

“Introduction to Poisson geometry”

Yvette Kosmann-Schwarzbach

Plan of lectures, 24-28 June, 2019

Lecture 1. Poisson manifolds and Poisson algebras

Introduction: What is Poisson geometry?

1.1 Review

- Differential forms and the de Rham differential
- Multivectors and the Schouten-Nijenhuis bracket

1.2 Definitions

- Poisson bivectors
- Poisson manifolds

1.3 Example: the dual of a Lie algebra

1.4 The Poisson bracket of functions, the Jacobi identity, Poisson algebras

1.5 Poisson manifolds generalize symplectic manifolds

Theorem. Non-degenerate closed 2-forms and non-degenerate Poisson bivectors are in 1-1 correspondence.

1.6 If time permits: Topics in the history of Poisson brackets

Lecture 2. The symplectic foliation. Poisson–Lichnerowicz cohomology

2.1 The symplectic leaves of a Poisson manifold

- Generalized foliations
- The generalized foliation of a Poisson manifold

2.2 Poisson maps

2.3 More examples of Poisson manifolds

2.4 Poisson–Lichnerowicz cohomology

2.5 The modular class of a Poisson manifold

Lecture 3. Poisson Lie groups, Lie bialgebras and Lie algebroids

3.1 Poisson–Lie groups and Lie bialgebras

- Poisson–Lie groups
- The infinitesimal of a Poisson–Lie group
- The double of a Lie bialgebra
- The classical Yang–Baxter equation

3.2 Lie algebroids

3.3 The Lie algebroid structure of the cotangent bundle of a Poisson manifold

3.4 Lie bialgebroids

3.5 Examples and applications

Lecture 4. Elements of the theory of Lie groupoids

4.1 Definition of Lie groupoids, Examples

4.2 Homomorphisms of Lie groupoids

4.3 The infinitesimal of a Lie groupoid

4.4 The tangent and cotangent groupoids of a Lie groupoid

Lecture 5. Multiplicative multivectors on Lie groups and Lie groupoids

5.1 Multiplicative multivectors on Lie groups

5.2 Multiplicativity as a cocycle condition

5.3 The infinitesimal of a multiplicative multivector

5.3 Multiplicative multivectors on Lie groupoids

5.4 Multiplicative forms on Lie groupoids

5.5 Symplectic and Poisson groupoids

Conclusion: An active field of research.