CORRIGENDUM TO "BETTI TABLES OF MONOMIAL IDEALS FIXED BY PERMUTATIONS OF THE VARIABLES"

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The statement of Proposition 2.7 in [Mu] contains a mistake. In the proposition, it was stated "$\beta_{i,\mathbf{a}}(I) \in \{0,1\}$ for all $\mathbf{a} \in \mathbb{Z}^n$", but this is false. The correct statement is as follows.

Proposition 2.7. Let $\mathcal{I}$ and $(p_1,q_1),\dots,(p_t,q_t)$ be as above. Fix $n \geq 2$ and let $I_n = \mathcal{I} \cap S_n$. Then

$$\begin{aligned}&\big\{\big(i,(a_1,\dots,a_n)\big) \in \mathbb{Z}\times\mathbb{Z}^n : \beta_{i,(a_1,\dots,a_n)}(I_n) \neq 0, a_1 \geq \cdots \geq a_n\big\}\\ &= \left(\bigcup_{k=1}^{t} \big\{\big(i,(p_k,q_k^{i+1},0^{n-i-2})\big) : i=0,1,\dots,n-2\big\}\right)\\ &\quad\cup \left(\bigcup_{k=1}^{t-1} \big\{\big(i+1,(p_k,q_{k+1}^{i+1},0^{n-i-2})\big) : i=0,1,\dots,n-2\big\}\right) \cup E,\end{aligned}$$

where $E = \big\{\big(i+1,(p_t^{i+2},0^{n-i-2})\big) : i=0,1,\dots,n-2\big\}$ if $p_t > q_t$ and $E=\emptyset$ if $p_t = q_t$. Moreover, if $p_t \neq q_t$ then $\beta_{i,\mathbf{a}}(I) \in \{0,1\}$ for any $\mathbf{a} \in \mathbb{Z}^n_{\geq 0}$.

The error only appears in Case (I) of the proof of the statement. In Case (I), it is necessary to consider the case when $p_k > q_k$ and the case when $p_k = q_k$ separately, but the latter case was missing. Below is the corrected version of Case (I).

Case (I). Suppose $\mathbf{a} = (p_k, q_k^{\ell}, 0^{n-\ell-1})$ with $1 \leq k \leq t$. Suppose $p_k > q_k$. Then

$$\begin{aligned}\Delta_{\mathbf{a}}^{I_n} &= \{F \subset [\ell+1] : x_1^{p_k-1}x_2^{q_k-1}\cdots x_{\ell+1}^{q_k-1}x^{[\ell+1]\setminus F} \in I_n\}\\ &= \{F \subset [\ell+1] : 1 \notin F \text{ and } ([\ell+1]\setminus F)\cap\{2,\dots,\ell+1\} \neq \emptyset\}\\ &= \{F \subset \{2,\dots,\ell+1\} : F \neq \{2,\dots,\ell+1\}\}.\end{aligned}$$

Thus $\Delta_{\mathbf{a}}^{I_n}$ is the boundary of the $(\ell-1)$-simplex and we have $\beta_{i,\mathbf{a}}(I_n) = \dim_{\Bbbk} \widetilde{H}_{i-1}(\Delta_{\mathbf{a}}^{I_n}) \neq 0$ if and only if $i = \ell-1$, and we also have $\widetilde{H}_{\ell-2}(\Delta_{\mathbf{a}}^{I_n}) \cong \Bbbk$.

Suppose $p_k = q_k$. Then k must be equal to t and

$$\Delta_{\mathbf{a}}^{I_n} = \{F \subset [\ell+1] : (x_1x_2\cdots x_{\ell+1})^{p_t-1}x^{[\ell+1]\setminus F} \in I_n\} = \{F \subset [\ell+1] : |F| \leq \ell-1\}.$$

Thus $\beta_{i,\mathbf{a}}(I_n) = \dim_{\Bbbk} \widetilde{H}_{i-1}(\Delta_{\mathbf{a}}^{I_n}) \neq 0$ if and only if $i = \ell-1$.

References

[Mu] S. Murai, Betti tables of monomial ideals fixed by permutations of the variables, *Trans. Amer. Math. Soc.* **373** (2020), 7087–7107

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