DIGITAL TECHNOLOGIES
FOR CONTROLLING ROOM ACOUSTICS

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ABSTRACT

One benefit brought by introducing digital technologies to controlling room acoustics seems that great many achievements of discrete mathematics, such as number theory and linear algebra, can straightforwardly be exploited as signal-processing programs for digital computers and digital signal processors. From this viewpoint, some applications of Bezout identities [1] to inverse filtering of room acoustics are mentioned.

INTRODUCTION

Consider a single-input single-output linear acoustic system shown in Fig. 1. $G_1(z)$ denotes the sound propagation path (room transfer function) from loudspeaker S_1 to microphone M. In order to control sound observed by microphone M, inverse filtering of $G_1(z)$ seems easy and effective. This processing is performed by inverse filter H(z) given as follows.

$$H(z) = z^{-d}/G_1(z) = 1/g_1(z), \qquad (1)$$

where
$G_1(z) = z^{-d}\, g_1(z)$,
z^{-d}: sound propagation delay from S_1 to M, and
$g_1(z)$: polynomial of degree "m," which represents direct sound as well as reflective sounds.

Since polynomial $g_1(z)$ is usually non-minimum phase [2-3], however, H(z) cannot be obtained as a causal filter.

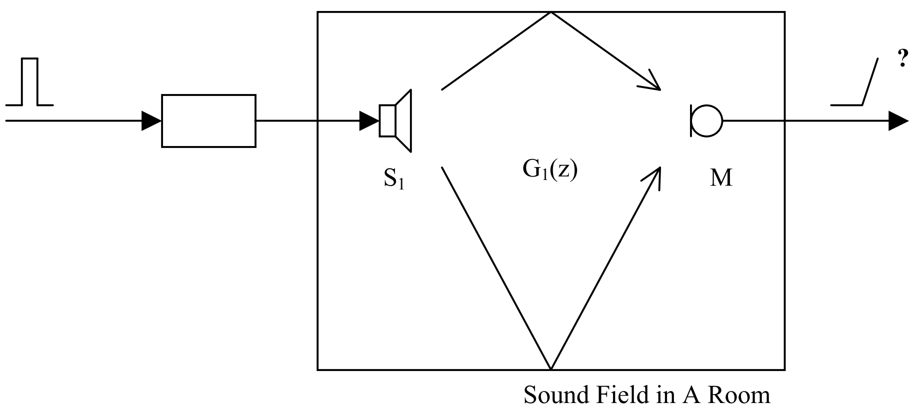

Fig. 1 Inverse filtering for single-input single-output linear acoustic system.

APPLYING BEZOUT IDENTITIES TO INVERSE FILTERING

Principle

Causal inverse filtering may be performed in a two-input single-output system shown in Fig. 2, where the sound propagation path from loudspeaker S_2 to microphone M is added to the previous system in Fig. 1. This processing is formalized as a following Bezout identity (multiple-input/output inverse-filtering theorem, MINT)[3-4].

$$H_1(z)G_1(z) + H_2(z)G_2(z) = z^{-d},$$
$$\leftrightarrows \quad H_1(z)g_1(z) + H_2(z)\, zg_2(z) = 1, \qquad (2)$$

where
$G_2(z) = z^{-d}g_2(z)$,
z^{-d} : sound propagation delay common to $G_1(z)$ and $G_2(z)$, and
$g_2(z)$: polynomial of degree "n."

Inverse filter set $\{H_1(z), H_2(z)\}$ exists if and only if polynomials $g_1(z)$ and $g_2(z)$ are co-prime (in other wards, $g_1(z)$ and $g_2(z)$ have no common zero) . $H_1(z)$ and $H_2(z)$ are given as follows [5].

$$H_1(z) = H_{1,min}(z) + g_2(z)Q(z) \text{ and } H_2(z) = H_{2,min}(z) - g_1(z)Q(z), \qquad (3)$$

where
$\{H_{1,min}(z), H_{2,min}(z)\}$: minimum-degree unique solution-set given as
$\deg H_{1,min}(z) < \deg g_2(z) = n$ and $\deg H_{2,min}(z) < \deg g_1(z) = m$, and
$Q(z)$: arbitrary polynomial.

Note that as far as $g_1(z)$ and $g_2(z)$ are co-prime, causal inverse filtering will be performed through $H_1(z)$ and $H_2(z)$ even when $g_1(z)$ or $g_2(z)$ is non-minimum phase.

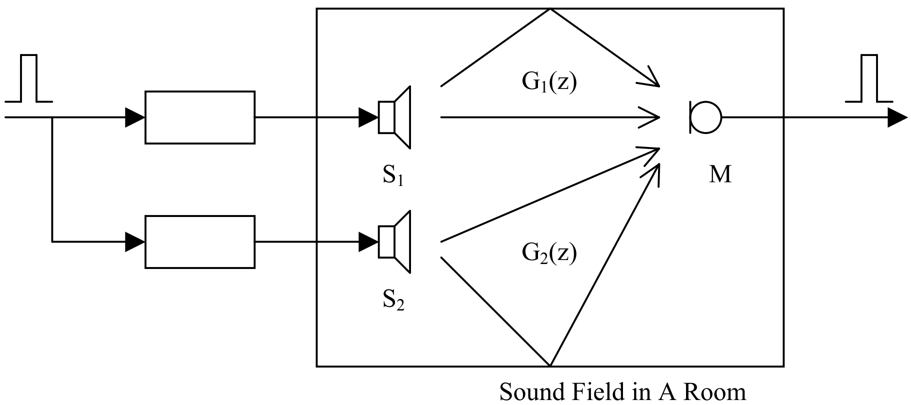

Fig. 2 Inverse filtering for two-input single-output linear acoustic system using a Bezout Identity.

Calculation of inverse filter set

Though matrix calculation may be exploited to obtain the minimum degree inverse filter set, $\{H_{1,min}(z), H_{2,min}(z)\}$, adaptive filtering sometimes shows better performance regarding calculation complexity as well as storage capacity. A schematic diagram of this filter-calculation, a Filtered-X algorithm [6], is shown in Fig. 3. Here, $C_1(z)$ and $C_2(z)$ respectively represent the replicas of sound propagation paths $G_1(z)$ and $G_2(z)$. Respective signals $y_1(k)$ and $y_2(k)$ (k: integer index) are produced though $C_1(z)$ and $C_2(z)$ from broadband audio signal x(k), and input to adaptive filters $H_1(z)$ and $H_2(z)$. Reference x(k-d) denotes the delayed version of x(k), which is synthesized by considering propagation delay z^{-d} common to $G_1(z)$ and $G_2(z)$. Error e(k) for calculating adaptive filters $H_1(z)$ and $H_2(z)$ is defined as follows.

$$e(k) = x(k-d) - \{[H_1(z)]y_1(k) + [H_2(z)]y_2(k)\},$$
$$\leftrightarrows\ e(k) = [1 - \{H_1(z)g_1(z) + H_2(z)\, z^{-w} g_2(z)\}]x(k). \qquad (4)$$

This relation is made equivalent to relation (2) when e(k) becomes zero. By defining the respective degrees of $H_1(z)$ and $H_2(z)$ as "n – 1" and "m – 1," therefore, minimum degree inverse filter set $\{H_{1,min}(z), H_{2,min}(z)\}$ may be obtained by using an adaptive algorithm minimizing the power of error e(k), such as LMS and RLS [7]. Loudspeakers S_1 and S_2 emit sounds processed through convolver set $\{P_1(z), P_2(z)\}$, which is a copy of $\{H_{1,min}(z), H_{2,min}(z)\}$, so that precise inverse filtering may be performed at microphone M.

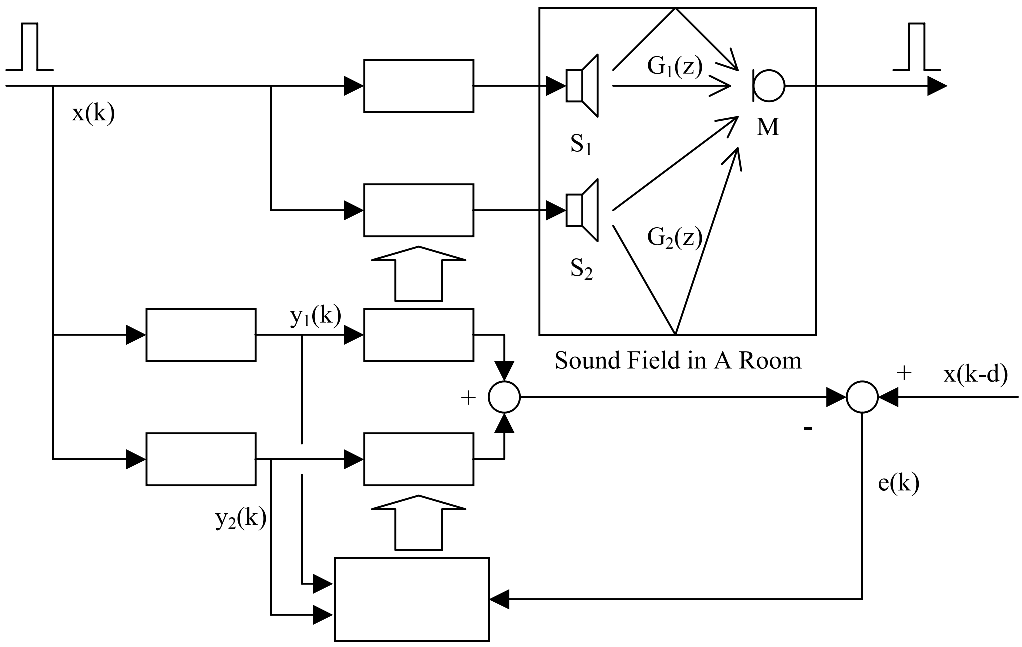

Fig. 3 Calculation of inverse filter set $\{H_1(z), H_2(z)\}$.

IDENTIFICATION OF SOUND PROPAGATION PATHS

Identification using two loudspeakers

The principle described in the previous section can also be exploited to identify sound propagation paths $G_1(z)$ and $G_2(z)$ from loudspeakers S_1 and S_2 to microphone M as shown in Fig. 4 [8]. The respective output signals of convolvers $P_1(z)$ and $P_2(z)$, which are produced from broadband audio signal x(k), are fed to the loudspeakers as well as adaptive filters $H_1(z)$ and $H_2(z)$. Sound r(k) observed with the microphone can be denoted as follows.

$$r(k) = [P_1(z)G_1(z) + P_2(z)G_2(z)]x(k),$$
$$\leftrightarrows \quad r(k) = [P_1(z)g_1(z) + P_2(z)g_2(z)]x(k)= [R(z)]x(k). \tag{5}$$

Summation o(k) of the signals output from $H_1(z)$ and $H_2(z)$ can also be denoted as follows.

$$o(k) = [P_1(z)H_1(z) + P_2(z)H_2(z)]x(k). \tag{6}$$

Here, if solution set $\{H_1(z), H_2(z)\}$ that satisfies the following relation can uniquely be determined, the solution set will obviously be equivalent to set of sound propagation paths $\{g_1(z), g_2(z)\}$.

$$R(z) = P_1(z)H_1(z) + P_2(z)H_2(z). \tag{7}$$

The strategies for uniquely determining $\{H_1(z), H_2(z)\}$ are summarized as follows.

s-1: Convolvers $P_1(z)$ and $P_2(z)$ should not have any common zero.
s-2: Respective degrees of $P_1(z)$ and $P_2(z)$ should be given as follows.
$\deg P_1(z) > \deg H_2(z) \geqq \deg g_2(z)$ and
$\deg P_2(z) > \deg H_1(z) \geqq \deg g_1(z)$, or
$\deg P_1(z) = \deg P_2(z) = L + 1$ and $\deg H_1(z) = \deg H_2(z) = L$,
where $L = \max(\deg g_1(z), \deg g_2(z))$.

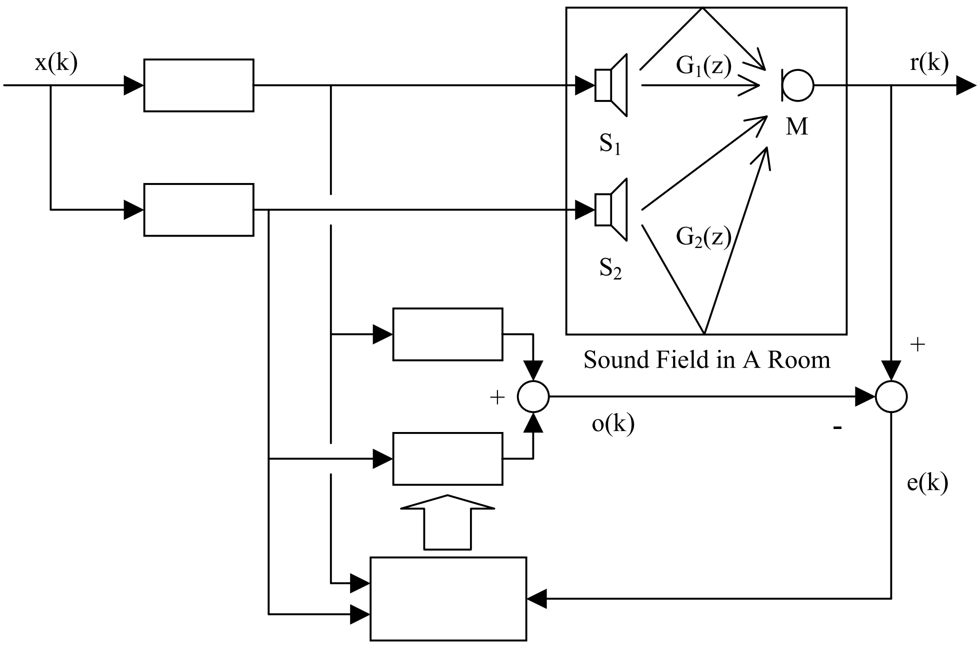

Fig.4 Identification of sound propagation paths $G_1(z)$ and $G_2(z)$.

By using $P_1(z)$ and $P_2(z)$ designed along these strategies, and minimizing the power of error e(k) given by the following relation, $\{H_1(z), H_2(z)\}$ will converge to $\{g_1(z), g_2(z)\}$.

$$e(k) = r(k) - o(k). \tag{8}$$

Blind identification using two microphones

Linear prediction methods combined with Bezout idenities [9-10] may achieve blind identification of the sound propagation paths between sound sources and microphones. Consider a single-input two-output linear acoustic system shown in Fig. 5. Broadband audio signals are emitted from sound source S to microphones M_1 and M_2 through unknown sound propagation paths $G_1(z)$ and $G_2(z)$. Their maximum degree, max (deg $g_1(z)$, deg $g_2(z)$), is only assumed to be known. Adaptive filters $H_1(z)$ and $H_2(z)$ are calculated by using the delayed versions of the respective microphone output-signals, $y_1(k-1)$ and $y_2(k-1)$, so as to minimize the power of error e(k) given as follows.

$$\begin{aligned} & e(k) = y_1(k) - \{[H_1(z)]y_1(k-1) + [H_2(z)]y_2(k-1)\}, \\ \leftrightarrows\ & e(k) = [g_1(z)]u(k) - [g_1(z)H_1(z) + g_2(z)H_2(z)]u(k-1), \\ \leftrightarrows\ & e(k) = g_{1,0}\,u(k) + [g_{1,rest}(z) - \{g_1(z)H_1(z) + g_2(z)H_2(z)\}]u(k-1), \end{aligned} \tag{9}$$

where

$y_i(k) = [G_i(z)]u(k) \leftrightarrows y_i(k) = [g_i(z)]u(k)$ (i = 1 or 2), and

$g_1(z) = g_{1,0} + z^{-1}g_{1,rest}(z)$.

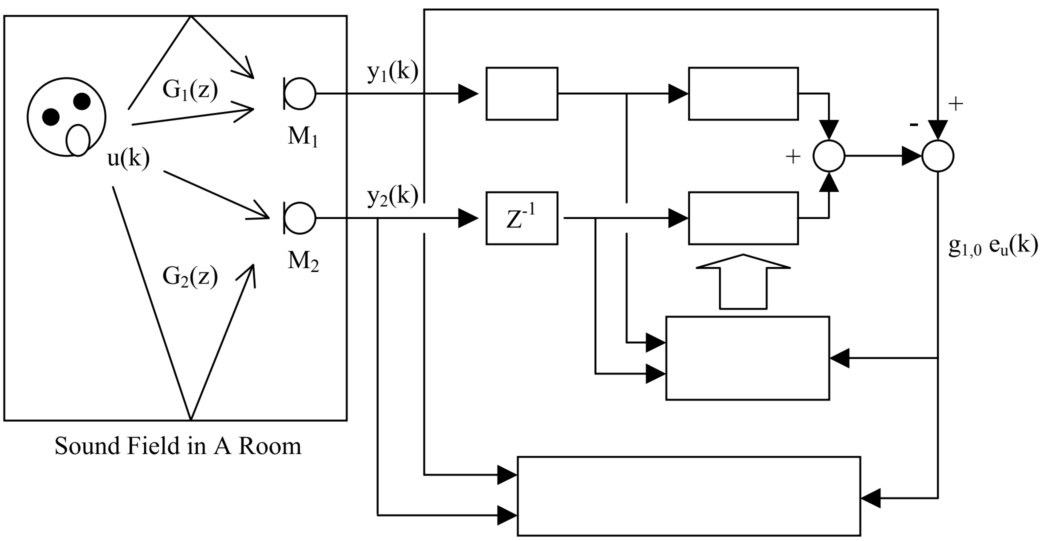

Fig.5 Blind identification of sound propagation paths $G_1(z)$ and $G_2(z)$

Suppose the following relationship.

$$u(k) = F(z)u(k-1) + e_u(k), \tag{10}$$

where

F(z): forward linear prediction filter whose degree satisfies;

$\deg F(z) \leqq \deg \{g_1(z)H_1(z) + g_2(z)H_2(z)\}$, and

$e_u(k)$: non-zero prediction error that satisfies the relation;
$E[e_u(k)u(k)] = E[\,|\,e_u(k)\,|^2]$,
where E[*] denotes the expectation of *.

Hence, Eq. (9) can be rewritten as follows.

$$e(k) = g_{1,0}\{F(z)\,u(k-1)+e_u(k)\}+[g_{1,rest}(z) - \{g_1(z)H_1(z)+g_2(z)H_2(z)\}]u(k-1),$$
$$\leftrightarrows\quad e(k) = [\{g_{1,0}\,F(z)+g_{1,rest}(z)\} - \{g_1(z)H_1(z)+g_2(z)H_2(z)\}]u(k-1)+g_{1,0}\,e_u(k)\}. \qquad (11)$$

By minimizing the power of e(k), prediction error $g_{1,0}\,e_u(k)$ will be obtained. Sound propagation paths $g_1(z)$ and $g_1(z)$ can be estimated up to factor $g_{1,0}/|\,g_{1,0}\,|^2$, therefore, from simple cross-correlation calculations as follows.

$$\left.\begin{aligned} E[g_{1,0}\,e_u(k)y_1(k)] \,/\, E[\,|g_{1,0}\,e_u(k)|^2] &= g_{1,0}[g_{\,1}(z)]E[\,|e_u(k)\,|^2]/\{|g_{1,0}\,|^2E[\,|e_u(k)\,|^2]\} \\ &= (g_{1,0}/|\,g_{1,0}\,|^2)g_1(z), \text{ and} \\ E[g_{1,0}\,e_u(k)y_2(k)] \,/\, E[\,|g_{1,0}\,e_u(k)|^2] &= g_{1,0}[g_{\,2}(z)]E[\,|e_u(k)\,|^2]/\{|g_{1,0}\,|^2E[\,|e_u(k)\,|^2]\} \\ &= (g_{1,0}/|\,g_{1,0}\,|^2)g_2(z). \end{aligned}\right\} \quad (12)$$

Though these estimated paths are still ambiguous in their amplitudes, this incompleteness will not cause any difficulty to some needs such as de-reverberation on audio signals received by microphones M_1 and M_2.

SUMMARY

Some applications of Bezout Identities to inverse filtering are mentioned as successful and promising examples of digital technologies exploited in controlling room acoustics.

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