may be lower for higher β, since given $m_1^{S_1} = m_1^{S_2} = B$, the conditional probability of visit and hence the benefit of lying are higher. This indicates the credibility effect of multiple reports we discussed earlier, which alone has a negative effect on the accuracy of the reports.

On the other hand, as w and thus τ^C also increase, cross-checking substantially reduces the benefit of lying. Hence for the intermediate values of w, β has a significant effect on τ^C, although in all cases truth-telling is feasible for w large enough. If β is large, truth-telling is already feasible for moderate values of w, since $\tau^C = 1$ implies that independent reporting essentially *always* leads to the lies being exposed. The case where $\beta = 0.6$ features a discontinuous jump from $\tau^C < 1$ to truth-telling whereas we observes a continuous transition to truth-telling in the cases of $\beta = 0.3$ and 0.4.[22]

[22]For $\beta = 0.6$ and similar high values of β, the truth-telling equilibrium may coexist with up to two noisy equilibria, as two of the three solutions to (8) give $\tau^C \in [0,1]$. We focus on the truth-telling equilibrium when it exists.



(a) *R*'s total expected payoff

(b) *S*'s total expected *accounting* profit

Figure 4: Equilibrium payoff with respect to w ($\gamma = 0.9$, $x = 0.05$)

Figure 3 suggests that when access to news reports is cheap, not only is the accuracy of the reports low, but also competition is less effective in increasing the credibility of news reports. This is because a media firm is less concerned with cross-checking by the audience as the opponent is less likely to reveal truthfully. When the audience's decision to visit is highly manipulable due to a cheap cost of access, the temptation to lie to induce future visits, which both firms are susceptible to, outweighs the discipline from cross-checking.

We present R's expected payoff and S's expected accounting profit in Figure 4. Here we focus only on two values of β because the scale of the payoffs varies significantly across different β's. For the two values of β considered in the Figure, we see that the expected payoff of R and the expected accounting profit of each individual firm as functions of w in the most informative equilibrium. The expected payoff of R is strictly increasing in w as long as $\tau^C < 1$, after which it decreases in w as informativeness is fixed. This is partly in contrast to Figure 2a, where R's payoff is decreasing in w when w is relatively small. In fact, as β becomes close to 0 R's expected payoff exhibits the same feature that it is decreasing in w when w is relatively small. This indicates that a higher degree of independence β enhances the positive effect of an increase in w (through an increase in τ^C) on R's expected payoff.

The expected accounting profit of the firms increases in w as long as there is a probability of visit in period 1 and $\tau^C < 1$, similarly to what we saw in Section 2. In addition, we see that the firms are better off with a higher degree of independence β because it functions as a commitment device for reporting truthfully more often, which in turn increases the probability of visit. However, while the media firms benefit from ex ante independence, once one firm observes the other firm's report in sequential communication, the former has no incentive to send a contradicting report, as we saw earlier. In reality, especially for online news items that spread very fast, committing to independent reporting may be more difficult, since news outlets would often observe each other's reports before readers access them.

4 Remarks on Pricing

So far we have assumed away pricing. In reality, accessing the full contents of a newspaper often requires some payment of a fee. In the online news market, this typically takes the form of a subscription fee that grants unlimited access to news articles for a specific period of time. Practices differ significantly across outlets. Tabloid newspapers tend to offer significantly more freely available contents than quality newspapers (e.g. New York Times). The latter however regularly offer drastically discounted subscription fees. Another interesting feature of the online news market is that subscription readers generate revenue via more than just the subscription payment. Even clicks by subscribers may generate advertising revenue well as valuable data (from navigating the website) which can be used or sold to a third party. Thus Media firms would still have an incentive to induce actual visits after a subscription has been purchased.

In what follows, we discuss simple extensions of our single- and multiple firm models by incorporating endogenously priced subscriptions. A subscription, once bought, allows readers to access reports without any additional payment from that point onwards. Subscribers however still incur the fixed and random costs $w+v_t$ for each visit, which capture non-monetary costs (opportunity cost, cognitive effort to process information). For technical convenience, in the following we explicitly interpret R as a continuum of receivers with mass one, where the distribution of v_t is i.i.d. across all receivers and periods. The receivers are thus all ex ante identical. For both extensions (single and two firm models), except for the assumptions on pricing and purchasing decisions we will explicitly describe below, all the other aspects of the model are the same as in the corresponding original model without pricing we saw earlier.

The key observation with respect to subscription pricing is that, as long as the media firms obtain revenue from clicks, a paid subscription service does not fundamentally change the incentive to misreport from what we saw in the previous sections, since ounce purchased, a subscription does not alter the marginal cost of visit for receivers/consumers or the marginal revenue from visit for the media firm. Using this feature, we will see that the media firm extracts surplus from consumers in the case of single firm, and Bertrand competition leads both firms to charge a zero price for subscription in the two firm model.

First, let us add a subscription fee to the single firm model introduced in Section 2. At the beginning of period 1, before S and receivers learn their private information (ω_1 for S, and v_1 for the receivers), S offers a subscription at price p and each receiver decides whether to buy it. We assume for simplicity that this is the only point at which receivers can buy a subscription. Unless a receiver purchases the subscription he cannot acquire m_t before choosing a_t in either period. As in the original model, every receiver observes m_t at the end of period t after they have chosen a_t, regardless of whether they have acquired m_t before choosing a_t.[23]

Remark 1. *There exists an equilibrium in which S sets $p^* > 0$, every receiver buys the subscription, and the subsequent behaviour of the players is as in the equilibrium described in Section 2.*

This is surplus extraction by a monopolist, where p^* makes the (ex ante identical) consumers

[23]The proofs of the following Remarks are in Appendix A.

indifferent between purchasing and not purchasing the subscription, which may apply to an outlet that provides highly differentiated news articles. On the other hand, if we assume the consumers facing the firm are heterogeneous, the media firm may charge a very low or zero price to attract a large readership, especially if the relative size of the advertising revenue is large. The main observation from Remark 1 however is that our results regarding the relationship between the informativeness of reports and the (non-monetary) cost of access to news continue to hold.

Let us now add subscription services with Bertrand pricing to the two-firm model discussed in Section 3. At the beginning of period 1, before the two media firms and receivers learn their private information, they simultaneously offer subscriptions at respective prices p_1 and p_2, while each (ex ante identical) receiver decides whether to buy one subscription or none (they cannot buy two, for simplicity). As earlier, this is the only point at which a receiver can buy a subscription. A receiver can only visit the firm that he subscribes to before choosing a_t. Each receiver observes $m_t^{S_1}$ and $m_t^{S_2}$ at the end of period t after they have chosen a_t, regardless of whether he acquired a report beforehand. Reports can thus be cross-checked at the end of period t.

Remark 2. *There exists a symmetric equilibrium in which, on the equilibrium path, firms charge a subscription price of zero, every receiver buys a subscription with probability 1/2 from each firm, and the subsequent behaviour of the players is as in the equilibrium described in Section 3.*

As already mentioned, Bertrand competition ensures that subscriptions are offered for free, and our results in 3 continue to hold. As a technical note, it is easy to see that the symmetric equilibrium that we describe above in Remark 2 still exists if we allow receivers to purchase two subscriptions, as long as they can only visit one firm in each period. While the subscriptions are free, the receivers are indifferent between holding one and two subscriptions. On the other hand, allowing receivers to visit multiple outlets in period $t$would require the calculation of the marginal value of a second consultation, which is complex and beyond the scope of the current extension.

5 Conclusion

The driving force behind information distortion in our model is the media firm's incentive to generate uncertainty about tomorrow's state in order to encourage future clicks by potential readers. We have found that for an intermediate cost level, a higher cost of consultation increases the informativeness of communication by dampening a reader's responsiveness to uncertainty generating messages and thereby discouraging the media firm to distort its private information. Moreover, in this parameter region, both the media firm and the readers might be better off with a higher cost of access. We argue that this may explain why there is a widespread concern that media reports have become less accurate in recent years. According to our model, this would be an indirect consequence of the reduced cost of access stemming from the rise of online news.

The main insights of our paper appear to extend beyond the context of media. Similar incentives to misreport private information may well be present in many repeated relationships between an uninformed party and an expert who sells his reports over time (for example doctor-patient relations, or financial advisers, etc.).

6 Appendix A: Proofs and Calculations

For expositional convenience, we first prove Proposition 2 for the intermediate range of w so that the structure of the equilibrium clearly understood and presented. We then give proofs for Proposition 1 and the other remaining Propositions.

6.1 Preliminary Results for Proposition 2

Before proving Proposition 2 for the intermediate range of w as specified in (1), we present a series of lemmas in order to narrow down the class of strategy profiles we need to consider for the derivation of the informative equilibrium.

Lemma 1. *In any equilibrium S reports truthfully in period 2.*

Proof. In period 2, S has a strict incentive to report truthfully (i.e. to send $m_2 = \omega_2$) in any putative equilibrium. Since it is the last period, there is no incentive to induce a future visit by misreporting, and sending $m_2 = \omega_2$ yields an extra truth-telling reward. This observation allows us to focus on the informativeness of the report in period 1. □

Lemma 2. *a) No equilibrium features uninformative communication in period 1. b) No equilibrium features fully informative communication in period 1.*

Proof. **Step 1** The proof of part a) is by contradiction. There are two cases of uninformative communication in period 1 to consider. The first is the case where both $m_1 = A$ and $m_1 = B$ are sent with strictly positive probability. In this case, both $m_1 = A$ and $m_1 = B$ yield the same probability of consultation in period 2 since both are uninformative. However if it is the case, since $R's$ consultation behaviour in period 2 is independent of m_1, S is strictly better off reporting truthfully with probability 1, as this yields the extra truth-telling benefit. Therefore, this case is excluded.

The second case is such that either $m_1 = A$ or $m_1 = B$ is sent with ex ante probability one in period 1. Assume that S always sends the same message $\widetilde{m}_1$=$J \in \{A,B\}$ in period 1. Now, the out-of-equilibrium belief in period 1 that maximizes S's incentive to choose $\widetilde{m}_1$ in period 1 is that, given $m_1' \neq \widetilde{m}_1$, R assigns probability one to $\omega_1 = A$. Consider thus a putative equilibrium in which there is some $J \in A,B$ such that S always sends message $\widetilde{m}_1 = J$ in period 1 and if R observes $m_1' = -J = \{A,B\} \setminus J$ in period 1, he assigns probability 1 to $\omega_1 = A$. We now show that in such a putative equilibrium, S has a deviation incentive in period 1 when $\omega_1 = -J$. Let $\omega_1 = -J$. Note that sending m_1'= $-J$ yields the expected payoff $2fx$ across the two periods if $w > 1-\gamma$ as implied by (1). Indeed, S obtains the truth-telling payoff in period 1, and anticipates that he will receive it as well in period 2 though R will not consult in period 2. On the other hand, sending $\widetilde{m}_1 = J$ yields

$$f\,(1+\theta-2\theta\gamma-2w)+fx.$$

Note that $(1+\theta-2\theta\gamma-2w)$ is positive. After receiving message $\widetilde{m}_1 = J$ in period 1, R's belief about ω_1 remains unchanged, i.e. R considers that he has not learned anything concerning the state in period 1. In consequence, he visits with probability $(1+\theta-2\theta\gamma-2w)$ in period 2. The second element fx in the above expression is the expected truth-telling payoff in period 2. Now, note that

$$fx > f(1+\theta-2\theta\gamma-2w)$$

rewrites as

$$w > \frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2},$$

as assumed in (1). It follows that in this putative equilibrium, S has a strict incentive to send $m_1' = -J$ in period 1 if $\omega_1 = -J$. We may conclude that case 2 is excluded, i.e. there exists no equilibrium such that either $m_1 = A$ or $m_1 = B$ is sent with ex ante probability 1 in period 1.

Step 2 The proof of part b) is also by contradiction and there are again two cases to consider. Consider first the case in which S's equilibrium strategy in period 1 is such that $m_1 = \omega_1$ for any realization of ω_1. In such an equilibrium, the assumption that w is in the intermediate range (1) implies that R consults with probability zero in period 2 after $m_1 = A$ and with positive probability after $m_1 = B$. Let us now show that S then strictly prefers to deviate to sending $m_1 = B$ given $\omega_1 = A$. Let $\omega_1 = A$. Message $m_1 = A$ leads to a visiting probability of zero in period 2, and thus yields an expected payoff of fx. In contrast, $m_1 = B$ leads to a strictly positive probability of visit in period 2, namely probability $1-2w$. Message $m_1 = B$ thus yields expected payoff $f(1-2w)$. Now, note that $f(1-2w) > fx$ is equivalent to $\frac{1-x}{2} > w$, which is consistent with the assumption on w given in (1). S thus has a strict incentive to deviate to $m_1 = B$ when $\omega_1 = A$, so the equilibrium breaks down.

Consider next the case in which S's equilibrium strategy in period 1 is such that $m_1 = A$ if $\omega_1 = B$ and $m_1 = B$ if $\omega_1 = A$. In this case, the incentive to deviate given $\omega_1 = A$ is further reinforced, relative to the first case. That is, deviating to $m_1 = A$ given $\omega_1 = A$ indeed not only yields a positive probability of consultation tomorrow (as opposed to $m_1 = B$) , but the deviation also yields the extra truth-telling benefit. So the considered equilibrium breaks down. Hence we conclude that there cannot be an equilibrium with fully informative communication in period 1. □

The intuition for part a) is that in a putative equilibrium with uninformative communication in period 1, the report in period 1 should not have any influence on R's decision to visit in period 2. However, given that the report has no influence, S would strictly prefer to reveal truthfully in period 1 for the truth-telling reward. The intuition for part b) is that if x is small and communication is fully informative in period 1, then S must have an incentive to deviate in the period to the message indicating state B even when the state is A, in order to induce a future visit. Our next two Lemmas concern R's consultation behaviour in equilibrium.

Lemma 3. *No equilibrium features zero probability of consultation in period 2.*

Proof. The proof is by contradiction. Assume first that the probability of consultation in period 2 is zero, and that both messages ($m_1 = A$ and B) are sent with strictly positive probability in equilibrium. In this case, m_1 does not affect R's consultation decision on-the-equilibrium path, since the probability of visit remains zero. This implies that S is always strictly better off sending $m_1 = \omega_1$ in period 1, as this yields the truth-telling benefit. Given truth-telling in period 1, the probability of visit in period 2 after $m_1 = B$ is $1-2w > 0$, which contradicts the assumption that the probability of consultation in period 2 is zero.

Second, assume that the probability of consultation in period 2 is zero, and that either $m_1 = A$ or B is sent with probability one in period. However, this implies an equilibrium with uninformative communication, which we have ruled out in Lemma 2.

Thus we conclude that the probability of visit is strictly positive in equilibrium. □

Lemma 3 implies that some equilibrium message in period 1 has to lead to consultation with positive probability in period 2. This opens up the possibility that one of the messages available to S in period 1 makes consultation in period 2 more likely than the other, thereby creating an incentive to send the former message in order to maximize the probability of consultation in period 2. The next Lemma demonstrates that R's consultation decision in period 2 is affected significantly by m_1, if S's communication strategy for period 1 is what we call a *simple* communication strategy.

Lemma 4. *In any equilibrium featuring a simple communication strategy in period 1, R consults with probability zero in period 2 if* $m_1 = A$, *and with strictly positive probability if* $m_1 = B$.

Proof. Consider an equilibrium in which S's strategy in period 1 is a simple communication strategy. From Lemma 2, any equilibrium features an informative communication strategy in period 1. Therefore, we must have $\tau_A^1 > 0$. This implies that both $m_1 = A$ and $m_1 = B$ are sent with strictly positive probability in period 1. We also know from Lemma 3 that any equilibrium must feature a positive probability of consultation in period 2. In the following we consider R's consultation behaviour in period 2 when $m_1 = A$ and $m_1 = B$, respectively.

First, if $m_1 = A$ under a simple communication strategy, R believes $P(\omega_1 = A \mid m_1 = A) = 1$. Note that if R does not consult in period 2, then he chooses action A in the period. Consulting in period 2 guarantees the correct action (as S reports truthfully) but comes at the cost $w + v_2$. Thus R consults in period 2 if and only if $1 - w - v_2 \geq P(\omega_2 = A \mid m_1 = A) = \gamma$. The assumption that w is in the intermediate range (1) implies $w + \gamma > 1$, which implies that R consults in period 2 with probability zero after $m_1 = A$.

Let $m_1 = B$. We know from Lemma 3 that any equilibrium must feature a strictly positive probability of consultation in period 2. We furthermore know that R consults with probability zero after $m_1 = A$. It follows that R must consult with strictly positive probability after $m_1 = B$. □

Since the fixed cost of visit w is in the intermediate range (1), R never makes a costly visit in period 2 if he is sure that $\omega_1 = A$ (entailing low uncertainty about ω_2), which is the case here given $m_1 = A$.

Our last Lemma for Proposition 2 establishes that the equilibrium communication strategy in period 1 must indeed be a simple communication strategy, which implies that focusing on simple strategies in both periods is without loss of generality.

Lemma 5. *In any equilibrium that features a positive probability of consultation in period 2, the communication strategy in period 1 is a simple communication strategy.*

Proof. The proof is organized as follows. We show by contradiction that in equilibrium, $m_1 = B$ must induce a weakly higher probability of consultation in period 2 than $m_1 = A$. In turn, we show that this immediately implies that S reports $m_1 = B$ with probability 1 when $\omega_1 = B$.

Note that the report is truthful in period 2 in any equilibrium. Lemma 2 indicates that any equilibrium involves informative communication in period 1. If the report is informative in period 1, R is (weakly) more likely to make a costly visit in period 2, if the message $m_1 \in \{A, B\}$ shifts R's posterior towards $\omega_1 = B$ than the other message.

Assume that $m_1 = A$ induces a weakly higher probability of consultation in period 2 than $m_1 = B$. If this is the case, then S reports $m_1 = A$ with probability 1 when $\omega_1 = A$, since by doing so S obtains both the truth-telling reward and induces a weakly higher probability of consultation in period 2 than by sending $m_1 = B$. However, if an equilibrium is such that S sends $m_1 = A$ for sure whenever $\omega_1 = A$, then the report $m_1 = B$ induces a strictly higher posterior that the state is $\omega_1 = B$ than $m_1 = A$. Indeed, note that if $\tau_A^1 = 1$, then

$$P(\omega_1 = B \mid m_1 = A) = \frac{(1-\theta)(1-\tau_B^1)}{(1-\theta)(1-\tau_B^1)+\theta} < 1$$

while

$$P(\omega_1 = B \mid m_1 = B) = \frac{(1-\theta)\tau_B^1}{(1-\theta)\tau_B^1+\theta(0)} = 1.$$

Consequently, the report $m_1 = B$ must induce a higher probability of consultation in period 2 than $m_1 = A$, but this contradicts the assumption that $m_1 = A$ induces a weakly higher probability of consultation. We have thus shown by contradiction that $m_1 = B$ must induce a strictly higher probability of consultation in period 2 than $m_1 = A$. Note furthermore that this fact immediately implies that S reports $m_1 = B$ with probability 1 when $\omega_1 = B$, since by doing so S obtains both the truth-telling reward and induces a strictly higher probability of consultation in period 2 than by sending $m_1 = A$. □

The Lemmas presented so far considerably narrow down the class of strategy profiles we need to consider. We know that if there exists an equilibrium, it satisfies the following description. The equilibrium must feature a simple partially informative communication strategy in period 1 (Lemmas 2 and 5) and truth-telling in period 2 (Lemma 1). Furthermore, R consults with strictly positive probability in period 2 if $m_1 = B$ and with probability 0 if $m_1 = A$ (Lemmas 3 and 4). The Proposition shows the unique equilibrium that pins down the value of τ_A^1 and the probability of consultation in period 2 given $m_1 = B$.

6.2 Proposition 2

Proof. **Part a)** We know from the previous Lemmas that if there exists an equilibrium, it satisfies the following description. It features a simple partially informative communication strategy in period 1 and the fully informative simple communication strategy in period 2. Furthermore, R consults with strictly positive probability in period 2 if $m_1 = B$ and with probability 0 if $m_1 = A$. In the following proof, we show that there exists a unique equilibrium satisfying the above description, and characterize the value of τ_A^1 as well as the consultation probability in period 2 given $m_1 = B$. We first derive the probability of visit by R in period 2 given $m_1 = B$. Given this probability, we consider S's choice between reporting truthfully ($m_1 = A$) or sending $m_1 = B$ when $\omega_1 = A$. We then examine the parameter conditions under which there exists an equilibrium of the above described type. We show that it must be unique, if it exists, and obtain the explicit solution for τ_A^1.

Assume in what follows that S uses a simple strategy in both periods, where the strategy in period 2 is fully informative. Note that regardless of the message in period 1, if R does not visit in period 2, he chooses $a_2 = A$ in period 2. Therefore, given $m_1 = B$, a visit by R in period 2 requires

$$1 - w - v_2 > P(\omega_1 = A \mid m_1 = B)\gamma + P(\omega_1 = B \mid m_1 = B)\frac{1}{2}, \tag{9}$$

where $P(\omega_1 = B \mid m_1 = B) = \frac{1-\theta}{1-\theta+\theta(1-\tau_A^1)}$. Hence, (9) is equivalent to

$$1 - w - v_2 > \left(1 - \frac{1-\theta}{1-\theta+\theta(1-\tau_A^1)}\right)\gamma + \frac{1-\theta}{1-\theta+\theta(1-\tau_A^1)}\frac{1}{2},$$

which rewrites as

$$v_2 < \frac{\theta - 2\theta\gamma - 2\theta\tau_A^1 + 2\theta\gamma\tau_A^1 + 1}{2\left(1-\theta\tau_A^1\right)} - w.$$

Thus, given $m_1 = B$, R makes a costly visit in period 2 with positive probability if and only if

$$\frac{\theta - 2\theta\gamma - 2\theta\tau_A^1 + 2\theta\gamma\tau_A^1 + 1}{2\left(1-\theta\tau_A^1\right)} - w > 0. \tag{10}$$

Let

$$\varphi_B(\tau_A^1, w, \gamma) \equiv \frac{\frac{\theta-2\theta\gamma-2\theta\tau_A^1+2\theta\gamma\tau_A^1+1}{2\left(1-\theta\tau_A^1\right)} - w}{\frac{1}{2}} = \frac{\theta - 2\theta\gamma - 2\theta\tau_A^1 + 2\theta\gamma\tau_A^1 + 1}{1-\theta\tau_A^1} - 2w. \tag{11}$$

If (10) holds, then conditional on $m_1 = B$, R makes a visit in period 2 with probability

$$\max\left\{\min\left\{\varphi_B(\tau_A^1, w, \gamma), 1\right\}, 0\right\}.$$

Next, let us examine the incentives of S in period 1. Recall that S reports truthfully when $\omega_1 = B$. Suppose that instead $\omega_1 = A$. Sending $m_1 = A$ yields the truth-telling payoff xf but zero probability of visit in period 2. Therefore, given $\omega_1 = A$, S is indifferent between $m_1 = A$ and $m_1 = B$ in period 1 if and only if $fx + 0 = 0 + f\varphi_B(\tau_A^1, w, \gamma)$, which is satisfied when

$$\varphi_B(\tau_A^1, w, \gamma) = x. \tag{12}$$

Note that $\varphi_B(\tau_A^1, w, \gamma)$ is strictly increasing in τ_A^1, since from (10)

$$\frac{\partial \varphi_B(\tau_A^1, w, \gamma)}{\partial \tau_A^1} = \frac{\theta\,(1-\theta)\,(2\gamma-1)}{\left(1-\theta\tau_A^1\right)^2} > 0,$$

which implies that there exists a unique solution to (12) such that $\tau_A^1 \in (0,1)$ if $\varphi_B(1,w,\gamma) > x$ and $\varphi_B(0,w,\gamma) < x$. Note that $\varphi_B(1,w,\gamma) > x$ rewrites as $w < \frac{1-x}{2}$, which is consistent with the assumption that w is in the intermediate range (1). On the other hand, $\varphi_B(0,w,\gamma) < x$ is rewritten $w > \frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}$. Thus, there exists a unique equilibrium in which S uses a simple strategy in both periods, the strategy in period 2 being fully informative, if w satisfies $w \in \left(\frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}, \frac{1-x}{2}.\right)$. The interval is well-defined since $\frac{\theta(2\gamma-1)}{2} > 0$. The left hand side inequality is as stated in the Proposition, while (1) implies the right hand side inequality. Solving the equality (12) for τ_A^1, we obtain the unique solution

$$\tau_A^{1*} = \frac{2w+\theta\,(2\gamma-1)+x-1}{\theta\,(2\gamma+2w+x-2)}$$

as stated.

Part b) Recall that $\gamma > \frac{1}{2}$ and that the assumption on w (1) implies $1-x-2w > 0$. We have

$$\begin{aligned}
\frac{\partial \tau_A^{1*}}{\partial f} &= 0, \\
\frac{\partial \tau_A^{1*}}{\partial w} &= \frac{2(2\gamma-1)(1-\theta)}{\theta(2\gamma+2w+x-2)^2} > 0, \\
\frac{\partial \tau_A^{1*}}{\partial \gamma} &= \frac{2(1-x-2w)(1-\theta)}{\theta(2\gamma+2w+x-2)^2} > 0, \\
\frac{\partial \tau_A^{1*}}{\partial x} &= \frac{(2\gamma-1)(1-\theta)}{\theta(2\gamma+2w+x-2)^2} > 0.
\end{aligned}$$

□

6.3 Proposition 1

Proof. **Outline:** Recall that throughout, as in the main text, we keep the assumption that x is small so that $x < (1-\theta)\,(2\gamma-1)$, which implies $1-\gamma < \frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}$.

Step 1 proves the following three useful properties of equilibrium behaviour of both S and R. First, any equilibrium must feature truth-telling in period 2 regardless of the state. Second, any equilibrium must feature a positive probability of consultation in period 2. Third, any equilibrium with informative communication in period 1 must feature a *simple* communication strategy, that is, if $m_1 = B$ then $\omega_1 = B$ (truth-telling when the state is B).

Step 2 shows that under the conditions imposed on part a), there exists no equilibrium featuring truth-telling in both periods.

Step 3 examines the case where $w \in \left[1-\gamma, \frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}\right]$ and shows that there exists no equilibrium that features an informative simple communication strategy in period 1.

Step 4 examines the case where $w < 1-\gamma$ and shows the same result as that of Step 3 holds.

Step 5 shows that there exists an equilibrium that features uninformative communication such that $m_1 = B$ regardless of the state in period 1 and truth-telling in period 2.

Step 6 confirms the statement.

Step 1 The fact that any equilibrium must feature truth-telling in period 2 regardless of the state follows from the same argument as that of Lemma 1. The fact that any equilibrium must feature a positive probability of consultation in period 2 follows from the same argument as in the proof of Lemma 3.

We now show that any equilibrium with informative communication in period 1 must feature a simple communication strategy, that is, truth-telling in period 1 if $\omega_1 = B$. We invoke the same argument as in the proof of Lemma 5 and we repeat it for convenience in what follows. We show by contradiction that in equilibrium, $m_1 = B$ must induce a weakly higher probability of consultation in period 2 than $m_1 = A$. In turn, we show that this immediately implies that S reports $m_1 = B$ with probability 1 when $\omega_1 = B$.

Suppose that the message in period 1 is informative in period 1. Clearly, R is (weakly) more likely to make a costly visit in period 2, if the message $m_1 \in \{A,B\}$ shifts R's posterior towards $\omega_1 = B$ relative to the other message.

Assume that $m_1 = A$ induces a weakly higher probability of consultation in period 2 than $m_1 = B$. If this is the case, then S reports $m_1 = A$ with probability 1 when $\omega_1 = A$, since by doing so S obtains both the truth-telling benefit and induces a weakly higher probability of consultation in period 2 than by sending $m_1 = B$. However, if an equilibrium is such that S sends $m_1 = A$ for sure whenever $\omega_1 = A$, then the report $m_1 = B$ induces a strictly higher posterior that the state is $\omega_1 = B$ than $m_1 = A$. Indeed, note that if $\tau_A^1 = 1$, then

$$P(\omega_1 = B \mid m_1 = A) = \frac{(1-\theta)(1-\tau_B^1)}{(1-\theta)(1-\tau_B^1)+\theta} < 1$$

while

$$P(\omega_1 = B \mid m_1 = B) = \frac{(1-\theta)\tau_B^1}{(1-\theta)\tau_B^1+\theta(0)} = 1.$$

Consequently, if communication in period 1 is informative, then $m_1 = B$ must induce a higher probability of consultation in period 2 than $m_1 = A$, but this contradicts the assumption that $m_1 = A$ induces a weakly higher probability of consultation. We conclude then by contradiction that if communication in period 1 is informative, then $m_1 = B$ must induce a strictly higher probability of consultation in period 2 than $m_1 = A$. Note furthermore that this fact immediately implies that, if communication in period 1 is informative, then S reports $m_1 = B$ with probability 1 when $\omega_1 = B$, since by doing so S obtains both the truth-telling benefit and induces a strictly higher probability of consultation in period 2 than by sending $m_1 = A$.

Step 2 Let us show that there exists no equilibrium that features truth-telling in period 1. Assume that there is an equilibrium with truth-telling in both periods. The visiting probabilities of R in period 2 satisfy the following. We have

$$P(d_2 = 1 \mid m_1 = A) = \frac{1-\gamma-w}{\frac{1}{2}}$$

if $w \leq 1-\gamma$ and $P(d_2 = 1 | m_1 = A) = 0$ if $w > 1-\gamma$. On the other hand,

$$P(d_2 = 1 \mid m_1 = B) = \frac{\frac{1}{2}-w}{\frac{1}{2}}.$$

We now consider the incentive of S in period 1. Let $\omega_1 = B$, in which case S strictly prefers to report truthfully in period 1, as this yields the truth-telling benefit and also maximizes the visiting probability in period 2. Let $\omega_1 = A$. At period 1, the expected payoff of sending $m_1 = A$ is given by fx if $w \in (1-\gamma, \frac{1}{2} - x - \frac{\theta(2\gamma-1)}{2}]$ since the probability of visit in period 2 is zero, and the corresponding expected payoff for $w \leq 1-\gamma$ is given by

$$fx + 2(1-\gamma-w)f.$$

Meanwhile, the expected payoff when sending $m_1 = B$ is given by $2(1-2w)f$.

Therefore, when $w \leq 1-\gamma$, S strictly prefers to report $m_1 = B$ when $\omega_1 = A$ (i.e. deviating from truth-telling) if and only if

$$fx + 2(1-\gamma-w)f < 2(1-2w)f,$$

which reduces to $x < 2\gamma - 1$. This inequality is readily implied by

$$x < \left(\frac{1}{2} - \frac{\theta(2\gamma-1)}{2}\right) - (1-\gamma),$$

for $\theta \in [\frac{1}{2}, 1)$.

If on the other hand $w \in (1-\gamma, \frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}]$, S strictly prefers sending $m_1 = B$ when $\omega_1 = A$ if and only if

$$fx < (1-2w)f,$$

which is equivalent to $x < 1-2w$. The inequality rewrites as $w < \frac{1-x}{2}$, which holds true given $w \in (1-\gamma, \frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}]$.

Thus from this step we conclude that no equilibrium can feature truth-telling in both periods.

Step 3 In this step, we consider $w \in \left[1-\gamma, \frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}\right]$, where R never visits in period 2 if he is certain that the state is A. We refer to our equilibrium characterization (in the proof of Proposition 1) that assumed

$$w \in \left(\frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}, \frac{1-x}{2}\right).$$

The proof shows that given $w \geq 1-\gamma$, an informative equilibrium that features a simple communication strategy exists only if

$$w > \frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}.$$

The inequality clearly does not hold for the range of w we consider and thus there is no informative equilibrium.

Step 4 Let us focus on the case where $w < 1-\gamma$. Consider a putative equilibrium with a simple communication strategy ($m_1 = B$ when $\omega_1 = B$). R's visiting probability in period 2 after $m_1 = A$ is given by

$$P(d_2 = 1 \mid m_1 = A) = \frac{1-\gamma-w}{\frac{1}{2}}.$$

If $m_1 = B$, a positive probability of visiting in period 2 requires

$$1 - w - v_2 > P(\omega_1 = A \mid m_1 = B)\gamma + P(\omega_1 = B \mid m_1 = B)\frac{1}{2}.$$

Note that

$$P(\omega_1 = B | m_1 = B) = \frac{1-\theta}{1-\theta+\theta(1-\tau_A^1)}.$$

Thus given $m_1 = B$, R visits in period 2 if and only if

$$1 - w - v_2 > \left(1 - \frac{1-\theta}{1-\theta+\theta(1-\tau_A^1)}\right)\gamma + \frac{1-\theta}{1-\theta+\theta(1-\tau_A^1)}\frac{1}{2},$$

which is equivalent to $\frac{1}{2(1-\theta\tau_A^1)}\left(\theta - 2\theta\gamma - 2\theta\tau_A^1 + 2\theta\gamma\tau_A^1 + 1\right) - w > v_2$. Thus we have

$$P(d_2 = 1 \mid m_1 = B) = \frac{\frac{\theta - 2\theta\gamma - 2\theta\tau_A^1 + 2\theta\gamma\tau_A^1 + 1}{2(1-\theta\tau_A^1)} - w}{1/2},$$

provided that the numerator in the above expression is positive (note that it is never larger than 1/2).

We now examine S's choice of message in period 1. Assume that $\omega_1 = A$. Sending A yields the truth-telling benefit fx, and R makes visit in period 2 with probability $\frac{1-\gamma-w}{\frac{1}{2}}$. If

$$\frac{\theta - 2\theta\gamma - 2\theta\tau_A^1 + 2\theta\gamma\tau_A^1 + 1}{2\left(1-\theta\tau_A^1\right)} - w > 0,$$

then R visits with probability

$$\varphi_B(\tau_A^1, w, \gamma) = \frac{\frac{1}{2(1-\theta\tau_A^1)}\left(\theta - 2\theta\gamma - 2\theta\tau_A^1 + 2\theta\gamma\tau_A^1 + 1\right) - w}{\frac{1}{2}} \tag{13}$$

after $m_1 = B$. If $\omega_1 = A$, S is thus indifferent between $m_1 = A$ and $m_1 = B$ in period 1 if and only if

$$fx + f\frac{1-\gamma-w}{\frac{1}{2}} = f\frac{\frac{1}{2\left(1-\theta\tau_A^1\right)}\left(\theta-2\theta\gamma-2\theta\tau_A^1+2\theta\gamma\tau_A^1+1\right)-w}{\frac{1}{2}}. \tag{14}$$

Note first that $\varphi_B(\tau_A^1, w, \gamma) \leq 1$ for any $\tau_A^1 \in [0,1]$. This is because the marginal value of consulting, ignoring the cost of consultation, is at most $\frac{1}{2}$. Note furthermore that

$$\frac{\partial \varphi_B(\tau_A^1, w, \gamma)}{\partial \tau_A^1} = \frac{\frac{1}{2}\theta\left(1-\theta\right)\frac{2\gamma-1}{\left(\theta\tau_A^1-1\right)^2}}{\frac{1}{2}} > 0, \tag{15}$$

so that $\varphi_B(\tau_A^1, w, \gamma)$ is strictly increasing in τ_A^1. Note also that $\varphi_B(\tau_A^1, w, \gamma)$ is a continuous function of τ_A^1.

For the rest of this step, let us use a monotonicity argument to study the existence of an equilibrium in simple strategies. Because of the continuity and monotonicity in (15), there exist a unique solution to (14) and hence an equilibrium if

$$\varphi_B(1, w, \gamma) = \frac{\frac{1}{2}-w}{\frac{1}{2}} \geq x + \frac{1-\gamma-w}{\frac{1}{2}}$$

and

$$\varphi_B(0, w, \gamma) = \frac{1+\theta-2w-2\theta\gamma}{2\times\frac{1}{2}} < x + \frac{1-\gamma-w}{\frac{1}{2}},$$

since (14) requires

$$\varphi_B(\tau_A^1, w, \gamma) = x + \frac{1-\gamma-w}{\frac{1}{2}} \tag{16}$$

in equilibrium. However, our assumption $w < 1-\gamma < \frac{1-x}{2}$ implies

$$\varphi_B(1, w, \gamma) = \frac{\frac{1}{2}-w}{\frac{1}{2}} < x + \frac{1-\gamma-w}{\frac{1}{2}},$$

which together with the monotonicity shown in (15) also implies (16) never holds since

$$\varphi_B(\tau_A^1, w, \gamma) < x + \frac{1-\gamma-w}{\frac{1}{2}}$$

for any $\tau_A^1 \in [0,1]$. Thus we conclude that if $w < 1-\gamma$ there is no informative equilibrium.

Step 5 We here show that if

$$x < (1-\theta)(2\gamma-1),$$

there exists an equilibrium that features uninformative communication in period 1 and truth-telling in period 2. The equilibrium is constructed as follows. S always sends $m_1 = B$ in period 1, and if R receives the off-the-equilibrium message $m_1 = A$ in period 1, he assigns probability one to $\omega_1 = A$. We now check that S has no incentive to deviate from $m_1 = B$ in period 1.

Let us focus on $\omega_1 = A$. Given the off-the-equilibrium belief, $m_1 = A$ yields the expected payoff

$$fx + f\left(\frac{1-\gamma-w}{\frac{1}{2}}\right)$$

if $w < 1-\gamma$, while $m_1 = A$ yields fx for $w \in \left[1-\gamma, \frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}\right]$.

On the other hand, whether $w < 1-\gamma$ or $w \in \left[1-\gamma, \frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}\right]$, $m_1 = B$ yields

$$f\left(\frac{\frac{1}{2}+\frac{1}{2}\theta-\theta\gamma-w}{\frac{1}{2}}\right).$$

Recall that given $m_1 = B$ in the putative equilibrium, R does not update his belief about the state. Therefore, whether $w < 1-\gamma$ or $w \in \left[1-\gamma, \frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}\right]$, given $\omega_1 = A$, R sends $m_1 = B$ with probability 1 if

$$fx + f\left(\frac{1-\gamma-w}{\frac{1}{2}}\right) < f\left(\frac{\frac{1}{2}+\frac{1}{2}\theta-\theta\gamma-w}{\frac{1}{2}}\right), \tag{17}$$

where the left hand side represents the expected payoff from truth-telling when $w < 1-\gamma$ which is larger than the expected payoff from truth-telling when $w \in \left[1-\gamma, \frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}\right]$. Note that (17) reduces to

$$x < (1-\theta)(2\gamma-1),$$

as assumed. Thus we conclude that there exists an uninformative equilibrium where S reports $m_1 = B$ regardless of the state.

Step 6 Using the observation that any informative equilibrium communication in period 1 must feature a simple communication strategy (Step 1), we have now shown that if $w \leq \frac{1-x}{2} - \frac{\theta(2\gamma-1)}{2}$ and $\frac{x}{2} < \left(\frac{1}{2} - \frac{\theta(2\gamma-1)}{2}\right) - (1-\gamma)$, there exists no equilibrium featuring truth-telling in both periods or no equilibrium that features informative communication in period 1. We have then shown that under these same conditions, there exists an equilibrium with uninformative communication ($m_1 = B$ regardless of the state) in period 1. □

6.4 Proposition 3

We present a proof for each part (Part 1-a), Part 1-b), and Part 2) separately below.

6.4.1 Proposition 3: Part 1-a)

Proof. Recall that $\gamma > \frac{1}{2}$, and that the assumption on w in (1) implies $1-x-2w > 0$.

Step 1 Let r_1 denote the probability that R visits in period 1. Note that R, when deciding whether or not to visit in period 1, only considers his expected payoff in period 1, as he will observe m_1 in the beginning of period 2 whether or not he visits in period 1. R's expected payoff for period 1 if he does not visit is $\theta = \frac{1}{2}$. If R visits and $v_1 = 0$, then his expected payoff for period 1 is $\theta\tau_A + (1-\theta) - w$. The increase in the payoff by visiting in period 1 is thus given by

$(\theta\tau_A+(1-\theta))-w-\theta=\frac{1}{2}\tau_A+(1-\frac{1}{2})-w-\frac{1}{2}$. Therefore we have

$$r_1\left(w,x,\gamma,\frac{1}{2}\right)=P\left(v_1\leq\frac{1}{2}\tau_A+(1-\frac{1}{2})-w-\frac{1}{2}\right)$$
$$=\frac{\frac{1}{2}\tau_A+(1-\frac{1}{2})-w-\frac{1}{2}}{\frac{1}{2}}.$$

Given the equilibrium value of τ_A, it follows that

$$r_1\left(w,x,\gamma,\frac{1}{2}\right)=-\frac{4w\gamma-2x-2\gamma-8w+2wx+4w^2+3}{2w+x+2\gamma-2}.$$

Step 2 Note that

$$\frac{\partial r_1\left(w,x,\gamma,\frac{1}{2}\right)}{\partial w}=\frac{2(8w-4w^2-4wx-8w\gamma-x^2-4x\gamma+4x-4\gamma^2+10\gamma-5)}{(2w+x+2\gamma-2)^2}.$$

Let the expression in the parentheses in the numerator be

$$G\equiv 8w-4w^2-4wx-8w\gamma-x^2-4x\gamma+4x-4\gamma^2+10\gamma-5.$$

Solving $G=0$ for w, the unique relevant solution is given by $w=\frac{1}{2}\sqrt{2\gamma-1}-\gamma-\frac{1}{2}x+1$. Now we have

$$\left.\frac{\partial G}{\partial w}\right|_{w=\frac{1}{2}\sqrt{2\gamma-1}-\gamma-\frac{1}{2}x+1}=-4\sqrt{2\gamma-1}<0$$

so that $\frac{\partial r_1\left(w,x,\gamma,\frac{1}{2}\right)}{\partial w}>0$ for $w<\frac{1}{2}\sqrt{2\gamma-1}-\gamma-\frac{1}{2}x+1$. Note that $\frac{1-x}{2}<\frac{1}{2}\sqrt{2\gamma-1}-\gamma-\frac{1}{2}x+1$, since

$$\left(\frac{1}{2}\sqrt{2\gamma-1}-\gamma-\frac{1}{2}x+1\right)-\frac{1-x}{2}=\frac{1}{2}\sqrt{2\gamma-1}-\gamma+\frac{1}{2}>0$$

since we assume $\gamma\in(\frac{1}{2},1)$. Therefore we have $\frac{\partial r_1\left(w,x,\gamma,\frac{1}{2}\right)}{\partial w}>0$ for $w<\frac{1-x}{2}<\frac{1}{2}\sqrt{2\gamma-1}-\gamma-\frac{1}{2}x+1$.

Step 3 Solving $r_1\left(w,x,\gamma,\frac{1}{2}\right)=0$ with respect to w, the unique relevant solution is given by $w=\frac{4-2\gamma-x-\sqrt{x^2+4x\gamma+4\gamma^2-8\gamma+4}}{4}$. Meanwhile, we have $r_1\left(\frac{1-x}{2},x,\gamma,\frac{1}{2}\right)=x>0$. This and $\frac{\partial r_1\left(w,x,\gamma,\frac{1}{2}\right)}{\partial w}>0$ for $w<\frac{1-x}{2}$ as shown above, imply $\frac{4-2\gamma-x-\sqrt{x^2+4x\gamma+4\gamma^2-8\gamma+4}}{4}<\frac{1-x}{2}$.

Step 4 We now conclude that there exists some

$$w\in\left(\max\left\{\frac{4-2\gamma-x-\sqrt{x^2+4x\gamma+4\gamma^2-8\gamma+4}}{4},1-\gamma,\frac{1-x}{2}-\frac{(2\gamma-1)}{4}\right\},\frac{1-x}{2}\right)\qquad(18)$$

such that $r_1\left(w,x,\gamma,\frac{1}{2}\right)\in(0,1)$ in equilibrium, as stated. □

6.4.2 Proposition 3: Part 1-b)

Proof. We know from part 1-a) that R visits with positive probability in period 1 if w satisfies (18). In what follows we explicitly derive the expression for the expected payoff of R in period 1. Let K_1 denote the expected cost of visiting in period 1, conditional on R deciding to visit. We have

$$\begin{aligned} K_1 &= w + E\left(v_2 \middle| v_2 \leq \left(\frac{1}{2}\tau_A + (1-\frac{1}{2})\right) - w - \frac{1}{2}\right) \\ &= w + \frac{\left(\frac{1}{2}\tau_A + (1-\frac{1}{2})\right) - w - \frac{1}{2}}{2}. \end{aligned}$$

The expected consultation cost incurred in period 1 is

$$\begin{aligned} & P(d_1 = 1)K_1 \\ &= \left(\frac{\left(\frac{1}{2}\tau_A + (1-\frac{1}{2})\right) - w - \frac{1}{2}}{\frac{1}{2}}\right)\left(w + \frac{\left(\frac{1}{2}\tau_A + (1-\frac{1}{2})\right) - w - \frac{1}{2}}{2}\right). \end{aligned}$$

On the other hand, in equilibrium, the probability that R's action matches the state in period 1 is given by

$$\begin{aligned} & P(\omega_1 = A)(1 - r_1) \\ & + P(\omega_1 = A) r_1 [\tau_A(1) + (1 - \tau_A)(0)] \\ & + P(\omega_1 = B)(1 - r_1)(0) \\ & + P(\omega_1 = B) r_1 (1). \end{aligned}$$

Summarizing, R's expected payoff for period 1 is given by

$$\begin{aligned} \Pi_{R1}(w,x,\gamma,\theta) = {} & \frac{1}{2}(1 - r_1(w,x,\gamma,\theta)) + \frac{1}{2}(r_1(w,x,\gamma,\theta))\,\tau_A(w,x,\gamma,\theta) + \frac{1}{2} r_1(w,x,\gamma,\theta) \\ & - \left(\frac{\left(\frac{1}{2}\tau_A + (1-\frac{1}{2})\right) - w - \frac{1}{2}}{\frac{1}{2}}\right)\left(w + \frac{\left(\frac{1}{2}\tau_A + (1-\frac{1}{2})\right) - w - \frac{1}{2}}{2}\right). \end{aligned}$$

Let us now consider the derivatives of this expected payoff in period 1. Substituting $\theta = \frac{1}{2}$ into the above, we have

$$\begin{aligned} & \Pi_{R1}\left(w,x,\gamma,\frac{1}{2}\right) \\ &= \frac{1}{4(2w+x+2\gamma-2)^2} \\ &\times \left(\begin{array}{c} 16w^4 + 16w^3x + 32w^3\gamma - 64w^3 + 4w^2x^2 + 16w^2x\gamma - 48w^2x + 16w^2\gamma^2 - 80w^2\gamma \\ +96w^2 - 8wx^2 - 24wx\gamma + 52wx - 16w\gamma^2 + 72w\gamma - 64w + 6x^2 + 16x\gamma - 20x + 12\gamma^2 - 28\gamma + 17 \end{array}\right) \end{aligned}$$

Thus we obtain the following derivative with respect to w

$$\frac{\partial \Pi_{R1}\left(w, x, \gamma, \frac{1}{2}\right)}{\partial w}$$
$$= \frac{1}{(2w+x+2\gamma-2)^3}$$
$$\times \left(\begin{array}{c} 16w^4+24w^3x+48w^3\gamma-64w^3+12w^2x^2+48w^2x\gamma-72w^2x+48w^2\gamma^2-144w^2\gamma+96w^2 \\ +2wx^3+12wx^2\gamma-24wx^2+24wx\gamma^2-92wx\gamma+70wx+16w\gamma^3-88w\gamma^2+140w\gamma-64w \\ -2x^3-10x^2\gamma+11x^2-16x\gamma^2+40x\gamma-22x-8\gamma^3+32\gamma^2-40\gamma+15 \end{array} \right).$$

It is easy to see that $(2w+x+2\gamma-2)^3 > 0$ under our assumptions. Let H be the expression in the large parentheses above and find values of w that constitute roots of the expression ($H = 0$). The two only admissible roots are $w' \equiv \frac{4-2\gamma-x-\sqrt{x^2+4x\gamma+4\gamma^2-8\gamma+4}}{4}$ and $w'' \equiv \frac{1}{2}\sqrt{2\gamma-1}-\gamma-\frac{1}{2}x+1$. As shown in the proof of part a), we have $w' < \frac{1}{2}-\frac{x}{2} < w''$. Also, we have

$$\left.\frac{\partial H}{\partial w}\right|_{w=w''} = -4(2\gamma-1)\left(x+2\gamma-2\sqrt{2\gamma-1}\right) < 0$$

since $\gamma \in (1/2, 1)$. Therefore, $H > 0$ for $w \in (w', w'')$, which implies $\Pi_{R1}\left(w, x, \gamma, \frac{1}{2}\right)$ is increasing in w, as stated. □

6.4.3 Proposition 3: Part 2

Proof. Let us define $E[k]$ as the expected number of times that S reports truthfully over the two periods. Given τ_A, we have $E[k] = \frac{1}{2}(\pi_A+1)+\frac{1}{2}(1+1)$. The accounting profit from period 2 is given by

$$\pi_{S2} = fx\left(\frac{1}{2}(1-\tau_A)+\frac{1}{2}\right),$$

where $\frac{1}{2}(1-\tau_A)+\frac{1}{2}$ is the probability that $m_1 = B$, and x is the probability of visit in period 2 given $m_1 = B$ in equilibrium. Therefor, S's the total payoff over two periods when there is no first period visit is given by

$$\Pi_S = \pi_{S2} + fxE[k] = fx\left(\frac{1}{2}(1-\tau_A)+\frac{1}{2}\right) + fx\left(\frac{1}{2}(\pi_A+1)+\frac{1}{2}(1+1)\right) = \frac{5fx}{2},$$

which is independent of w. In other words, when the probability of visit is 0 in period 1, S's total payoff is unaffected by w.

Suppose that the probability of visit in period 1 is strictly positive. The accounting profit in the first period in this case is given by

$$\pi_{S1} = f\frac{\left(\frac{1}{2}\tau_A+\frac{1}{2}-w-\frac{1}{2}\right)}{\frac{1}{2}}.$$

We have substituting the equilibrium τ_A into this, we have

$$\frac{\partial \pi_{S1}}{\partial w} = 2f\left(\frac{2\gamma-1}{(2w+x+2\gamma-2)^2}-1\right) > 0$$

since the expression in the large parenthesis is positive, $w < \frac{1-x}{2}$ that follows from (1) and $\gamma \in (1/2,1)$ imply $(2w+x+2\gamma-2)^2 < (2\gamma-1)^2 < (2\gamma-1) < 1$.

Thus we conclude that S's total expected payoff is unaffected by w when the probability of visit in period 1 is 0, and increasing in w when the probability of visit in period 1 is positive. □

6.5 R's ex ante total expected payoff in Figure 2a

We here derive R's total expected payoff for the parameter values such that R visits with positive probability in period 1.

Step 1 Let r_1 denote the probability that R visits in period 1. Let r_J, for $J \in \{A,B\}$, denote the probability that R visits in period 2 conditional on the period 1 message being J. Note that $r_A = 0$ in equilibrium. Let K_1 denote the expected visiting cost incurred by R in period 1, conditional on actually visiting. Let K_J, for $J \in \{A,B\}$, denote the expected visiting cost incurred by R in period 2, conditional on $m_1 = J$. Given r_1, r_A and r_B, R's expected total payoff is given by

$$\begin{aligned}
&P(\omega_1 = A, \omega_2 = A)(1-r_1)\left[2\tau_A + 2(1-\tau_A)\right] + \\
&P(\omega_1 = A, \omega_2 = A)r_1\left[2\tau_A + (1-\tau_A)\right] + \\
&P(\omega_1 = A, \omega_2 = B)(1-r_1)\left[\tau_A + (1-\tau_A)(1+r_B)\right] + \\
&P(\omega_1 = A, \omega_2 = B)r_1\left[\tau_A + (1-\tau_A)r_B\right] + \\
&P(\omega_1 = B, \omega_2 = A)\left[(1-r_1) + 2r_1\right] + \\
&P(\omega_1 = B, \omega_2 = B)\left[(1-r_1)r_B + r_1(1+r_B)\right] \\
&\quad - r_1 K_1 \\
&\quad - \left[P(\omega_1 = A)(1-\tau_A) + P(\omega_1 = B)\right] r_B K_B,
\end{aligned}$$

where $-r_1K_1 - [P(\omega_1 = A)(1-\tau_A) + P(\omega_1 = B)]\, r_B K_B$ represents the ex ante expected consultation cost and the rest represents the ex ante expected action-payoff. Let us provide more details below.

Step 2 The total expected consultation cost incurred by R is

$$\begin{aligned}
&r_1 K_1 \\
&+ \underbrace{[P(\omega_1 = A)\tau_A]}_{P(m_1=A)} r_A K_A + \underbrace{[P(\omega_1 = A)(1-\tau_A) + P(\omega_1 = B)]}_{P(m_1=B)} r_B K_B.
\end{aligned}$$

The first term, namely the expected consultation cost incurred in period 1 rewrites as

$$r_1 K_1 = \underbrace{\frac{\left(\frac{1}{2}\tau_A + (1-\frac{1}{2})\right) - w - \frac{1}{2}}{\frac{1}{2}}}_{r_1} \underbrace{\left(w + \frac{\left(\frac{1}{2}\tau_A + (1-\frac{1}{2})\right) - w - \frac{1}{2}}{2}\right)}_{K_1}.$$

Let us then compute the expected consultation cost incurred in period 2. We have

$$r_B = \frac{(\gamma-1)\frac{\tau_A-2}{\tau_A+2\gamma-3} - w}{\frac{1}{2}} = x$$

and

$$\begin{aligned} K_B &= w + E\left(v_2 \middle| v_2 \leq (\gamma-1)\frac{\tau_A-2}{\tau_A+2\gamma-3} - w\right) \\ &= w + \frac{(\gamma-1)\frac{\tau_A-2}{\tau_A+2\gamma-3} - w}{2} = w + \frac{x}{4}. \end{aligned}$$

The probability of receiving $m_1 = B$ is given by

$$P(\omega_1 = A)(1-\tau_A) + P(\omega_1 = B) = \left(\frac{1}{2}(1-\tau_A) + \frac{1}{2}\right).$$

Recall that $r_A = 0$ in the equilibrium we consider. Thus the ex ante expected consultation cost incurred in period 2 rewrites as

$$\left[P(\omega_1 = A)(1-\tau_A) + P(\omega_1 = B)\right] r_B K_B = \underbrace{\left(\frac{1}{2}(1-\tau_A) + \frac{1}{2}\right)}_{P(\omega_1=A)(1-\tau_A)+P(\omega_1=B)} \underbrace{x}_{r_B} \underbrace{\left(w + \frac{x}{4}\right)}_{K_B}.$$

Step 3 R's expected action-payoff for both periods is given by

$$\begin{aligned} &P(\omega_1 = A, \omega_2 = A)(1-r_1)\left[2\tau_A + 2(1-\tau_A)\right] + \\ &P(\omega_1 = A, \omega_2 = A) r_1 \left[2\tau_A + (1-\tau_A)\right] + \\ &P(\omega_1 = A, \omega_2 = B)(1-r_1)\left[\tau_A + (1-\tau_A)(1+r_B)\right] + \\ &P(\omega_1 = A, \omega_2 = B) r_1 \left[\tau_A + (1-\tau_A) r_B\right] + \\ &P(\omega_1 = B, \omega_2 = A)\left[(1-r_1) + 2r_1\right] + \\ &P(\omega_1 = B, \omega_2 = B)\left[(1-r_1) r_B + r_1 (1+r_B)\right], \end{aligned}$$

where

$$P(\omega_1 = A, \omega_2 = A) = \frac{1}{2}\gamma$$
$$P(\omega_1 = A, \omega_2 = B) = \frac{1}{2}(1-\gamma)$$
$$P(\omega_1 = B, \omega_2 = A) = \frac{1}{4}$$
$$P(\omega_1 = B, \omega_2 = B) = \frac{1}{4}$$

and τ_A, r_1, r_B as above.

6.6 Proposition 4

Proof. Assume that $w \in \left(\frac{1-x}{2}, \frac{1}{2}\right)$ and $\frac{x}{2} < \left(\frac{1}{2} - \frac{\theta(2\gamma-1)}{2}\right) - (1-\gamma)$. Consider a putative equilibrium with truth-telling in both periods. The visiting probability in period 2 given $m_1 = B$ is given by $\frac{\frac{1}{2}-w}{\frac{1}{2}} = 1-2w$, which is clearly positive. Given our assumptions, the visiting probability given $m_1 = A$ is on the other hand is zero. We now examine the incentive of S. If $\omega_1 = A$, sending $m_1 = A$ yields the expected payoff fx whereas $m_1 = B$ yields $(1-2w)f$. The truth-telling condition in this case

$$fx \geq (1-2w)f$$

is equivalent to $\frac{x}{2} \geq \frac{1}{2} - w$. Note that we are considering the case where $w > \frac{1-x}{2}$, which implies $\frac{1}{2} - w < \frac{1}{2} - (\frac{1-x}{2}) = \frac{x}{2}$, which in turn leads to $\frac{x}{2} \geq \frac{1}{2} - w$. Thus we have established that there is an equilibrium that features truth-telling in period 1 (and period 2) strictly positive probability of consultation in period 2. □

6.7 Proposition 5

The proof is given in the main text.

6.8 Proposition 6

The proof is given in the main text.

6.9 *R*'s ex ante total expected payoff in Figure 4a

The calculation is similar to that for the ex ante total expected payoff in Figure 2a. *R*'s ex ante expected payoff in the model in Section (3) is given by

$$
\begin{aligned}
&P(\omega_1 = A, \omega_2 = A)(1-r_1)\left[2\tau^C + 2\left(1-\tau^C\right)\right] + \\
&P(\omega_1 = A, \omega_2 = A)r_1\left[2\tau^C + \left(1-\tau^C\right)\right] + \\
&P(\omega_1 = A, \omega_2 = B)(1-r_1)\left[(1-(\beta(1-\tau^C)^2 + (1-\beta)(1-\tau^C)) + (\beta(1-\tau^C)^2 + (1-\beta)(1-\tau^C))(1+r_B)\right] + \\
&P(\omega_1 = A, \omega_2 = B)r_1\left[\tau^C + \left(1-\tau^C\right)r_B\right] + \\
&P(\omega_1 = B, \omega_2 = A)\left[(1-r_1) + 2r_1\right] + \\
&P(\omega_1 = B, \omega_2 = B)\left[(1-r_1)r_B + r_1(1+r_B)\right] \\
&\quad - r_1 K_1 \\
&\quad - \left[P(\omega_1 = A)(\beta(1-\tau^C)^2 + (1-\beta)(1-\tau^C)) + P(\omega_1 = B)\right] r_B K_B.
\end{aligned}
$$

The differences from the corresponding calculation for Figure 2a are as follows, Given $\omega_1 = A$, the probability that *R* is uncertain about the state due to $m_1^{Si} = m_1^{Sj} = B$ is now $\beta(1-\tau^C)^2 + (1-\beta)(1-\tau^C)$, with the complementary probability $(1-(\beta(1-\tau^C)^2 + (1-\beta)(1-\tau^C))$ that *R* correctly infers $\omega_1 = A$ from $m_1^{Si} = m_1^{Sj} = A$ or $m_1^{Si} \neq m_1^{Sj}$. Those are reflected in the third line of the expression above. The probability of visit r_1 and the conditional expected cost of visit K_1 are calculated in the same way as in Figure 2a, simply by replacing τ_A with τ^C since *R* acquires at most one message before choosing a_1. In the last line of the above expression, the unconditional probability that *R* receives $m_1^{Si} = m_1^{Sj} = B$ and thus is uncertain about the state at the end of period 1 given by

$$P(\omega_1 = A)(\beta(1-\tau^C)^2 + (1-\beta)(1-\tau^C)) + P(\omega_1 = B).$$

The probability of visit in period 2 given $m_1^{Si} = m_1^{Sj} = B$ denoted by r_B, and the expected cost of visit denoted by K_B are calculated from (6).

6.10 Remark 1

Proof. In the equilibrium strategy profile above, if *S* deviates to $p < p^*$ off the equilibrium, then all players still subsequently behave as in 2. If *S* instead deviates to $p > p^*$, the receiver acquires no subscription and *S* reports truthfully. We show below that the strategy profile is sequentially rational.

Let p^* be such that an individual receiver's equilibrium expected payoff when purchasing the subscription at $p = p^*$ equals his expected payoff when he does not purchase the subscription and *S* reports truthfully in both periods (and thus the receiver observes a truthful report at the end of each period). The following argument shows that the value of p that achieves this indifference is positive and unique. Note that if a receiver does not purchase the subscription, despite truthful reports in both periods, his expected payoff is the same as if he would not observe a report in either period, since he chooses $a_1 = A$ in period 1 (recall $\theta \in [\frac{1}{2}, 1)$) and $a_2 = A$ in period 2 regardless of whether $m_1 = A$ or $= B$. That is, the report the receiver observes at the end of period 1 is irrelevant to his action or payoff.

Next, note that if a receiver purchases the subscription at $p = 0$ and S's strategies are the same as those in Section 2, the receiver's expected payoff is by construction strictly larger than the expected payoff when he does not purchase the subscription. This is because the subscription itself is free and R visits voluntarily with positive probability (recall that the report in period 2 is fully informative), at a cost that is furthermore always strictly smaller than the marginal value of a visit. Therefore we must have $p^* > 0$ to make the receiver indifferent between purchasing the subscription and not purchasing.

We now verify that players' strategies are sequentially rational. Note first that the subscription fee, once paid, alters neither the receiver's marginal cost of visit $w + v_t$ nor S's marginal benefit from a visit. Therefore, once the subscription fee has been paid, incentives of both S and the receiver are identical to those arising in the single firm model in Section 2.

Consider first the receiver's incentives. If $p \leq p^*$, recall that a receiver purchases the subscription while his subsequent decision to visit in both periods is as described in Section 2. If the receiver does not purchase the subscription, he cannot visit in either period 1 or 2. If $p \leq p^*$, the receiver's deviation expected payoff when he does not buy the subscription (in which case the accuracy of the report is the one as described in Section 2 since this individual deviation does not affect S's incentives) is the same as the hypothetical expected payoff that the receiver would obtain if he chose not to buy the subscription and S reveals truthfully. This is because, as noted earlier, the report the receiver observes at the end of period 1 is irrelevant to his action or payoff if he never visits in period 2. Thus if $p \leq p^*$, an individual receiver always weakly prefers buying the subscription to not buying. If instead $p > p^*$, then by construction (given the definition of p^*) the receiver prefers not to purchase the subscription.

Consider now S's incentives. Note first that if no receiver has purchased the subscription, S strictly prefers to report truthfully in both periods and thereby obtain the truth-telling benefit, since there is no incentive for S to misreport. The following argument shows that S is better off setting $p \leq p^*$ (where all receivers the subscription) than $p > p^*$ (where no receiver purchases the subscription and S reports truthfully in both periods). If S sets $p > p^*$ and no subscription is sold, then S has no visit in either period but enjoys the truth-telling benefit in both periods. Suppose instead that S sets $p \leq p^*$ and all receivers purchase the subscription. If $\omega_1 = B$, then S subsequently enjoys the truth-telling benefit in period 1 and the revenue through visits from a fraction of the receivers in period 2. If instead $\omega_1 = A$, S has to choose between the truth-telling benefit (by $m_1 = A$) and revenue from visits (by $m_1 = B$) in period 2, but S's conditional expected payoff in equilibrium must be equal to (when $\tau_A^{1*} \in (0,1)$ and $\tau_A^{1*} = 1$) or larger than (when $\tau_A^{1*} = 0$) the truth-telling benefit only. We may thus conclude that firm i has no incentive to deviate to $p > p^*$. Finally, simply note that among any price $p \leq p^*$, S strictly prefers higher p. S's optimal choice of subscription price is thus $p = p^*$. □

6.11 Remark 2

Proof. In the equilibrium strategy profile that we consider if one firm deviates to a positive subscription price off the equilibrium, all receivers buy a subscription from the other firm and both firms communicate truthfully.

We now verify that the strategies are sequentially rational. We first examine firms' incentives. Consider first the firms' incentives *after* firm i unilaterally deviates to $p_i > 0$ from $p_i = p_j = 0$. Suppose that given the deviation all receivers purchase a subscription from firm j, so that firm i sells no subscription. Then after the deviation firm i has no incentive to induce future visits and strictly prefers to report truthfully in both periods to obtain the truth-telling benefit. In addition, given that firm i reports truthfully, firm j has no incentive to deviate from truth-telling in period 1 since any misreporting by firm j would be exposed through firm i's report.

Consider now firm i's incentive to unilaterally deviate to $p_i > 0$. After the deviation, firm i sells no subscription and thus receives no visits but obtains only the truth-telling benefit in both periods. Suppose instead that firm i sets $p_i = 0$ and sells a subscription to half of the receivers. If $\omega_1 = B$, then firm i subsequently enjoys the truth-telling benefit in period 1 and the revenue from visits in period 2. If instead $\omega_1 = A$, firm i has to choose between the truth-telling benefit (by $m_1^{Si} = A$) and the revenue from visits (by $m_1^{Si} = B$) in period 2, but firm i's conditional expected payoff in equilibrium must be equal to (when $\tau^C \in (0,1)$ and $\tau^C = 1$) or larger than (when $\tau^C = 0$) the truth-telling benefit only. We thus conclude that firm i has no incentive to deviate to $p_i > 0$, given that the receivers do not purchase at that price.

Finally, consider now each individual receiver's incentives after firm i unilaterally deviates to $p_i > 0$. Note that both firms now communicate truthfully (and thus identically), so that any receiver is willing to buy a subscription from j at $p_j = 0$ with probability 1. Consider a receiver's incentives on the equilibrium path, after he buys a subscription from one of the firms at $p_i = p_j = 0$. Clearly, the incentives regarding the decision whether to visit on this equilibrium path are the same as in the equilibrium described in Section 3. Also, each receiver is strictly better off buying a subscription at $p_i = p_j = 0$, since the subscription is free and visiting is voluntary and occurs only when it increases his expected payoff. We may conclude that price competition for subscription leads to zero price, and we can resort to our analysis in Section 3. □

7 Appendix B: Infinite Horizon

In this Appendix we extend our two-period model to an infinite horizon setup and demonstrate that our key results and intuitions do not depend on the feature that truth-telling occurs in equilibrium in the final period. In reality, the underlying state often evolves over time without a specific end date, and an infinite horizon model captures the situation where in every period there is an incentive for S to induce future visits by misreporting. Technically, both players now face a recursive problem. The state ω_t in the infinite horizon model corresponds to a broad topic (politics, economy, sports, etc.) followed by readers over the long run, rather than a particular news event that develops and ends over a short period of time. An example could be that states A and B correspond to the stability of the current condition of the economy. In state A, the condition is very stable (either being good or bad) and likely to remain the same for a while, whereas in state B it is highly volatile and thus difficult to forecast.

7.1 Model

Let the time period be $t = 0, 1, 2, ..., +\infty$. Both S and R discount their future payoffs at a common discount factor δ and they maximize the discounted future stream of payoffs in every period. The transition matrix of the state, the players, their action sets and per-period payoffs, are the same as in Section 2.

Let $\tilde{x} \equiv x/\delta$ for convenience, since the discount factor δ affects the players' incentives only via x. Specifically, a higher discount factor means that S values revenue from future visits more, which in turn implies that the incentive to misreport becomes relatively higher, resulting in the lower effective benefit of truth telling x/δ. We focus on the interesting case of intermediate visiting costs and impose a slightly modified and simpler version of (1) as follows:

$$w \in \left(1-\gamma, \frac{1-\tilde{x}}{2}\right). \tag{19}$$

At the beginning of each period, S observes ω_t. By the beginning of period t, R observes neither ω_t nor ω_{t-1} but instead observes $\{\omega_{t-2}, m_{t-1}\}$. The assumption that R observes ω_{t-2} captures the notion that as time goes by, a past event is more clearly recognized and understood. The assumption that m_{t-1} is observed, as in the two periods model, captures diffusion of information. S chooses a message in each period from period 1 onwards, and R chooses whether to visit and what action to take from period 2 onwards. The timing of payoff realizations is irrelevant, as long as it does not allow R to infer ω_t and ω_{t-1} during period t separately from the messages received.

A communication strategy of S for the whole game is given by a sequence of one-shot communication strategies, one for each t. A stationary communication strategy for the whole game is such that S uses the same one-shot strategy in every period. A strategy for period t is said to belong to class n if it conditions the message sent in period t on neither more nor less than $\{\omega_{t-n+1}, ..., \omega_{t-2}, \omega_{t-1}, \omega_t\}$. In other words, a class n strategy is conditioned on the history of the state up to $n-1$ periods ago.[24] Note that a class n strategy is conditioned only on realized states, but not on S's own messages in the past. A *basic class n* strategy for period t is a strategy for period t such that S always sends $m_t = B$ if $\omega_t = B$.

For concreteness, let us illustrate a basic class 2 strategy for period t in some detail. First, the probability of sending $m_t \in \{A, B\}$ in period t is a function of (and only of) $\{\omega_{t-1}, \omega_t\}$. Such a strategy for period t is therefore described by four truth-telling probabilities at period t, and we denote by $\tau^t_{\omega_{t-1}\omega_t}$ the probability that $m_t = \omega_t$ given $\{\omega_{t-1}, \omega_t\}$. Second, a basic class 2 strategy for period t features $\tau^t_{\omega_{t-1}B} = 1$ for any ω_{t-1}. A basic class 2 strategy for period t is *partially informative* if neither $\tau^t_{AA} = \tau^t_{BA} = 1$ nor $\tau^t_{AA} = \tau^t_{BA} = 0$. If $\tau^t_{AA} = \tau^t_{BA} = 1$ then the basic class 2 strategy for period t is *fully informative* (and also features truth-telling). Finally, if $\tau^t_{AA} = \tau^t_{BA} = 0$ then the basic class 2 strategy for period t is *uninformative* since $m_t = B$ regardless of the state. A stationary strategy of S for the whole game that involves the infinite repetition of a basic class n strategy for each period is called a basic class n stationary strategy. In what follows, for simplicity we restrict ourselves to basic class 1 and 2 stationary strategies of S. We will see shortly that no basic class 1 strategy can form part of a stationary equilibrium, so that the simplest class of basic stationary equilibrium strategies of S is class 2.

A strategy of R for the whole game is given by a sequence of one-shot strategies, one for each t. R's strategy for each period has two components, a consultation rule and an action rule. A stationary strategy of R is such that R uses the same one-shot strategy in each period t. A strategy of R is said to belong to class n if R's choices are conditioned on neither more nor less than $\{\omega_{t-n+1}, ..., \omega_{t-2}, m_{t-1}, m_t\}$, where we (abusively) denote $m_t = \emptyset$ if m_t was not observed. Given our exogenous restriction to class 1 and class 2 stationary communication strategies of S, it is without loss of generality to focus on stationary strategies of R of class no larger than $n = 3$. A class 3 strategy for period t, as is the case for a strategy for period t of any class, involves a consultation rule that takes the form of a set of threshold rules. For each possible observed history (ω_{t-2}, m_{t-1}), there is a threshold value of the random cost $v_t \in [0, \frac{1}{2}]$, denoted by $v(\omega_{t-2}, m_{t-1})$, such that R visits in the beginning of period t if and only if $v_t \leq v(\omega_{t-2}, m_{t-1})$. We denote by $\varphi_{\omega_{t-2}m_{t-1}}$ the conditional probability that R consults in period t given (ω_{t-2}, m_{t-1}), which is simply $P(v_t \leq v(\omega_{t-2}m_{t-1}))$. A class 3 strategy of R for period t also involves an action rule which conditions R's chosen action at t on $\{\omega_{t-2}, m_{t-1}, m_t\}$.

We focus on equilibria featuring stationary strategies for both S and R, and we call such equilibria stationary. In an equilibrium where R acquires m_t with positive probability p after the observed history $\{\omega_{t-n+1}, ..., \omega_{t-2}, m_{t-1}\}$, we simply say that R consults with probability p in period t after this observed history. We say that a stationary equilibrium features a positive probability of consultation if there is some $(n-1)$-elements history $\{\omega_{t-n+1}, ..., \omega_{t-2}, m_{t-1}\}$ which has a positive stationary probability and the strategy of R specifies a positive probability of consultation in period t given this $(n-1)$-elements history. Recall that both S and R are unable to commit to future actions, so that the behaviour has to be incentive compatible at every point in time (i.e.

[24] We assume that the stochastic process governing the state starts in period 0, but that S only communicates from period 1 onwards while R only starts making choices (visiting choice and action choice) from period 2 onwards.

every information set) in equilibrium.

We conclude with a discussion of our focus on stationary strategy profiles. We do not consider non-stationary strategies (e.g. Grim-Trigger, Stick and Carrot) that might achieve more informative reporting through punishment for misreporting (provided a sufficiently high discount factor). Such strategies would in principle be feasible since R observes past states (though with a lag), so that R could detect misreporting and thus choose not to visit for a while after a misreport as punishment. Note that the following simple and intuitive class of such strategy profiles would not support truth-telling in equilibrium. Consider the class where punishment by R is accompanied by uninformative messages by S during the punishment phase, since otherwise R might have an incentive to visit during this phase and thus fail to impose punishment. The contradiction is that if S knows that R never visits in the next period(s), S reports truthfully to obtain the truth-telling benefit, which in turn induces R to visit, making the punishment by R ineffective. Equilibrium strategies to support truth-telling would thus have to be more sophisticated, involving e.g. punishment for not punishing. Such sophisticated equilibrium constructions featuring non-stationary strategies seem implausible in the context of communication between a media firm and potentially large audience. Insofar as a report is informative concerning the period-t state, punishment (committing not to visit) for at t is subject to a free-riding problem among receivers, which makes their coordinated punishment of the sender unlikely. Yet another problem with non-stationary strategies is that equilibria featuring sophisticated punishment strategies are typically not renegotiation proof, in contrast to the stationary equilibria that we study.

7.2 Equilibrium

The following Lemma outlines key features of R's equilibrium consultation.

Lemma 6. *a) No stationary equilibrium features zero probability of consultation. b) In any stationary equilibrium in which S uses a basic communication strategy, the probability of consultation in period t is zero* if $m_{t-1} = A$, *regardless of* $\omega_{t-2} \in \{A, B\}$.

All proofs in Appendix B are relegated to Section 7.3. The Lemma is a consequence of the assumption that w is in the intermediate range (19): w is low enough for consultation to occur in equilibrium, but also high enough that the consultation decision is conditioned on the previous period's message. The next Lemma concerns S's equilibrium communication strategy.

Lemma 7. *a) No stationary equilibrium features an uninformative or a fully informative communication strategy. b) No stationary equilibrium features a basic class 1 communication strategy.*

Babbling cannot be an equilibrium strategy. In such a putative equilibrium, R never consults and S deviates to truth-telling to reap the truth-telling benefit x. Equally, perfectly informative communication cannot be part of an equilibrium. In such a putative equilibrium, the visiting probability in $t+1$ is such that S deviates from truth-telling and sends $m_t = B$ even if $\omega_t = A$ (note that $m_t = A$ yields a visiting probability of 0). So if there exists a basic class 1 equilibrium, it must be such that S randomizes when $\omega_t = B$. This in turn implies that R must consult with the same probability at $t+1$ given the alternative observed histories $\omega_{t-1}=A, m_t=B$ and $\omega_{t-1}=B, m_t=B$. This, however, cannot be true.

The next Proposition presents our equilibrium characterization. In a search space containing all equilibria featuring either basic class 1 or basic class 2 stationary communication strategies, there exists a unique equilibrium. It features a basic class 2 strategy. The argument leading to the result builds on our Lemmas. By Lemma 6, any stationary equilibrium must feature a positive probability of consultation, as well as a probability zero of consultation at t if $m_{t-1}=A$. By Lemma 7, any stationary equilibrium features a partially informative communication strategy, which can furthermore not be a basic class 1 strategy. It follows that if we find a stationary equilibrium featuring either a basic class 1 or 2 communication strategy, then the equilibrium must feature an imperfectly informative basic class 2 communication strategy and R must consult with positive probability at t if and only if $m_{t-1}=B$. The last part of the characterization is to prove that there exists an equilibrium satisfying the above description, and this is the focus of the proof provided in the Appendix.

Proposition 7. *There exists no stationary equilibrium featuring a basic class 1 communication strategy. There exists a unique stationary equilibrium featuring a basic class 2 communication strategy. In this equilibrium, R plays a class 3 strategy. Furthermore, S's communication strategy features $\tau^*_{AA}, \tau^*_{BA} \in (0,1)$ and $\tau^*_{AA} \neq \tau^*_{BA}$, and R consults with probability $\tilde{x}$ if $m_{t-1}=B$ and with probability zero if $m_{t-1}=A$.*

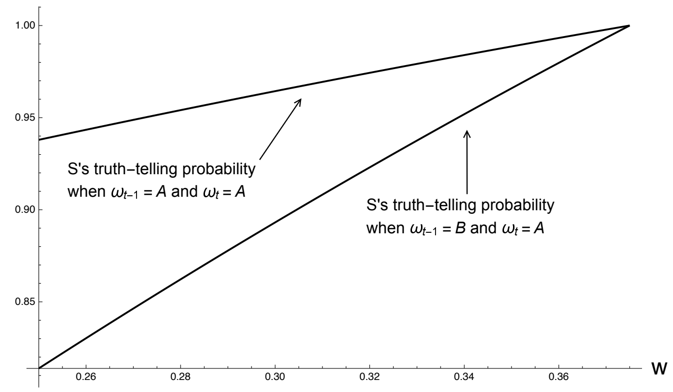
The intuition behind the equilibrium is very similar to that obtained for the analysis of the two-period model. To see that players best respond to each other, consider the following. Assume that S plays a class 2 strategy and that R plays a class 3 strategy, i.e. m_t is conditioned on both ω_t and ω_{t-1}, while R's consultation decision at $t+1$ depends only on m_t and ω_{t-1}.

Let us consider the strategy of S. The message m_t affects S's expected payoff in two ways, namely through i) the truth-telling benefit in period t, which depends only on the current state ω_t; and ii) the consultation probability in period $t+1$, which is induced by m_t given ω_{t-1} and fully described by R's consultation strategy $\varphi_{AA}=0$, $\varphi_{BA}=0$, φ_{AB} and φ_{BB}. Let us look at the second channel ii) in more detail. When choosing m_t, S acknowledges that in period $t+1$, R will condition his visit only on ω_{t-1} and m_t, and thus m_{t-1} does not affect R's visiting probability in period $t+1$. It follows in turn that S conditions m_t only on $\{\omega_{t-1}, \omega_t\}$. Note in particular that S has no incentive to deviate and condition m_t on m_{t-1}, since m_{t-1} affects neither the truth-telling payoff in period t nor the visiting probability in period $t+1$.

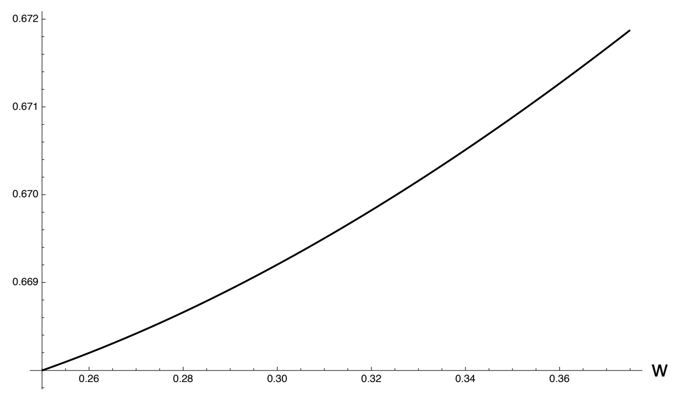
We now turn to R's equilibrium strategy. The equilibrium features $\tau^*_{AA}, \tau^*_{BA} \in (0,1)$, and note that $\varphi_{AB}=\varphi_{BB} \in (0,1)$ implies $\tau^*_{AA} \neq \tau^*_{BA}$. Indeed, if $\tau^*_{AA}=\tau^*_{BA}$, then we cannot have $\varphi_{AB}=\varphi_{BB} \in (0,1)$ since the conditional distribution of ω_t at the end of period t given $\{\omega_{t-1}=A, m_t=B\}$ and $\{\omega_{t-1}=B, m_t=B\}$ has to differ, which in turn implies that the expected payoff from visiting in period $t+1$ differs depending on ω_{t-1}. The states prior to ω_{t-1} are irrelevant to the stationary equilibrium behaviour because the assumption that the state follows a Markov process implies that only ω_{t-1} matters for the distribution of ω_t. We now present key comparative statics properties of equilibrium communication.

Proposition 8. *In the equilibrium identified in Proposition 7, τ^*_{AA} and τ^*_{BA} are constant in f, increasing in w and decreasing in $\tilde{x}(= x/\delta)$.*

The comparative statics results are qualitatively the same as in the two-period model. Clearly if τ^*_{AA} and τ^*_{BA} remained unchanged, an increase in w would lower the probability of visit at t given $m_t = B$. This implies that the equilibrium informativeness of communication must increase to keep the consultation probability equal to $\tilde{x}$. Equivalently, a higher cost of visit w makes misreporting (i.e. sending $m_t = B$ when $\omega_t = A$) less attractive since R's future visits become less sensitive to the current report. As S becomes more patient (δ higher and $\tilde{x}$ lower) the incentive to misreport increases, since a future visit induced by misreporting becomes more valuable relative to the truth-telling reward in the current period. Also, f does not enter τ^*_{AA} or τ^*_{BA} as the benefit (higher revenue) and the cost (foregoing the truth-telling benefit) of lying offset each other. Proposition 8 demonstrates that the main insights obtained in the two-period setup do *not* depend on the feature that truth-telling occurs in the final period.



(a) S's equilibrium behaviour



(b) R's average per-period payoff

Figure 5: Equilibrium behaviour and payoff with respect to w, where $\gamma = 3/4, x = 1/5$ and $\delta = 4/5$

We now consider how R's equilibrium payoff varies with w. Let the average per-period payoff of R be the limit of his average payoff across periods 2 to T, for T tending to infinity. The realized frequency of state A over this time interval thus coincides for sure with the stationary distribution.[25]

[25]We derive the exact expression for this average per-period payoff in the Appendix.

A numerical example is presented in Figure 5. Figure 5a illustrates Proposition 8 showing that τ^*_{AA} and τ^*_{BA} are increasing in w. We see $\tau^*_{AA} \geq \tau^*_{BA}$, that is, the current report is more informative when the previous state involves less uncertainty ($\omega_{t-1} = A$). When making his consultation choice at period $t+1$ after observing $m_t = A$, R also considers the (observed) state in period $t-1$, and $\omega_{t-1} = A$ implies lower uncertainty about ω_{t+1} than $\omega_{t-1} = B$. This means R is less manipulable when $\omega_{t-1} = A$, which leads to a lower incentive for S to misreport. Figure 5b shows that R's per-period payoff may be increasing in w.[26] That is, as in the two-period model, the benefit of an increase in w (more accurate reports) may outweigh the cost (increased average costs of visit) so that R may suffer overall from a cheaper cost of access to reports.

S's per-period average accounting profit on the other hand is decreasing in w. Note that an increase in w leads to a higher probability of truth-telling when $\omega_t = A$ as we saw in Proposition 8 while the probability of visit $\tilde{x}$ after $m_t = B$ remains unchanged (Proposition 7). This implies that the average visiting probability decreases, because $m_t = A$, after which R never visits, becomes more likely. The effect of w on S's accounting profit thus contrasts with what we found in the two period model, where the accounting profit may be increasing in w. The difference is due to the high initial uncertainty in the two-period model (we set the maximal initial uncertainty $\theta = 1/2$ for the numerical example in Section 2.2). From Figure 1, we can see that the probability of visit in period 1 may be *increasing* in w since the high uncertainty in that period makes the decision to visit very sensitive to the informativeness of m_1.

7.3 Proofs and Calculations

7.3.1 Lemma 6

Proof. Part a) Consider a putative stationary equilibrium with no consultation. In such an equilibrium, S would have a strict incentive to send m_t=ω_t in every period. So such an equilibrium would have to feature a fully informative basic communication strategy of S. But then, given our parameter assumptions, R would have a strict incentive to consult with positive probability after m_t= B.

Part b) In an equilibrium where $m_{t-1} = A$ implies that $\omega_{t-1} = A$ for sure, a necessary condition for R to choose to consult with positive probability in period t given $m_{t-1} = A$ is $1 - w > \gamma$, or equivalently $w < 1 - \gamma$ since the gross benefit of consultation (ignoring consultation costs) is at most $1 - \gamma$. This benefit corresponds to a scenario where S's communication is fully informative at t. A fortiori, if S's report is not fully informative in period t, R never visits in period t given $m_{t-1} = A$ if $w > 1 - \gamma$. Now, simply note that states $w > 1 - \gamma$ according to the assumption 19. □

7.3.2 Lemma 7

Proof. **Step 1** Proof of a). Consider a putative stationary equilibrium featuring an uninformative communication strategy of S. Such an equilibrium must feature no consultation. But in a putative

[26] We give the calculation of the payoff in Section 7.3.5. The parameter values are the same as in Figure 2 produced for the two-period model, except for the discount factor δ and the relevant range of w.

stationary equilibrium featuring no consultation, S would strictly favour sending $m_t = \omega_t$ for sure for any value of ω_t, thereby deviating from his assumed uninformative stationary strategy.

Step 2 Proof of a) continued. Consider a putative stationary equilibrium featuring a fully informative stationary communication strategy of S. Consider first the case where S's strategy is such that S always sends m_t=ω_t. Assume now that $\omega_t = A$. Message $m_t = A$ leads to a visiting probability of 0 in period $t+1$ and thus yields a period $t+1$ expected payoff over t and $t+1$ of fx. In contrast, $m_t = B$ leads to a strictly positive probability of visit of $\frac{\frac{1}{2}-w}{\frac{1}{2}} = 1-2w$ in period $t+1$. It thus yields an expected payoff over t and $t+1$ of $\delta f(1-2w)$. Now, note that $\delta f(1-2w) \leq fx$ is equivalent to $\frac{1-\widetilde{x}}{2} \leq w$, which contradicts the assumption that w is in the intermediate range 19.

Consider now the case where S's strategy is such that S always sends $m_t \neq \omega_t$. Assume now that $\omega_t = A$. Message $m_t = B$ leads to a visiting probability of 0 in period $t+1$ and thus yields an expected payoff of 0 over t and $t+1$. In contrast, $m_t = A$ yields the truth-telling reward fx in period t and also leads to a strictly positive probability of visit of $1-2w$ in period $t+1$. It follows trivially that in this putative equilibrium, S strictly prefers to deviate to $m_t = A$ given $\omega_t = A$.

Step 3 Proof of b). Consider a putative stationary equilibrium featuring a stationary basic communication strategy of S that belongs to class 1. Consider two different cases, 1 and 2. In case 1, the stationary strategy of S builds on a pure strategy for period t. It is thus either fully informative or uninformative. We have already shown in Steps 1 and 2 above that this cannot be true in equilibrium. The second case to consider is that the stationary strategy of S builds on a mixed strategy of S in each period (which must be identical in all periods by the stationarity assumption). By definition it must be true that S sends $m_t = B$ whenever $\omega_t = B$. So the randomization must take place when the state is A. Recall that $m_t = \omega_t$ yields the immediate truth-telling reward fx. Note also that if $m_t = A$, then R is thus sure that $\omega_t = A$. Given our assumptions on parameter values, it follows immediately that R consults with probability zero at $t+1$ if $m_t = A$. Note furthermore that if given ω_t, S randomizes between messages A and B at t, then it must be true that both messages yield the same expected payoff for S over t and $t+1$. Using the fact that R consults with probability zero at $t+1$ if $m_t = A$, the required indifference condition is equivalent to stating that it needs to be true that

$$P(d_{t+1} = 1\,|m_t = B, \omega_{t-1} = A) = P(d_{t+1} = 1\,|m_t = B, \omega_{t-1} = B) = \widetilde{x}.$$

But this in turn requires that

$$P(\omega_{t+1} = B\,|m_t = B, \omega_{t-1} = A) = P(\omega_{t+1} = B\,|m_t = B, \omega_{t-1} = B),$$

which guarantees that the benefit of consulting at $t+1$ after $m_t = B$ is the same, regardless of the observed value of ω_{t-1}. This equality can however not be satisfied given the Markov process faced and the assumed communication strategy of S. □

7.3.3 Proposition 7

Proof. **Step 1** Let us first show that there is no equilibrium that features $\tau_{AA} = 1$ or $\tau_{BA} = 1$ (a fortiori there is no truth-telling equilibrium, where $\tau_{AA} = \tau_{BA} = 1$). If there is such an equilibrium, it has to be that S is better off reporting $\omega_t = A$ truthfully, namely either

$$x + \delta P(d_t = 1 \mid m_{t-1} = A) f \geq 0 + \delta P(d_t = 1 \mid \omega_{t-2} = A, m_{t-1} = B) f \tag{20}$$

or

$$x + \delta P(d_t = 1 \mid m_{t-1} = A) f \geq 0 + \delta P(d_t = 1 \mid \omega_{t-2} = B, m_{t-1} = B) f. \tag{21}$$

Part b) of Lemma 8 states that $\varphi_{AA} = \varphi_{BA} = 0$ while Part a) of the Lemma states that $\varphi_{AB} > 0$ or $\varphi_{BB} > 0$. Meanwhile, $\tau_{AA} = 1$ or $\tau_{BA} = 1$ implies $\widetilde{v}(A,B) = \frac{1}{2} - w$ or $\widetilde{v}(B,B) = \frac{1}{2} - w$; and hence $\varphi_{AB} = 1 - 2w$ or $\varphi_{BB} = 1 - 2w$, respectively. Recall that $\tilde{x} \equiv x/\delta$. From (20) and (21), if $\varphi_{JB} = 1 - 2w$, S reports truthfully when $\omega_{t-1} = J$ and $\omega_t = A$ if

$$\tilde{x} f \geq (1 - 2w) f. \tag{22}$$

However, (22) simplifies to $w \geq \frac{1-\tilde{x}}{2}$, which contradicts the assumption 19. Therefore, neither $\tau_{AA} = 1$ nor $\tau_{BA} = 1$ can be supported in equilibrium.

Step 2 Let us prove the existence of an equilibrium that features $\tau_{AA} \in (0,1)$ and $\tau_{BA} \in (0,1)$. First let us consider R's decision to consult. In order to calculate his expected payoffs when he consults and when he does not consult, we need to consider his conditional expectations about the previous state ω_{t-1} and the current state ω_t, given ω_{t-2} and m_{t-1}. Note that

$$\begin{aligned} P(\omega_t = A \mid \omega_{t-2} = A, m_{t-1} = B) &= 1 - P(\omega_t = B \mid \omega_{t-2} = A, m_{t-1} = B) \\ &= \underbrace{\frac{1-\gamma}{\gamma(1-\tau_{AA}) + (1-\gamma)}}_{P(\omega_{t-1}=B \mid \omega_{t-2}=A, m_{t-1}=B)} \times \frac{1}{2} + \underbrace{\left(1 - \frac{1-\gamma}{\gamma(1-\tau_{AA}) + (1-\gamma)}\right)}_{P(\omega_{t-1}=A \mid \omega_{t-2}=A, m_{t-1}=B)} \times \gamma \end{aligned}$$

and

$$\begin{aligned} P(\omega_t = A \mid \omega_{t-2} = B, m_{t-1} = B) &= 1 - P(\omega_t = B \mid \omega_{t-2} = B, m_{t-1} = B) \\ &= \underbrace{\frac{\frac{1}{2}}{\frac{1}{2}(1-\tau_{BA}) + \frac{1}{2}}}_{P(\omega_{t-1}=B \mid \omega_{t-2}=B, m_{t-1}=B)} \times \frac{1}{2} + \underbrace{\left(1 - \frac{\frac{1}{2}}{\frac{1}{2}(1-\tau_{BA}) + \frac{1}{2}}\right)}_{P(\omega_{t-1}=A \mid \omega_{t-2}=B, m_{t-1}=B)} \times \gamma. \end{aligned}$$

Basic sender strategies imply $P(\omega_t = A \mid \omega_{t-2}, m_{t-1} = A) = 1$ and $P(\omega_t = B \mid \omega_{t-2}, m_{t-1} = A) = 0$ for $\omega_{t-2} \in \{A, B\}$.

The conditional joint distribution of the present and the previous state satisfies

$$P(\omega_{t-1} = A, \omega_t = A \mid \omega_{t-2}, m_{t-1}) = P(\omega_{t-1} = A \mid \omega_{t-2}, m_{t-1}) \gamma$$

and

$$P(\omega_{t-1}=B,\omega_t=A\,|\,\omega_{t-2},m_{t-1})=P(\omega_{t-1}=B\,|\,\omega_{t-2},m_{t-1})\frac{1}{2}.$$

Since R chooses A when $m_t=A$ and he chooses B when $m_t=B$, his expected payoff for period t conditional on consultation in period t (exclusive of the visiting costs) is given by

$$\begin{aligned}\pi_t^R(d_t=1\mid\omega_{t-2}=A,m_{t-1}=B)=&\,P(\omega_t=B\mid\omega_{t-2}=A,m_{t-1}=B)\\&+P(\omega_{t-1}=A,\omega_t=A\mid\omega_{t-2}=A,m_{t-1}=B)\tau_{AA}\\&+P(\omega_{t-1}=B,\omega_t=A\mid\omega_{t-2}=A,m_{t-1}=B)\tau_{BA},\end{aligned}$$

where $P(\omega_t=B\mid\omega_{t-2}=A,m_{t-1}=B)$ represents his payoff (of 1) multiplied by the conditional probability that $\omega_t=B$. If he does not consult, his expected payoff for period t is given by

$$\pi_t^R(d_t=0\mid\omega_{t-2}=A,m_{t-1}=B)=P(\omega_t=A\mid\omega_{t-2}=A,m_{t-1}=B)$$

since he chooses A. Thus the gross benefit of visiting given $\omega_{t-2}=A$ and $m_{t-1}=B$ is given by

$$\Delta_t^R(\omega_{t-2}=A,m_{t-1}=B)\equiv\pi_t^R(d_t=1\mid\omega_{t-2}=A,m_{t-1}=B)-\pi_t^R(d_t=0\mid\omega_{t-2}=A,m_{t-1}=B).$$

Similarly, for $\omega_{t-2}=B$ and $m_{t-1}=B$ we have

$$\begin{aligned}\pi_t^R(d_t=1\mid\omega_{t-2}=B,m_{t-1}=B)=&\,P(\omega_t=B\mid\omega_{t-2}=B,m_{t-1}=B)\\&+P(\omega_{t-1}=A,\omega_t=A\mid\omega_{t-2}=B,m_{t-1}=B)\tau_{AA}\\&+P(\omega_{t-1}=B,\omega_t=A\mid\omega_{t-2}=B,m_{t-1}=B)\tau_{BA},\end{aligned}$$

and

$$\pi_t^R(d_t=0\mid\omega_{t-2}=B,m_{t-1}=B)=P(\omega_t=A\mid\omega_{t-2}=B,m_{t-1}=B).$$

Thus the gross benefit of consulting given $\omega_{t-2}=B$ and $m_{t-1}=B$ is given by

$$\Delta_t^R(\omega_{t-2}=B,m_{t-1}=B)\equiv\pi_t^R(d_t=1\,|\,\omega_{t-2}=B,m_{t-1}=B)-\pi_t^R(d_t=0\,|\,\omega_{t-2}=B,m_{t-1}=B).$$

The above gross benefits, together with w and the realized value of v_t, determine R's best response given τ_{AA} and τ_{BA}.

Step 3 An equilibrium featuring a basic communication strategy as well as $\tau_{AA}\in(0,1)$ and $\tau_{BA}\in(0,1)$ requires two indifference conditions to hold simultaneously (one for $\omega_{t-2}=A$ and the other for $\omega_{t-2}=B$), namely

$$\underbrace{xf}_{\text{truth-telling benefit + no visit}}=\underbrace{\delta\left(\frac{\Delta_t^R(\omega_{t-2}=A,m_{t-1}=B)-w}{\frac{1}{2}}\right)f}_{\text{no truth-telling benefit + positive prob of visit in }t+1}\tag{23}$$

and

$$xf = \delta \left(\frac{\Delta_t^R (\omega_{t-2} = B, m_{t-1} = B) - w}{\frac{1}{2}} \right) f. \tag{24}$$

Solving simultaneously for τ_{AA} and τ_{BA}, we obtain three pairs of solutions.

The first pair is given by $\tau_{AA} = \frac{\tilde{x}+2w+4\gamma-2}{2\gamma}$ and $\tau_{BA} = \tilde{x} + 2w$. The second pair of solutions is given by

$$\tau_{AA} = \frac{(\tilde{x}+2w+6\gamma-1)\gamma + \sqrt{8\gamma^2(1-\tilde{x}-2w-4\gamma^2) + (6\gamma^2+\tilde{x}\gamma-\gamma+2w\gamma)^2}}{4\gamma^2},$$

and

$$\tau_{BA} = \frac{(3+\tilde{x}+2w-2\gamma)\gamma + \sqrt{8\gamma^2(1-\tilde{x}-2w-4\gamma^2) + (6\gamma^2+\tilde{x}\gamma-\gamma+2w\gamma)^2}}{4\gamma(1-\gamma)}.$$

The third pair is given by:

$$\tau_{AA}^* = \frac{(\tilde{x}+2w+6\gamma-1)\gamma - \sqrt{8\gamma^2(1-\tilde{x}-2w-4\gamma^2) + (6\gamma^2+\tilde{x}\gamma-\gamma+2w\gamma)^2}}{4\gamma^2} \tag{25}$$

and

$$\tau_{BA}^* = \frac{(3+\tilde{x}+2w-2\gamma)\gamma - \sqrt{8\gamma^2(1-\tilde{x}-2w-4\gamma^2) + (6\gamma^2+\tilde{x}\gamma-\gamma+2w\gamma)^2}}{4\gamma(1-\gamma)}. \tag{26}$$

For $\tau_{AA} \in (0,1)$ and $\tau_{BA} \in (0,1)$, the first solution requires $w < 1-\gamma$, which is at odds with the assumption that w is in the intermediate range 19. Similarly, the second solution requires $\tilde{x} < 0$ while we have assumed $\tilde{x} > 0$. The third solution gives $\tau_{AA}^* \in (0,1)$ and $\tau_{BA} \in (0,1)$ for $\gamma \in (1/2, 1)$ and

$$w \in \left(-\frac{\tilde{x}}{2}, \frac{1-\tilde{x}}{2} \right),$$

which is satisfied under our assumptions. Thus we conclude that the third pair of solutions (25) and (26) pins down the unique equilibrium in stationary basic strategies. □

7.3.4 Proposition 8

Proof. We use the closed form expressions for τ_{AA}^* and τ_{BA}^* appearing in (25) and (26). We obtain

$$\frac{\partial \tau_{AA}^*}{\partial w} = \frac{2 - \frac{\partial}{\partial w} F(w, \tilde{x}, \gamma)}{4\gamma}, \tag{27}$$

$$\frac{\partial \tau_{BA}^*}{\partial w} = \frac{2 - \frac{\partial}{\partial w} F(w, \tilde{x}, \gamma)}{4(1-\gamma)}, \tag{28}$$

$$\frac{\partial \tau_{AA}^*}{\partial \tilde{x}} = \frac{1 - \frac{\partial}{\partial x} F(w, \tilde{x}, \gamma)}{4\gamma} \tag{29}$$

$$\frac{\partial \tau_{BA}^*}{\partial \tilde{x}} = \frac{1 - \frac{\partial}{\partial x} F(w, \tilde{x}, \gamma)}{4(1-\gamma)} \tag{30}$$

where

$$F(w,\tilde{x},\gamma)=\sqrt{8\gamma^2(1-\tilde{x}-2w-4\gamma^2)+(6\gamma^2+\tilde{x}\gamma-\gamma+2w\gamma)^2}.$$

Partially differentiating F, we have

$$\frac{\partial F(w,\tilde{x},\gamma)}{\partial w}=\frac{2\gamma^2(\tilde{x}+6\gamma+2w-5)}{\sqrt{\gamma^2(\tilde{x}^2+2\tilde{x}(6\gamma+2w-5)+(3-2\gamma)^2+4w^2+4(6\gamma-5)w)}},$$

and

$$\frac{\partial F(w,\tilde{x},\gamma)}{\partial \tilde{x}}=\frac{\gamma^2(\tilde{x}+6\gamma+2w-5)}{\sqrt{\gamma^2(\tilde{x}^2+2\tilde{x}(6\gamma+2w-5)+(3-2\gamma)^2+4w^2+4(6\gamma-5)w)}}.$$

The partial derivatives above are all non-positive when $\tilde{x}+6\gamma+2w-5\leq 0$. The inequality can be written for $\gamma\in(1/2,1)$ as $w\leq\frac{1-\tilde{x}}{2}$, which is consistent with the assumption that w is in the intermediate range 19. Therefore, from (27) to (30) we obtain $\frac{\partial\tau^*_{AA}}{\partial w}>0$, $\frac{\partial\tau^*_{BA}}{\partial w}>0$, $\frac{\partial\tau^*_{AA}}{\partial\tilde{x}}>0$, and $\frac{\partial\tau^*_{BA}}{\partial\tilde{x}}>0$. □

7.3.5 *R*'s per-period average payoff

Step 1 Let $\overline{\Pi}_R$ be R's average per-period payoff in the equilibrium. We then have

$$\begin{aligned}\overline{\Pi}_R=&\sum_{\omega_{t-2},\omega_{t-1},m_{t-1}}P(\omega_{t-2},\omega_{t-1},m_{t-1})\\&\times\left[P(\omega_t=B\,|\,\omega_{t-1})\varphi_{\omega_{t-2}m_{t-1}}+P(\omega_t=A\,|\,\omega_{t-1})(1-\varphi_{\omega_{t-2}m_{t-1}}(1-\tau_{\omega_{t-1}A})\right]\\&-\sum_{\omega_{t-2},\omega_{t-1},m_{t-1}}P(\omega_{t-2},\omega_{t-1},m_{t-1})\varphi_{\omega_{t-2}m_{t-1}}\left(\frac{v(\omega_{t-2}m_{t-1})}{2}+w\right),\end{aligned}$$

where the first term is the average action-payoff and the second term is the average per-period consultation cost. In what follows we will derive each term separately.

Step 2 Let us give explicit expressions for some of the variables above. In the equilibrium we have $\varphi_{BA}=\varphi_{AA}=0$ and $\varphi_{AB}=\varphi_{BB}=\widetilde{x}$. The fact that $\varphi_{AB}=\frac{v(AB)}{\frac{1}{2}}=\widetilde{x}$ and $\varphi_{BB}=\frac{v(BB)}{\frac{1}{2}}=\widetilde{x}$ implies $v(AB)=v(BB)=\frac{\widetilde{x}}{2}$. We also have $\tau_{AB}=\tau_{BB}=1$, and

$$\tau_{AA}(w,\widetilde{x},\gamma)=\frac{(\widetilde{x}+2w+6\gamma-1)\gamma-\sqrt{8\gamma^2(1-\widetilde{x}-2w-4\gamma^2)+(6\gamma^2+\widetilde{x}\gamma-\gamma+2w\gamma)^2}}{4\gamma^2}$$

and

$$\tau_{BA}(w,\widetilde{x},\gamma)=\frac{(3+\widetilde{x}+2w-2\gamma)\gamma-\sqrt{8\gamma^2(1-\widetilde{x}-2w-4\gamma^2)+(6\gamma^2+\widetilde{x}\gamma-\gamma+2w\gamma)^2}}{4\gamma(1-\gamma)}.$$

Note also that

$$
\begin{aligned}
&P(\omega_{t-2}=A,\omega_{t-1}=A,m_{t-1}=A)=\mu_A\gamma\tau_{AA},\\
&P(\omega_{t-2}=B,\omega_{t-1}=A,m_{t-1}=A)=\mu_B\frac{1}{2}\tau_{BA},\\
&P(\omega_{t-2}=A,\omega_{t-1}=A,m_{t-1}=B)=\mu_A\gamma(1-\tau_{AA}),\\
&P(\omega_{t-2}=B,\omega_{t-1}=A,m_{t-1}=B)=\mu_B\frac{1}{2}(1-\tau_{BA}),\\
&P(\omega_{t-2}=A,\omega_{t-1}=B,m_{t-1}=B)=\mu_A(1-\gamma),\\
&P(\omega_{t-2}=B,\omega_{t-1}=B,m_{t-1}=B)=\mu_B\frac{1}{2},\\
&P(\omega_{t-2}=A,\omega_{t-1}=B,m_{t-1}=A)=\mu_A(1-\gamma)(0)=0,\\
&P(\omega_{t-2}=B,\omega_{t-1}=B,m_{t-1}=A)=\mu_B\frac{1}{2}(0)=0,
\end{aligned}
$$

where $\mu_A=\frac{1}{3-2\gamma}$.

Step 3 Let us derive the average per-period action-payoff. Note that the average action-payoff corresponds to the sum of the probabilities of the three events (denoted by Events 1, 2 and 3, respectively) in which R chooses the correct action to match the state. Event 1 is one where the state is B and R visits (, in which case R receives message B with probability 1). Event 2 is one where the state is A, and R visits and receives message A. Event 3 is one where the state is A and R does not consult. It follows that the expected probability of R choosing the correct action is given by

$$
\sum_{\omega_{t-2},\omega_{t-1},m_{t-1}} P(\omega_{t-2},\omega_{t-1},m_{t-1})\left[P(\omega_t=B|\omega_{t-1})\varphi_{\omega_{t-2}m_{t-1}}+P(\omega_t=A|\omega_{t-1})(1-\varphi_{\omega_{t-2}m_{t-1}}(1-\tau_{\omega_{t-1}A}))\right].
$$

Using the fact that

$$
P(\omega_{t-2}=A,\omega_{t-1}=B,m_{t-1}=A)=P(\omega_{t-2}=B,\omega_{t-1}=B,m_{t-1}=A)=0,
$$

we may write

$$\sum_{\omega_{t-2},\omega_{t-1},m_{t-1}} P(\omega_{t-2},\omega_{t-1},m_{t-1}) \left[\begin{array}{c} P(\omega_t = B\,|\,\omega_{t-1})\varphi_{\omega_{t-2}m_{t-1}} \\ +P(\omega_t = A\,|\,\omega_{t-1})(1-\varphi_{\omega_{t-2}m_{t-1}}(1-\tau_{\omega_{t-1}A})) \end{array} \right]$$

$$\begin{aligned}
&= \\
&\mu_A \gamma \tau_{AA} \left[P(\omega_t = B\,|\,\omega_{t-1} = A)\varphi_{AA} + P(\omega_t = A\,|\,\omega_{t-1} = A)(1-\varphi_{AA}(1-\tau_{AA})\right] \\
&+ \mu_B \frac{1}{2}\tau_{BA} \left[P(\omega_t = B\,|\,\omega_{t-1} = A)\varphi_{BA} + P(\omega_t = A\,|\,\omega_{t-1} = A)(1-\varphi_{BA}(1-\tau_{AA})\right] \\
&+ \mu_A \gamma(1-\tau_{AA}) \left[P(\omega_t = B\,|\,\omega_{t-1} = A)\varphi_{AB} + P(\omega_t = A\,|\,\omega_{t-1} = A)(1-\varphi_{AB}(1-\tau_{AA})\right] \\
&+ \mu_B \frac{1}{2}(1-\tau_{BA}) \left[P(\omega_t = B\,|\,\omega_{t-1} = A)\varphi_{BB} + P(\omega_t = A\,|\,\omega_{t-1} = A)(1-\varphi_{BB}(1-\tau_{AA})\right] \\
&+ \mu_A (1-\gamma) \left[P(\omega_t = B\,|\,\omega_{t-1} = B)\varphi_{AB} + P(\omega_t = A\,|\,\omega_{t-1} = B)(1-\varphi_{AB}(1-\tau_{BA})\right] \\
&+ \mu_B \frac{1}{2} \left[P(\omega_t = B\,|\,\omega_{t-1} = B)\varphi_{BB} + P(\omega_t = A\,|\,\omega_{t-1} = B)(1-\varphi_{BB}(1-\tau_{BA})\right].
\end{aligned}$$

Using the observations in Step 2, the above expression in turn rewrites as

$$\begin{aligned}
&P(\omega_{t-2} = A, \omega_{t-1} = A, m_{t-1} = A)\left[P(\omega_t = A\,|\,\omega_{t-1} = A)\right] \\
&+P(\omega_{t-2} = B, \omega_{t-1} = A, m_{t-1} = A)\left[P(\omega_t = A\,|\,\omega_{t-1} = A)\right] \\
&+P(\omega_{t-2} = A, \omega_{t-1} = A, m_{t-1} = B)\left[P(\omega_t = B\,|\,\omega_{t-1} = A)\widetilde{x} + P(\omega_t = A\,|\,\omega_{t-1} = A)(1-\widetilde{x}(1-\tau_{AA})\right] \\
&+P(\omega_{t-2} = B, \omega_{t-1} = A, m_{t-1} = B)\left[P(\omega_t = B\,|\,\omega_{t-1} = A)\widetilde{x} + P(\omega_t = A\,|\,\omega_{t-1} = A)(1-\widetilde{x}(1-\tau_{AA})\right] \\
&+P(\omega_{t-2} = A, \omega_{t-1} = B, m_{t-1} = B)\left[P(\omega_t = B\,|\,\omega_{t-1} = B)\widetilde{x} + P(\omega_t = A\,|\,\omega_{t-1} = B)(1-\widetilde{x}(1-\tau_{BA})\right] \\
&+P(\omega_{t-2} = B, \omega_{t-1} = B, m_{t-1} = B)\left[P(\omega_t = B\,|\,\omega_{t-1} = B)\widetilde{x} + P(\omega_t = A\,|\,\omega_{t-1} = B)(1-\widetilde{x}(1-\tau_{BA})\right],
\end{aligned}$$

which further simplifies to

$$\begin{aligned}
&\left(\frac{1}{3-2\gamma}\right)\gamma\tau_{AA}(w,\widetilde{x},\gamma)\,\gamma + \left(1-\frac{1}{3-2\gamma}\right)\frac{1}{2}\tau_{BA}(w,\widetilde{x},\gamma)\,\gamma \\
&+\left(\frac{1}{3-2\gamma}\right)\gamma(1-\tau_{AA}(w,\widetilde{x},\gamma))\left[(1-\gamma)\widetilde{x} + \gamma(1-\widetilde{x}(1-\tau_{AA}(w,\widetilde{x},\gamma)))\right] \\
&+\left(1-\frac{1}{3-2\gamma}\right)\frac{1}{2}(1-\tau_{BA}(w,\widetilde{x},\gamma))\left[(1-\gamma)\widetilde{x} + \gamma(1-\widetilde{x}(1-\tau_{AA}(w,\widetilde{x},\gamma)))\right] \\
&+\left(\frac{1}{3-2\gamma}\right)(1-\gamma)\left[\frac{1}{2}\widetilde{x} + \frac{1}{2}(1-\widetilde{x}(1-\tau_{BA}(w,\widetilde{x},\gamma)))\right] \\
&+\left(1-\frac{1}{3-2\gamma}\right)\frac{1}{2}\left[\frac{1}{2}\widetilde{x} + \frac{1}{2}(1-\widetilde{x}(1-\tau_{BA}(w,\widetilde{x},\gamma)))\right].
\end{aligned}$$

Step 4 In order to write R's expected per-period visiting cost with respect to τ_{BA} above and the exogenous variables, using the observations in Step 2, let us first write

$$\sum_{\omega_{t-2},\omega_{t-1},m_{t-1}} P(\omega_{t-2},\omega_{t-1},m_{t-1})\varphi_{\omega_{t-2}m_{t-1}}\left(\frac{v(\omega_{t-2}m_{t-1})}{2}+w\right)$$
$$=\left[\begin{array}{c} P(\omega_{t-2}=A,\omega_{t-1}=A,m_{t-1}=B) \\ +P(\omega_{t-2}=B,\omega_{t-1}=A,m_{t-1}=B) \\ +P(\omega_{t-2}=A,\omega_{t-1}=B,m_{t-1}=B) \\ +P(\omega_{t-2}=B,\omega_{t-1}=B,m_{t-1}=B) \end{array}\right]\times\widetilde{x}\left(\frac{\widetilde{x}}{4}+w\right),$$

which in turn rewrites as

$$\left(\begin{array}{c} \frac{1}{3-2\gamma}\gamma(1-\tau_{BA}(w,\widetilde{x},\gamma)) \\ +\left(1-\frac{1}{3-2\gamma}\right)\frac{1}{2}(1-\tau_{BA}(w,\widetilde{x},\gamma)) \\ +\frac{1}{3-2\gamma}(1-\gamma)+\left(1-\frac{1}{3-2\gamma}\right)\frac{1}{2} \end{array}\right)\widetilde{x}\left(\frac{\widetilde{x}}{4}+w\right).$$

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