

A SPATIO-TEMPORAL FASTICA ALGORITHM FOR SEPARATING CONVOLUTIVE MIXTURES

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ABSTRACT

This paper presents a spatio-temporal extension of the well-known fastICA algorithm of Hyvärinen and Oja that is applicable to both convolutive blind source separation and multichannel blind deconvolution tasks. Our time-domain algorithm combines multichannel spatio-temporal prewhitening via multi-stage least-squares linear prediction with a fixed-point iteration involving a new adaptive technique for imposing paraunitary constraints on the multichannel separation filter. Our technique also allows for efficient reconstruction of individual signals as observed in the sensor measurements for single-input, multiple-output (SIMO) BSS tasks. Analysis and simulations verify the utility of the proposed methods.

1. INTRODUCTION

The fastICA algorithm of Hyvärinen and Oja [1] is one of the most well-known and popular procedures for both independent component analysis (ICA) and blind source separation. For an m -element linear non-Gaussian signal mixture, the procedure consists of a signal prewhitening stage followed by a set of m fixed-point iterative procedures that extract independent components using a non-Gaussianity signal measure. Coefficient vector orthogonality is used to guarantee uniqueness of the extracted components. The algorithm enjoys a number of useful properties, including fast convergence, guaranteed global convergence for certain mixing conditions and contrasts, and robust behavior when noise is present.

Recently, several researchers have explored spatio-temporal extensions of spatial-only ICA and BSS procedures to attack problems in which the sources are mixed both in space and in time. These convolutive signal mixtures appear in the design of wide-band antenna arrays for wireless communications and in the design of microphone arrays for audio source separation and localization, among other applications. The simplest of these extensions treat the separation task in the (discrete) Fourier domain and apply existing spatial-only complex-valued ICA and BSS methods within each frequency bin. These methods suffer from a number of problems, most notably permutation, amplitude, and scaling inconsistencies across different frequency bins in the separated outputs. Although progress is being made in addressing these issues through post-processing [2], such extensions require significant effort beyond the separation step to reconstruct the sources.

A more elegant solution is to develop convolutive BSS algorithms using a time-domain separation criterion. An example of this approach is the information-theoretic natural gradient convolutive BSS and multichannel blind deconvolution algorithm developed in [3]. While this procedure can be successful, the source distributions must be approximately known, and the number of

sources must be exactly known as they are simultaneously extracted. These issues make such information theoretic approaches less practical.

In this paper, we present a spatio-temporal extension of the aforementioned fastICA algorithm to both convolutive BSS and multichannel blind deconvolution tasks. The proposed time-domain algorithm combines multichannel whitening via multi-stage least-squares linear prediction with a fixed-point iteration involving a new adaptive technique for imposing paraunitary constraints on the multichannel separation filter. A unique feature of our approach is its ability to easily and individually reconstruct the sources as they appear in the observed signal mixtures for the single-input, multiple-output (SIMO) BSS separation task. Analytical results regarding the adaptive paraunitary constraint procedure are provided, and simulations indicating the usefulness of the proposed approach for convolutive BSS are given.

2. PROBLEM FORMULATION AND EXISTING WORK

We first describe the SIMO BSS task [4]. Let $s_i(k)$, $1 \leq i \leq m$ denote m spatially-independent source signals, such that $s_i(k)$ is independent of $s_j(l)$ for $i \neq j$. These sources are measured in an n -dimensional signal mixture with $n \geq m$ as

$$x_j(k) = \nu_j(k) + \sum_{i=1}^m \sum_{p=0}^{\infty} a_{jip} s_i(k-p) \quad (1)$$

for $1 \leq j \leq n$, where $\{a_{jip}\}$ are the coefficients of the multichannel mixing system and $\nu_j(k)$ is uncorrelated Gaussian sensor noise. The goal is to extract estimates of the sources as they appear in the signal mixtures, ideally given by the mn signal set

$$x_{ij}(k) = \sum_{p=0}^{\infty} a_{jip} s_i(k-p) \quad (2)$$

for $1 \leq i \leq m$ and $1 \leq j \leq n$. In practice, each $\hat{x}_{ij}(k)$ is estimated from a set of separated source signals $y_i(k)$ as

$$\hat{x}_{ij}(k) = \sum_{p=0}^M g_{jip} y_i(k-p) \quad (3)$$

where M is a filter length parameter and $\{g_{jip}\}$ must be estimated or calculated from the separation system, the extracted signals $\{y_i(k)\}$, and/or the original input signal mixtures $\{x_j(k)\}$.

Previous work has identified two strategies for finding $\{\hat{x}_{ij}(k)\}$ after the separated sources $\{y_i(k)\}$ have been found. The first

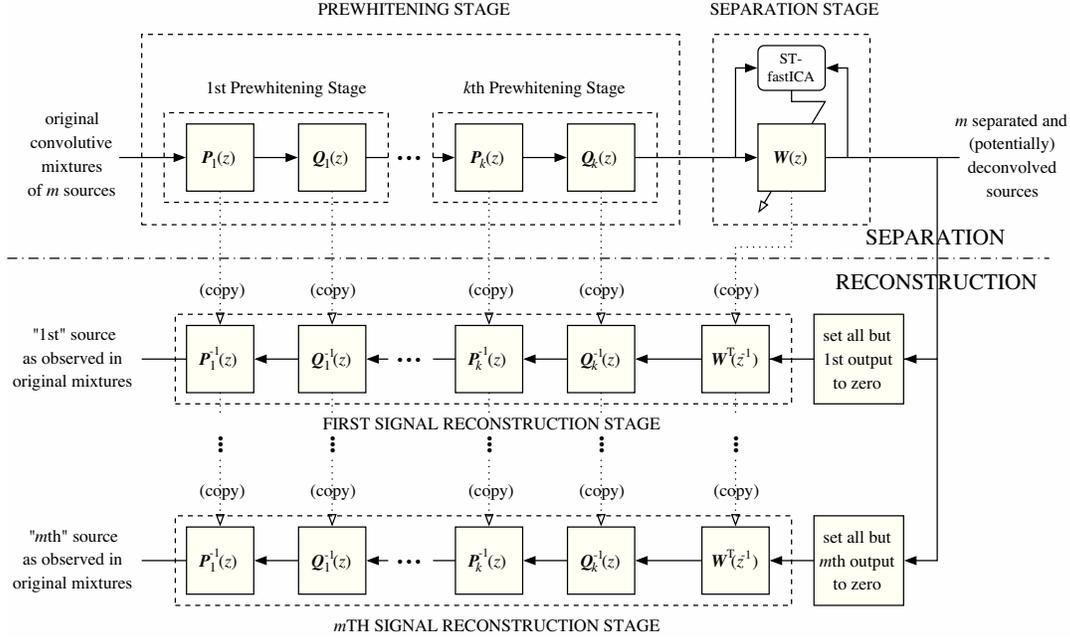


Fig. 1. Block diagram of the combined separation and signal reconstruction system.

strategy uses traditional linear estimation to calculate the $\{g_{ijp}\}$ coefficients, where the $\{x_{ij}(k)\}$ are the desired signals and the $\{y_j(k)\}$ are the reference signals [5]. This approach is complicated if the $\{y_i(k)\}$ are not uncorrelated in time, however, as it involves m disjoint $n(M+1)$ -dimensional estimation tasks. Moreover, it requires signal averaging between the $\{y_j(k)\}$ and the $\{x_i(k)\}$. The second calculates the inverse of the separation system for the $\{g_{ijp}\}$ [6]. This procedure is challenging due to the difficulty of calculating a multichannel system inverse that does not exploit a specific system structure. These procedures generally require filter lengths M that are longer than that of either the separation system or the original mixing channel.

3. THE PROPOSED METHOD

We propose a different strategy. Consider Fig. 1, which shows a signal processing architecture containing a prewhitening stage, a separation stage, and a signal reconstruction stage. Each of these processing stages is now described.

The goal of the prewhitening stage is to decorrelate the original signal mixtures in both space and time. We propose a multi-step prewhitening structure using pairs of multichannel linear systems with transfer function matrices given by

$$\mathbf{P}_i(z) = \mathbf{D}_i^{(P)} \begin{bmatrix} P_{11i}(z) & P_{12i}(z) & \cdots & P_{1ni}(z) \\ 0 & P_{22i}(z) & & \vdots \\ \vdots & & \ddots & \\ 0 & \cdots & 0 & P_{nni}(z) \end{bmatrix} \quad (4)$$

$$\mathbf{Q}_i(z) = \mathbf{D}_i^{(Q)} \begin{bmatrix} Q_{11i}(z) & 0 & \cdots & 0 \\ Q_{21i}(z) & Q_{22i}(z) & & \vdots \\ \vdots & & \ddots & 0 \\ Q_{n1i}(z) & \cdots & & Q_{nni}(z) \end{bmatrix} \quad (5)$$

where the causal $\{\mathbf{P}_i(z)\}$ and $\{\mathbf{Q}_i(z)\}$ are multichannel FIR filters of length K , the causal filters $\{P_{jji}(z)\}$ and $\{Q_{jji}(z)\}$ have unity zero-lag coefficient values, and $\mathbf{D}_i^{(P)}$ and $\mathbf{D}_i^{(Q)}$ are $(n \times n)$

diagonal scaling matrices. The coefficients for the j th row of the $\{\mathbf{P}_i(z)\}$ transfer function matrices are calculated by solving a least-squares multichannel forward linear prediction task, *e.g.* by minimizing the output power of the j th output signal. The diagonal entries of $\mathbf{D}_i^{(P)}$ are then calculated so that the scaled forward error residuals have unity variances. These scaled error residuals are used as inputs to the $\{\mathbf{Q}_i(z)\}$ multichannel system, in which the j th row of the $\{\mathbf{Q}_i(z)\}$ transfer function matrices are calculated by solving a second least-squares multichannel forward linear prediction task. The diagonal entries of $\mathbf{D}_i^{(Q)}$ are subsequently calculated so that these error residuals have unity variances. Note that this proposed method is a block-based procedure.

Several stages of this processing strategy are usually required because the estimation of $\mathbf{P}_i(z)$ and $\mathbf{Q}_i(z)$ is performed in a disjoint and sequential fashion. The exact number of stages k can be made adaptive, with a stopping criterion that depends on how much $\mathbf{P}_k(z)$ and $\mathbf{Q}_k(z)$ differ from identity.

The above prewhitening strategy has an important advantage: Both $\mathbf{P}_i(z)$ and $\mathbf{Q}_i(z)$ can be easily inverted without calculating any new filter coefficients, using the linear system equivalent of backsubstitution. Thus, so long as the linear system within the separation stage can be easily inverted, creating the inverse of the entire prewhitening-separation system is straightforward.

The goal of the second stage is to perform separation of the prewhitened signal mixtures based on non-Gaussianity. Here, we propose a novel extension of the fastICA algorithm that imposes structure on the separation system coefficients. Define the prewhitened input signal vector as

$$\mathbf{v}(k) = [\mathbf{v}_1^T(k) \ \mathbf{v}_2^T(k) \ \cdots \ \mathbf{v}_n^T(k)]^T \quad (6)$$

$$\mathbf{v}_j(k) = [v_j(k) \ v_j(k-1) \ \cdots \ v_j(k-L+1)]^T \quad (7)$$

Furthermore, define

$$\mathbf{w}_i = [\mathbf{w}_{i1}^T \ \mathbf{w}_{i2}^T \ \cdots \ \mathbf{w}_{in}^T]^T \quad (8)$$

$$\mathbf{w}_{ij} = [w_{ij0} \ w_{ij1} \ \cdots \ w_{ij(L-1)}]^T \quad (9)$$

as the separation system coefficient vector for the i th system out-

Table 1: MATLAB implementation of the proposed spatio-temporal fastICA algorithm.

<pre>function [y,W] = stfica(v,L,numiter); L = L + rem((L+1),2); [n,N] = size(v); W = kron(eye(n), [zeros((L-1)/2,1);1;zeros((L-1)/2,1)]); V = zeros(n*L,N); for i=1:n V((i-1)*L+1:i*L,:) = toeplitz([v(i,1);zeros(L-1,1)],v(i,:)); end y = zeros(n,N); for i=1:n Wold = zeros(n*L,1); k = 0; y(i,:) = W(:,i)'*V; crit = 1; while (crit*(k<numiter)) Wold = W(:,i); k = k+1; f = Y(i,:).^3; % OR f = tanh(20*y(i,:)); fp = 3*sum(y(i,:).^2); % OR fp = 20*sum(sech(20*y(i,:))); W(:,i) = V*f - fp*W(:,i); W(:,i) = orthW(W(:,1:i),n,L,10); y(i,:) = W(:,i)'*V; crit = (abs(abs(W(:,i)'*Wold)-1)>0.0001); end end</pre>	<pre>function [Wi] = orthW(W,n,L,numorth); i = size(W,2); Wi = W(:,i)/norm(W(:,i)); for k=1:numorth Wt = zeros(n*L,1); for j=1:i-1 Wt = Wt + gfun(Wi,W(:,j),n,L); end Wi = 3/2*Wi - 1/2*gfun(Wi,Wi,n,L) - Wt; end</pre> <hr/> <pre>function [G,C] = gfun(U,V,n,L); Wi = zeros(L,n); Wi(:) = U; Wj = zeros(L,n); Wj(:) = V; Ct = zeros((3*L-1)/2,1); Z = zeros((L-1)/2,1); ll = (L+1)/2:(3*L-1)/2; llr = L:-1:1; for i=1:n Ct = Ct + filter(Wi(llr,i),1,[Wj(:,i);Z]); end C = Ct(ll); Gt = filter(C(llr),1,[Wj;zeros((L-1)/2,n)]); G = Gt(ll,:); G = Gt(:);</pre>
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put. Then, we compute the i th separated signal sequence as

$$y_i(k) = \mathbf{w}_i^T \mathbf{v}(k) \quad (10)$$

for $1 \leq k \leq N$, assuming a data record length of N samples.

The coefficient vectors $\{\mathbf{w}_i\}$ are updated iteratively using a modified version of the fastICA algorithm as follows:

- *Step 1:* Compute $y_i(k)$ in (10) for $1 \leq k \leq N$.
- *Step 2:* Update the coefficient vector as

$$\mathbf{w}_i \leftarrow \frac{1}{N} \sum_{k=1}^N f(y_i(k)) \mathbf{v}(k) - f'(y_i(k)) \mathbf{w}_i, \quad (11)$$

where $f(y_i)$ is the contrast nonlinearity.

- *Step 3:* Normalize the length of the coefficient vector as

$$\mathbf{w}_i \leftarrow \frac{\mathbf{w}_i}{\sqrt{\mathbf{w}_i^T \mathbf{w}_i}} \quad (12)$$

- *Step 4:* While \mathbf{w}_i is not paraunitary with $\mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{w}_{i-1}$,

$$\mathbf{w}_i \leftarrow \frac{3}{2} \mathbf{w}_i - \frac{1}{2} \mathbf{g}(\mathbf{w}_i, \mathbf{w}_i) - \sum_{j=1}^{i-1} \mathbf{g}(\mathbf{w}_i, \mathbf{w}_j) \quad (13)$$

where $\mathbf{g}(\mathbf{w}_i, \mathbf{w}_j) = [\mathbf{g}_{ij1}^T \ \mathbf{g}_{ij2}^T \ \dots \ \mathbf{g}_{ijm}^T]^T$ with

$$\mathbf{g}_{ijk} = \mathbf{C}_{ij} \mathbf{w}_{jk} \quad (14)$$

and the (p, q) th element of \mathbf{C}_{ij} is given by

$$[\mathbf{C}_{ij}]_{pq} = \begin{cases} \sum_{k=1}^m \sum_{l=0}^{L-1} w_{jkl} w_{ik(l+p-q)} & \text{if } |p-q| < \frac{L-1}{2} \\ 0 & \text{otherwise.} \end{cases} \quad (15)$$

The first three steps of this procedure are identical in form to the single-unit fastICA algorithm. To understand Step 4, define the $(n \times 1)$ -dimensional i th system vector polynomial as

$$\underline{\mathcal{W}}_i(z) = \sum_{j=0}^{L-1} [w_{i1j} \ w_{i2j} \ \dots \ w_{inj}]^T z^{-j}. \quad (16)$$

Then, (13)–(15) can be expressed using polynomials as

$$\begin{aligned} \underline{\mathcal{W}}_i(z) \leftarrow & \frac{3}{2} \underline{\mathcal{W}}_i(z) - \left[\frac{1}{2} \left[\underline{\mathcal{W}}_i^T(z^{-1}) \underline{\mathcal{W}}_i(z) \right]_{-(L-1)/2}^{(L-1)/2} \underline{\mathcal{W}}_i(z) \right. \\ & \left. + \sum_{j=1}^{i-1} \left[\underline{\mathcal{W}}_j^T(z^{-1}) \underline{\mathcal{W}}_i(z) \right]_{-(L-1)/2}^{(L-1)/2} \underline{\mathcal{W}}_j(z) \right]_{-J}^{L-1} \end{aligned} \quad (17)$$

where $[\cdot]_{-J}^K$ denotes truncating the polynomials of its argument to order $-J$ through $-K$. Extensive simulations of this iterative sub-procedure indicate that (17) causes

$$\left[\underline{\mathcal{W}}_i^T(z^{-1}) \underline{\mathcal{W}}_i(z) \right]_{-(L-1)/2}^{(L-1)/2} \rightarrow 1 \quad (18)$$

$$\left[\underline{\mathcal{W}}_j^T(z^{-1}) \underline{\mathcal{W}}_i(z) \right]_{-(L-1)/2}^{(L-1)/2} \rightarrow 0 \quad \text{for } 1 \leq j < i. \quad (19)$$

The above constraints are a spatio-temporal extension of the orthonormality constraints imposed on \mathbf{w}_i in the original fastICA procedure and imply that the separation system is paraunitary.

As further justification of the iterative procedure for enforcing paraunitary constraints, let $L \rightarrow \infty$, and define

$$\underline{\mathbf{w}}_i = \underline{\mathcal{W}}_i(z)|_{z=e^{j\omega}} \quad (20)$$

$$\underline{\mathbf{W}}_i = [\underline{\mathcal{W}}_1(z) \ \underline{\mathcal{W}}_2(z) \ \dots \ \underline{\mathcal{W}}_{i-1}(z)]|_{z=e^{j\omega}} \quad (21)$$

Then, (17) can be rewritten as

$$\underline{\mathbf{w}}_{i,new} = \frac{3}{2} \underline{\mathbf{w}}_i - \frac{1}{2} \|\underline{\mathbf{w}}_i\|^2 \underline{\mathbf{w}}_i - \underline{\mathbf{W}}_i \underline{\mathbf{W}}_i^H \underline{\mathbf{w}}_i \quad (22)$$

where \cdot^H denotes Hermitian transpose. Define the variables

$$a_{0i} = \|\underline{\mathbf{w}}_i\|^2 - 1 \quad (23)$$

$$a_{1i} = \|\underline{\mathbf{W}}_i^H \underline{\mathbf{w}}_i\|^2 \quad (24)$$

where $\|\underline{\mathbf{w}}_i\|^2 = \underline{\mathbf{w}}_i^H \underline{\mathbf{w}}_i$. The condition $|a_{0i}| = a_{1i} = 0$ implies that $[\underline{\mathcal{W}}_1(z) \ \dots \ \underline{\mathcal{W}}_i(z)]$ form an m -dimensional paraunitary sequence if $[\underline{\mathcal{W}}_1(z) \ \dots \ \underline{\mathcal{W}}_{i-1}(z)]$ is already paraunitary. Then, (22) implies that these state variables evolve as

$$a_{0i,new} = \frac{1}{4} a_{0i}^2 (a_{0i} - 3) + (a_{0i} - 1) a_{1i} \quad (25)$$

$$a_{1i,new} = \frac{1}{4} a_{0i}^2 a_{1i}. \quad (26)$$

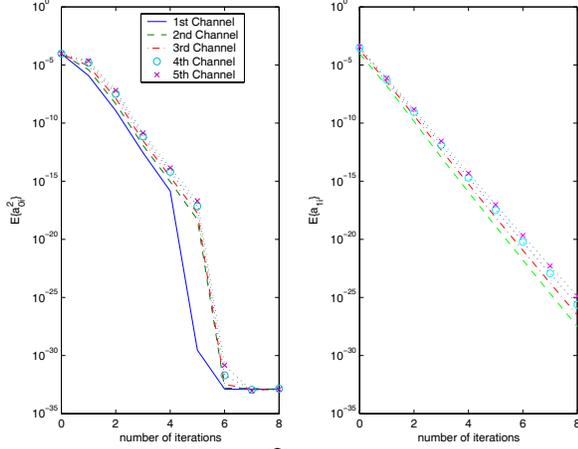


Fig. 2: Evolution of $E\{\widehat{a}_{0i}^2\}$ and $E\{\widehat{a}_{1i}\}$ for (13)–(15).

This pair of nonlinear coupled scalar equations can be easily simulated for different initial conditions, and such simulation studies show that a_{0i} and a_{1i} converge to zero for a wide range of initial value pairs. Empirically, we have observed convergence of this system if

$$\begin{aligned} -1 < a_{0i} < 2 \\ a_{1i} < a_{0i} + 1 \end{aligned} \quad \text{or} \quad \begin{aligned} 0 < \|\mathbf{w}_i\|^2 < 3 \\ \|\mathbf{W}_i^H \mathbf{w}_i\|^2 < \|\mathbf{w}_i\|^2, \end{aligned} \quad (27)$$

which are typically satisfied in practice.

Table 1 provides MATLAB code for implementing the spatio-temporal fastICA complete with appropriately chosen stopping criteria, where \mathbf{v} is the $(n \times N)$ prewhitened signal matrix, L is the separation system filter length, and `numiter` is the maximum number of iterations of the fastICA routine.

The goal of the last set of parallel stages, shown at the bottom of Fig. 1, is to reconstruct the individual sources as they appear in the original mixtures. The reconstruction of the i th separated signal involves setting all but the i th signal $y_i(k)$ to zero and then passing this signal through the inverse of the prewhitening and separation systems. For this calculation, note that

$$\begin{aligned} & [\mathbf{W}(z)\mathbf{Q}_k(z)\mathbf{P}_k(z) \cdots \mathbf{Q}_1(z)\mathbf{P}_1(z)]^{-1} \\ &= \mathbf{P}_1^{-1}(z)\mathbf{Q}_1^{-1}(z) \cdots \mathbf{P}_k^{-1}(z)\mathbf{Q}_k^{-1}(z)\mathbf{W}^T(z^{-1}) \end{aligned} \quad (28)$$

due to the paraunitariness of $\mathbf{W}(z)$ as constructed by the separation stage. Because of the triangular structures of the $\mathbf{P}_i(z)$ and $\mathbf{Q}_i(z)$ systems, they can be easily inverted.

The method we have described has a number of advantages over competing approaches:

1. No step size needs to be selected.
2. Knowledge of the source distributions is not needed, so long as their statistics imply a non-zero contrast value.
3. The number of non-Gaussian sources within the mixture need not be known *a priori*.
4. For SIMO BSS, the system inverse used for signal reconstruction is computed directly and exactly.
5. Convergence of the newly-developed spatio-temporal fastICA procedure appears to be as fast as its spatial-only counterpart; usually fewer than 10 iterations per unit are needed for i.i.d. sources.

4. SIMULATIONS

We now present numerical evaluations indicating the performance of the proposed methods. We first investigate the iterative paraunitary constraint scheme in (13)–(15). For these evaluations, we have chosen $n = 10$ and $L = 51$. For each simulation run, \mathbf{w}_i was initialized to a 510-element vector containing zero-mean uncorrelated Gaussian noise of variance 10^{-4} summed with a single non-zero unity-valued “center” tap at position $k = L(i-1) + (L+1)/2$. Shown in Fig. 2 for $1 \leq i \leq 5$ are the average evolutions of $E\{\widehat{a}_{0i}^2\}$ and $E\{\widehat{a}_{1i}\}$, computed from the elements of \mathbf{C}_{ij} for $1 \leq j \leq i$, as averaged over 100 different simulation runs. As can be seen, convergence to a paraunitary condition given by $E\{\widehat{a}_{0i}^2\} \approx 0$ and $E\{\widehat{a}_{1i}\} \approx 0$ is fast, approaching the machine precision of MATLAB in about 10 iterations.

We now demonstrate the ability of the proposed method to solve the SIMO BSS task. The measurements used for this evaluation were taken from a two-loudspeaker, two-microphone acoustic laboratory setup similar to that described in [7], in which the room reverberation time was approximately 130 msec. Mixtures containing a male and a female talker sampled at 8kHz were processed using the technique in Fig. 1, where $k = 4$, $K = 101$, $L = 81$, $N = 55200$, and `numiter` = 100. After processing, the mean-squared errors between the reconstructed sources and the original mixtures for the two-channel signals were between 10 dB and 11.8 dB below the powers of the original source mixtures for both talkers in the four output channels. In addition to these results, we have verified that our proposed method successfully deconvolves convolutive mixtures of i.i.d. signals with binary, uniform, and Laplacian distributions; these results are omitted due to space limitations.

5. CONCLUSIONS

In this paper, we have described a spatio-temporal extension of the well-known fastICA procedure. Our method employs least-squares prewhitening methods and a novel iterative scheme for maintaining paraunitary constraints on the separation system. The procedure enables the reconstruction of the individual sources within the signal mixtures for single-input, multiple-output BSS without explicitly calculating the inverse of the separation system’s impulse response. A theoretical study of the proposed method’s capabilities is the subject of current work.

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