UNDERDETERMINED DOA ESTIMATION BY THE NON-LINEAR MUSIC EXPLOITING HIGHER-ORDER MOMENTS

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ABSTRACT

This paper describes a new approach for extending MUltiple SIgnal Classification (MUSIC) to underdetermined direction-of-arrival (DOA) estimation with high resolution by exploiting higher-order moments. The proposed method maps the observed signals nonlinearly onto a space of expanded dimensions, in which signal statistics are analyzed. The covariance matrix in the higher-dimensional space corresponds to the higher-order cross moment matrix in the original space of the observed signals. Since the dimensionality of the noise subspace is increased by the mapping, the proposed method achieves higher resolution DOA estimation than the standard MUSIC, and also offers the ability to estimate DOAs in underdetermined conditions. We compared the characteristic of the proposed method with that of the conventional $2q$-MUSIC utilizing higher-order cumulants theoretically and experimentally.

Index Terms— MUSIC, Higher-order statistics, Microphone array, Underdetermined DOA estimation

1. INTRODUCTION

In array signal processing, direction of arrival (DOA) estimation is essential in various applications such as source separation and noise reduction. MUltiple SIgnal Classification (MUSIC) [1] is a popular high-resolution DOA estimation method which employs the subspace analysis of the observed signals. However, MUSIC is restricted by the dimensionality of the covariance matrix because MUSIC requires the identification of the noise subspace that is orthogonal to the transfer function vectors of the observed signals. To estimate N sound sources, $M > N$ sensors are required and the DOA estimation performance deteriorates as N approaches M. Thus, MUSIC has a problem in that an increase in the array scale is unavoidable when estimating a large number of sources.

To overcome this issue, several extensions of MUSIC have been proposed that increase the dimensionality of the observed signals virtually by using higher-order cumulants. For example, a MUSIC-like algorithm [2] exploits fourth-order cumulants, and its extension, $2q$-MUSIC [3], exploits cumulants of an arbitrary even order. These methods improve the resolution of DOA estimation and can also estimate DOAs in underdetermined conditions, where the number of sources exceeds the number of sensors, by increasing the signal expressiveness by way of subspace expansion.

A speech enhancement method called complementary beamforming [4] has been proposed for scenarios with more noise sources than sensors, and this approach was used for a DOA estimation problem [5]. Complementary beamforming has also been extended and explained as the mapping of a signal onto higher dimensional space using a kernel functions [6]. The approach used in these methods is especially effective for MUSIC to which dimensionality is critical. In this paper, utilizing mapping for MUSIC, we propose *mapped MUSIC* for underdetermined DOA estimation. By increasing the dimensionality of the noise subspace with the higher-dimensional mapping of the observed signal, the resolution of DOA estimation is improved and DOA estimation in underdetermined conditions is achieved. We describe a class of mapping suitable for mapped MUSIC, which allows us to analyze the cross moments of arbitrary even orders. Also we show an efficient way to calculate the cross moments for fourth and sixth orders. Moreover, we discuss the relation between the cross moment used in the proposed mapped MUSIC and the cross cumulant used in $2q$-MUSIC, and show that the analysis of the cross moment is more computationally efficient. Experimental results in a simulation of speech DOA estimation reveals that the proposed mapped MUSIC can achieve high resolution DOA estimation similar to $2q$-MUSIC but with much less computational complexity.

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2. PROBLEM STATEMENT

Throughout this paper, signals are expressed as complex amplitudes. Observed signals can be modeled as

$$\mathbf{x}(\omega,t) = [x_1(\omega,t),\cdots,x_M(\omega,t)]^{\mathrm{T}} = \sum_{i=1}^{N}\mathbf{a}_i(\omega)s_i(\omega,t)+\mathbf{n}(\omega,t), \quad (1)$$

$$\mathbf{n}(\omega,t) = [n_1(\omega,t),\cdots,n_M(\omega,t)]^{\mathrm{T}}, \quad (2)$$

$$\mathbf{a}_i(\omega) = [a_{1,i}(\omega),\cdots,a_{M,i}(\omega)]^{\mathrm{T}}, \quad (3)$$

where $t = 1,\cdots,L$ is a time frame index, ω is the angular frequency, M is the number of sensors, N is the number of sound sources, $[\cdot]^{\mathrm{T}}$ denotes transpose, $s_i(\omega,t)$ is the complex amplitude of the ith sound source, $x_j(\omega,t)$ is the complex amplitude of the signal observed with the jth sensor, $n_j(\omega,t)$ is the complex amplitude of the noise observed with the jth sensor, and $a_{j,i}(\omega)$ denotes the transfer function from the ith source to the jth sensor.

In the signal model expressed by Eq. (1), each sensor observes a mixture of source signals and noise signals. The problem in this paper is to estimate the DOAs of the source signals $s_1(\omega,t),\cdots,s_N(\omega,t)$ by finding steering vectors $\mathbf{b}(\omega;\theta_1),\cdots,\mathbf{b}(\omega;\theta_N)$ similar to the transfer function vectors $\mathbf{a}_1(\omega),\cdots,\mathbf{a}_N(\omega)$.

3. PROPOSED METHOD

3.1. DOA estimation algorithm of mapped MUSIC

Mapped MUSIC maps the M-dimensional observed vector $\mathbf{x}(\omega, t)$ onto M'-dimensional Euclidian space ($M' \geq M$) with a nonlinear function $\boldsymbol{\phi} : \mathbb{C}^M \to \mathbb{C}^{M'}$ and conducts a similar analysis to MUSIC with vectors $\boldsymbol{\phi}(\mathbf{x}(\omega, t))$. To estimate DOAs properly with mapped MUSIC, the information about the correlations between each microphone signal must be retained after mapping. This requirement is expressed as the following three conditions:

1. The magnitude relation of the norm is retained.
$$||\mathbf{x}||_2 \geq ||\mathbf{y}||_2 \to ||\boldsymbol{\phi}(\mathbf{x})||_2 \geq ||\boldsymbol{\phi}(\mathbf{y})||_2 \tag{4}$$
2. The origin remains intact.
$$\mathbf{x} = 0 \to \boldsymbol{\phi}(\mathbf{x}) = 0 \tag{5}$$
3. The orthogonality between vectors is preserved.
$$\mathbf{x}^{\mathrm{H}}\mathbf{y} = 0 \to \boldsymbol{\phi}^{\mathrm{H}}(\mathbf{x})\boldsymbol{\phi}(\mathbf{y}) = 0 \tag{6}$$

We describe the DOA estimation algorithm of mapped MUSIC with the mapping $\boldsymbol{\phi}$ satisfying Eqs. (4–6). The covariance matrix of $\boldsymbol{\phi}(\mathbf{x}(\omega, t))$ is expressed as

$$\mathbf{R}(\omega) = E[\boldsymbol{\phi}(\mathbf{x}(\omega, t))\boldsymbol{\phi}^{\mathrm{H}}(\mathbf{x}(\omega, t))], \tag{7}$$

where $E[\cdot]$ denotes the expectation of the argument, and $[\cdot]^{\mathrm{H}}$ denotes a complex conjugate transpose. The following equations are given by the eigen decomposition of covariance matrix $\mathbf{R}(\omega)$,

$$\mathbf{R}(\omega) = \mathbf{V}(\omega)\mathbf{E}(\omega)\mathbf{V}^{\mathrm{H}}(\omega), \tag{8}$$
$$\mathbf{V}(\omega) = [\mathbf{v}_1(\omega), \cdots, \mathbf{v}_{M'}(\omega)],\ \mathbf{V}^{\mathrm{H}}(\omega)\mathbf{V}(\omega) = \mathbf{I}_{M'}, \tag{9}$$
$$\mathbf{E}(\omega) = \mathrm{diag}[e_1(\omega), \cdots, e_{M'}(\omega)],$$
$$e_1(\omega) \geq \cdots \geq e_{M'}(\omega), \tag{10}$$
$$M' = \dim[\boldsymbol{\phi}(\mathbf{x}(\omega, t))], \tag{11}$$

where $\mathbf{v}_1(\omega), \cdots, \mathbf{v}_{M'}(\omega)$ are the eigenvectors for each eigenvalue $e_1(\omega), \cdots, e_{M'}(\omega)$, respectively, $\mathbf{I}_i$ denotes the i-dimensional identity matrix, $\mathrm{diag}[\cdot]$ is a diagonal matrix consisting of elements within the argument vector, and $\dim[\cdot]$ is the dimensionality of the argument. With N' denoting the number of large eigenvalues corresponding to the mappings of the directional sound source signals, we define the spanned subspace by $\mathbf{v}_1(\omega), \cdots, \mathbf{v}_{N'}(\omega)$ as the signal subspace $\mathcal{S}(\omega)$ in the mapping space. Incidentally, this paper assumes $N' = N$.

$$\mathcal{S}(\omega) \triangleq \mathrm{span}[\mathbf{v}_1(\omega), \cdots, \mathbf{v}_{N'}(\omega)], \tag{12}$$

where $\mathrm{span}[\cdot]$ denotes subspace spanned by argument vectors. Moreover, its orthogonal complement in $\mathrm{span}[\mathbf{v}_1(\omega), \cdots, \mathbf{v}_{M'}(\omega)]$ is defined as the noise subspace. Now, the following relation exists between the transfer function vectors $\mathbf{a}_1(\omega), \cdots, \mathbf{a}_N(\omega)$ and the vectors $\mathbf{v}_{N'+1}(\omega), \cdots, \mathbf{v}_{M'}(\omega)$ in the noise subspace.

$$\boldsymbol{\phi}^{\mathrm{H}}(\mathbf{a}_i(\omega))\mathbf{v}_j(\omega) = 0,$$
$$(i, j) \in \{i = 1, \cdots, N\ \ j = N' + 1, \cdots, M'\}. \tag{13}$$

Let us define the following *MUSIC score* of mapped MUSIC $f(\omega; \theta)$ utilizing the orthogonality in Eq. (13).

$$f(\omega; \theta) \triangleq \frac{1}{\sum_{i=N'+1}^{M'} |\boldsymbol{\phi}^{\mathrm{H}}(\mathbf{b}(\omega; \theta))\mathbf{v}_i(\omega)|}, \tag{14}$$

where $\mathbf{b}(\omega; \theta)$ ($||\mathbf{b}(\omega; \theta)||_2 = 1$) is a steering vector for direction θ. The score $f(\omega; \theta)$ takes a high value when the θ value is close to $\boldsymbol{\phi}^{\mathrm{H}}(\mathbf{b}(\omega; \theta))\boldsymbol{\phi}(\mathbf{a}_i(\omega)) \approx ||\boldsymbol{\phi}(\mathbf{a}_i(\omega))||_2$ for any $i = 1, \cdots, N$. Then, the conditions (4–6) result in relation $\mathbf{b}^{\mathrm{H}}(\omega; \theta)\mathbf{a}_i(\omega) \approx ||\mathbf{a}_i(\omega)||_2$. Mapped MUSIC utilizes this property and estimates DOA θ to make the value of $f(\omega; \theta)$ high. Here, note that mapped MUSIC with the following mapping with no signal modification is equivalent to MUSIC.

$$\boldsymbol{\phi}_1(\mathbf{x}) = \mathbf{x}. \tag{15}$$

3.2. Mapping for analysis of $2d$-th order moments

As shown in Sect. 3.1, mapped MUSIC can use any mapping function satisfying Eqs. (4–6), but its property is altered by the choice of mapping. In this paper, to evaluate the properties of mapping quantatively, we focus on the following mapping $\boldsymbol{\phi}_d : \mathbb{C}^M \to \mathbb{C}^{M^d}$, which gives a $2d$-th order cross moment matrix as its covariance matrix.

$$\boldsymbol{\phi}_d(\mathbf{x}) = \left[\prod_{l=1}^{d} x_{c_{1l}}^{l\oplus}, \cdots, \prod_{l=1}^{d} x_{c_{M^d l}}^{l\oplus} \right]^{\mathrm{T}}, \tag{16}$$
$$\mathbf{x} = [x_1, \cdots, x_M]^{\mathrm{T}}, \tag{17}$$
$$x^{l\oplus} = \begin{cases} x^* & (\text{if } l \text{ is odd}) \\ x & (\text{if } l \text{ is even}) \end{cases}, \tag{18}$$

where $\{\cdot\}^*$ denotes a complex conjugate. Then c_{kl} is an index specifying the element number of vector $\mathbf{x}$ for calculating $\boldsymbol{\phi}_d$, and this is the (k, l) element of the $M^d \times d$ index matrix $\mathbf{C}$ that arranges d-repeated-permutations of M in an arbitrary order as its row vectors:

$$\mathbf{C} = [c_{kl}]_{kl} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 2 \\ & & \vdots & \\ M & M & \cdots & M \end{bmatrix}, \tag{19}$$

where $[\cdot]_{ij}$ denotes a matrix consisting of the argument as its (i, j) element. In practice, the covariance matrix of the mapping of the observed signals $\boldsymbol{\phi}_d(\mathbf{x}(\omega, t))$ is expressed as a $2d$-th order cross moment matrix according to the following equation,

$$\mathbf{R}_d(\omega) = E[\boldsymbol{\phi}_d(\mathbf{x}(\omega, t))\boldsymbol{\phi}_d^{\mathrm{H}}(\mathbf{x}(\omega, t))] = [r_{ij}]_{ij},$$
$$(i, j = 1, \cdots, M^d), \tag{20}$$
$$r_{ij} = E\left[\left(\prod_{l=1}^{d} x_{c_{il}}^{\oplus} \right) \left(\prod_{l=1}^{d} x_{c_{jl}}^{\oplus} \right)^* \right]. \tag{21}$$

We can see the increase in the dimensionality of the covariance matrix from M to M^d by the definition of the mapping $\boldsymbol{\phi}_d : \mathbb{C}^M \to \mathbb{C}^{M^d}$. Then mapped MUSIC enables us to estimate the DOAs in underdetermined conditions due to the enhanced expressiveness of the noise subspace derived from the increase in dimensionality.

Equation (16) gives the mapping $\boldsymbol{\phi}_d$ for the analysis of the even-order cross moment, but these mappings can be replaced by an arbitrary mapping $\boldsymbol{\phi}'_d$ which gives the equivalent inner product:

$$\boldsymbol{\phi}_d^{\mathrm{H}}(\mathbf{x})\boldsymbol{\phi}_d(\mathbf{y}) = \boldsymbol{\phi}_d'^{\mathrm{H}}(\mathbf{x})\boldsymbol{\phi}'_d(\mathbf{y}), \tag{22}$$

and there are more useful mappings than that given by Eq. (16) from various viewpoints. For example, when the degree of the mapping function is two, the following mapping $\boldsymbol{\phi}'_2 : \mathbb{C}^M \to \mathbb{R}^{M^2}$ satisfying Eq. (22) simplifies the calculation of the covariance matrix and eigenvalue problem because its covariance matrix becomes real-valued.

$$\phi'_2(\mathbf{x}) \triangleq [\phi'_{\text{abs}}(\mathbf{x})^{\mathrm{T}}, \phi'_{\text{re}}(\mathbf{x})^{\mathrm{T}}, \phi'_{\text{im}}(\mathbf{x})^{\mathrm{T}}]^{\mathrm{T}}, \quad (23)$$

$$\phi'_{\text{abs}}(\mathbf{x}) \triangleq [\forall |x_i|^2 | 1 \le i \le M]^{\mathrm{T}}, \quad (24)$$

$$\phi'_{\text{re}}(\mathbf{x}) \triangleq \sqrt{2}[\forall \mathrm{Re}[x_i x_j^*] | 2 \le i \le M, 1 \le j \le i-1]^{\mathrm{T}}, \quad (25)$$

$$\phi'_{\text{im}}(\mathbf{x}) \triangleq \sqrt{2}[\forall \mathrm{Im}[x_i x_j^*] | 2 \le i \le M, 1 \le j \le i-1]^{\mathrm{T}}. \quad (26)$$

When the degree is three, there are several dependent elements with the same values in ϕ_3 so that the rank of $\mathbf{R}'_3$ becomes the same or less than $\frac{M^3+M^2}{2}$. Thus we can use the following mapping $\phi'_3 : \mathbb{C}^M \to \mathbb{C}^{\frac{M^3+M^2}{2}}$ which satisfies Eq. (22) and collects the same elements. Such mapping simplifies the calculation of the covariance matrix and the eigenvalue problem by omitting dependent rows and columns from the covariance matrix.

$$\phi'_3(\mathbf{x}) \triangleq [\phi'_{3a}(\mathbf{x})^{\mathrm{T}}, \phi'_{3b}(\mathbf{x})^{\mathrm{T}}, \phi'_{3c}(\mathbf{x})^{\mathrm{T}}, \phi'_{3d}(\mathbf{x})^{\mathrm{T}}]^{\mathrm{T}}, \quad (27)$$

$$\phi'_{3a}(\mathbf{x}) \triangleq [\forall |x_i|^2 x_i^* | 1 \le i \le M]^{\mathrm{T}}, \quad (28)$$

$$\phi'_{3b}(\mathbf{x}) \triangleq [\forall x_i x_j^{*2} | 1 \le i \le M, j \ne i]^{\mathrm{T}}, \quad (29)$$

$$\phi'_{3c}(\mathbf{x}) \triangleq \sqrt{2}[\forall |x_i|^2 x_j^* | 1 \le i \le M, j \ne i]^{\mathrm{T}}, \quad (30)$$

$$\phi'_{3d}(\mathbf{x}) \triangleq \sqrt{2}[\forall x_i x_j^* x_k^* | 1 \le i \le M, j \ne k \ne i]^{\mathrm{T}}. \quad (31)$$

We evaluate mapped MUSIC with mappings ϕ'_2 and ϕ'_3 in Sect. 4.

3.3. Comparison with $2q$-MUSIC exploiting cumulants

This section discusses the differences between mapped MUSIC based on an analysis of even order cross moments and $2q$-MUSIC based on an analysis of even order cross cumulants. The calculation of ith-order cumulants requires diverse multiplications of the moments, whose order is the same or less than i, and the computational cost will increase rapidly as the order of analysis increases. Thus mapped MUSIC, which can obtain a cross moment matrix with a single mapping, is simpler than $2q$-MUSIC and can more easily expand the statistics of analysis to a higher order. As an example, with the straightforward analysis of fourth and sixth order statistics, these methods require the computational costs shown in Table 1, where M is the number of sensors, L is the number of time frames of observed signals, and Dim. denotes dimension.

Table 1. Computational costs

	Multiplication (times) / Eigen decomposition
$2q$-MUSIC ($q=2$)	$L(18M^4 - 16M^2)$
	complex-valued matrix of M^2 Dim.
$2q$-MUSIC ($q=3$)	$L(210M^6 - 206M^3)$
	complex-valued matrix of M^3 Dim.
mapped MUSIC ($d=2$)	$\frac{L}{2}(M^4 + 5M^2 - 2M)$
	real-valued matrix of M^2 Dim.
mapped MUSIC ($d=3$)	$\frac{L}{8}(M^6 + 2M^5 + M^4 + 34M^3 - 6M^2)$
	complex-valued matrix of $\frac{M^3+M^2}{2}$ Dim.

While the $2q$-th order cross cumulant matrix in $2q$-MUSIC usually has the full rank of M^q, the dimensionality of the mapping ϕ_d when $d > 2$ is less than M^d as discussed in Sect. 3.2. In addition, there is often a further reduction in the rank of the cross moment matrix because of the environment. As a result, $2q$-MUSIC tends to have sharper peaks in the MUSIC score than mapped MUSIC if there are a sufficient number of observed signals. However, as we show in our experiments, mapped MUSIC performs as well as $2q$-MUSIC in practical conditions with a realistic number of observed signals.

4. EXPERIMENT

This section describes a simulation experiment to verify the efficiency of the proposed mapped MUSIC. We implemented MUSIC and $2q$-MUSIC ($q = 2, 3$) for comparison.

4.1. Experimental condition

We conducted an evaluation of the DOA estimation and computational complexity of each method by using a simulated recording of a speech mixture with a circular microphone array. We established three different reverberant conditions by using the image method [7] with different reflection coefficients. To emulate noisy observation, diffused pink noises [8] with three different SNRs were added to the observed signals, and to determine the influence of the reflection and number of sample frames, we also evaluated the performance with two different lengths of observed signals. Table 2 shows the other experimental conditions.

Table 2. Experimental conditions

Sensor array form		Circular array with radius of 0.1 m	
Sound sources		Speakers 1.5 m apart from array	
# of sound sources	3, 5	Sampling frequency	16 kHz
# of sensors	4	SNR	0,10,20 [dB]
Room size		5 m × 5 m	
Reverberation		T_{60} of 0,0.12,0.3 [s]	
Duration		3,5 [s]	
Frame length		1024 samples	
Frame shift length		512 samples	
Window function		Hamming window	

For the evaluation, we employed the root mean squared error (RMSE) between the estimated direction and the true sound direction. We evaluated 100 combinations of the positions of three or five sources selected randomly from the directions $\{0°, 30° \cdots, 330°\}$. When the MUSIC score had fewer peaks than sources, we added a penalty equal to the average error for all directions. And to compare the computational complexity, we recorded processing time of each method in every trial.

DOA estimation with MUSIC when $M \le N$ is performed by regarding the one-dimensional subspace concurrent with the minimum eigenvalue as the noise subspace in every frequency bin.

4.2. Experimental results

Figures 1–4 show the experimental results obtained under different conditions. The results reveal that the proposed mapped MUSIC and the $2q$-MUSIC perform similarly better than MUSIC under all conditions and demonstrate the high performance of DOA estimation even in underdetermined conditions. Under most conditions, mapped MUSIC with $d = 2$ performs as well as $2q$-MUSIC. Mapped MUSIC with $d = 3$ shows excellent accuracy in terms of DOA estimation with a high SNR and is the method that performs the best in this experiment. On the other hand, $2q$-MUSIC shows slightly poorer performance, although $2q$-MUSIC can be expected

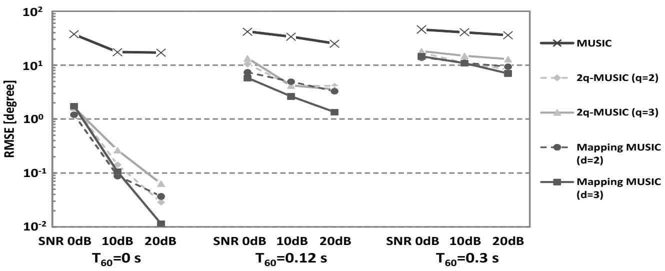

Fig. 1. Experimental results for 3 sources within 3 seconds.

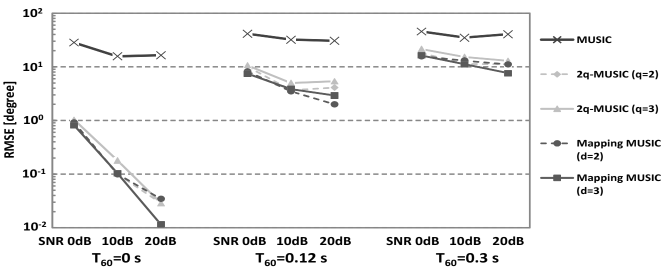

Fig. 2. Experimental results for 3 sources within 5 seconds.

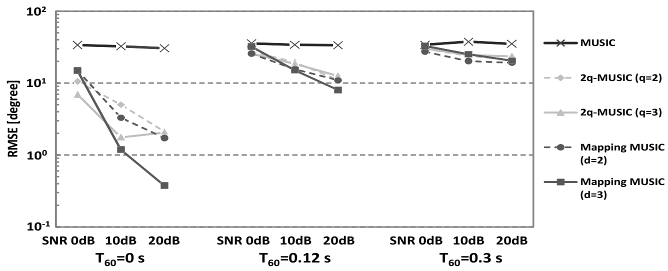

Fig. 3. Experimental results for 5 sources within 3 seconds.

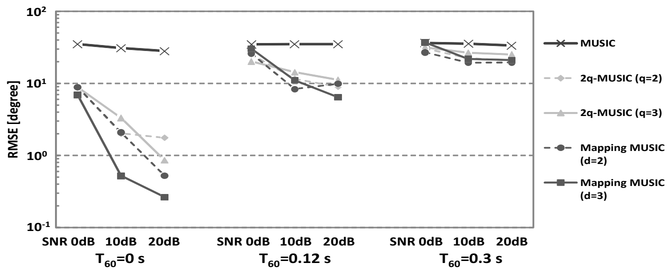

Fig. 4. Experimental results for 5 sources within 5 seconds.

to perform better than mapped MUSIC when these methods exploit the same order statistics and there are sufficient observed data. The reason for these results may be the statistical bias derived from the shortage of observed data, because cumulants tend to be affected by statistical bias more than moments. Figure 5 shows the average processing time under these conditions. As a result, our proposed mapped MUSIC is sufficiently efficient in practical conditions with a realistic number of observed signals and has much less computational complexity than $2q$-MUSIC.

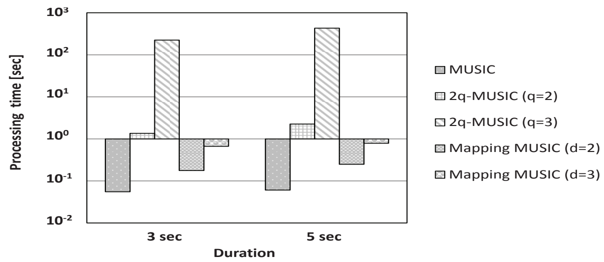

Fig. 5. Processing time.

5. CONCLUSION

In this paper, we proposed mapped MUSIC, a high-resolution DOA estimator, which is applicable even in underdetermined conditions. We described the mapped MUSIC algorithm and suitable mapping for DOA estimation. We also discussed the property of the mapping function with the degree of d to analyze the $2d$-th order cross moments, and proposed efficient algorithms with which to calculate fourth and sixth order moments. Furthermore, we compared the characteristics of the proposed method and the conventional $2q$-MUSIC utilizing $2q$-th order cumulants. We showed that the proposed methods has much less computational complexity than $2q$-MUSIC. We confirmed that DOA estimation with the proposed mapped MUSIC technique is as accurate as that obtained with the conventional $2q$-MUSIC technique but with less computational complexity.

6. REFERENCES

[1] R. O. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Trans. Antennas Propag.*, vol. AP-34, no. 3, pp. 276–280, 1986.

[2] B. Porat and B. Friedlander, "Direction finding algorithms based on higher order statistics," *IEEE Trans. Signal Process.*, vol. 39, no. 9, pp. 2016–2024, 1991.

[3] P. Chevalier, A. Ferreol, and L. Albera, "High resolution direction finding from higher order statistics: The 2q-music algorithm," *IEEE Trans. Signal Process.*, vol. 54, no. 8, pp. 2986–2997, 2006.

[4] H. Saruwatari, S. Kajita, K. Takeda, and F. Itakura, "Speech enhancement using nonlinear microphone array based on complementary beamforming," *IEICE Trans. Fundam.*, vol. E82-A, no. 8, pp. 1501–1510, 1999.

[5] H. Kamiyanagida, H. Saruwatari, K. Takeda, F. Itakura, and K. Shikano, "Direction of arrival estimation using nonlinear microphone array," *IEICE Trans. Fundam.*, vol. E84-A, no. 4, pp. 999–1010, 2001.

[6] S. Miyabe, Biing-Hwang (Fred) Juang, Hiroshi Saruwatari, and Kiyohiro Shikano, "Kernel-based nonlinear independent component analysis for underdetermined blind source separation," in *ICASSP*, 2009, pp. 1641–1644.

[7] J. A. Allen and D. A. Berkley, "Image method for efficiently simulating small-room acoustics," *J. Acoust. Soc. Am.*, vol. 65, no. 4, pp. 943–950, 1979.

[8] I. A. McCowan and H. Bourland, "Microphone array post-filter based on noise field coherence," *IEEE Trans. Speech Audio Process.*, vol. 11, no. 6, pp. 709–716, 2003.