A Ricardian model of trade and growth with endogenous trade status*

Takumi Naito†
Waseda University

November 15, 2011

Abstract

We formulate a two-country, continuum-good Ricardian model of trade and endogenous growth with endogenous trade status. After establishing the existence, uniqueness, and global stability of a balanced growth path, we show that, compared with the old balanced growth path, a permanent fall in the trade cost in any one country: (i) raises the growth rates of capital in all countries for all periods; (ii) increases both the range of the imported varieties and that of the exported varieties in all countries for all periods; and (iii) raises welfare in all countries. Our theoretical predictions are qualitatively consistent with the empirical evidence.

JEL classification: F13; F43

Keywords: Ricardian model; Endogenous growth; Endogenous trade status; Unilateral trade liberalization; Terms of trade

†Takumi Naito. Faculty of Political Science and Economics, Waseda University, 1-6-1 Nishiwaseda, Shinjuku-ku, Tokyo 169-8050, Japan. E-mail: tnaito@waseda.jp.

1 Introduction

How does trade liberalization affect long-run economic growth? There are two strands of theoretical literature tackling the fundamental question.[1] One strand of the literature (e.g., Osang and Pereira, 1996; Osang and Turnovsky, 2000; Naito, 2003, 2006a, 2006b) considers factor accumulation as the engine of endogenous growth in a small open economy, and concludes that a tariff reduction in the small country usually raises its growth rate. The other strand formulates endogenous growth based on R&D in a symmetric multi-country world with homogeneous firms (e.g., Dinopoulos and Segerstrom, 1999; Baldwin and Forslid, 2000; Peretto, 2003) or heterogeneous firms (e.g., Baldwin and Robert-Nicoud, 2008; Gustafsson and Segerstrom, 2010; Unel, 2010; Haruyama and Zhao, 2010; Dinopoulos and Unel, 2011), and reports that the growth effect of symmetric trade liberalization is generally ambiguous due to various market failures such as market power and externalities. The existing literature based on either the small-country or symmetric multi-country setting overlooks possible asymmetric effects that unilateral trade liberalization has on the liberalizing and partner countries. For example, suppose that a fall in the trade cost in a country, ceteris paribus, raises its growth rate. Then the relative increase in the export supply of the liberalizing country will change the terms of trade against itself but in favor of its partner country.[2] To see the consequences of these effects, we need to build an endogenous growth model that allows for heterogeneity across countries. The purpose of this paper is to propose such a theoretical framework.

We formulate a two-country, continuum-good Ricardian model of trade and endogenous growth with endogenous trade status. Our endogenous growth mechanism is based on the multi-country AK model of Acemoglu and Ventura (2002), where a continuum of countries trade only specialized varieties of intermediate goods.[3] In their model, faster growth in a country lowers its rental rate of capital in its capital market, which pulls down its growth rate through the deteriorated terms of trade. One feature of the Acemoglu-Ventura model is that the trade status of any variety is constant due to product differentiation, so it cannot capture the dynamic evolution of the industrial structure. In fact, as a country develops, a good that had been totally imported may be substituted by domestic production, and may even be exported at last. To describe such a process in the simplest possible setup, we choose the continuum-good Ricardian framework developed by Dornbusch et al. (1977).[4] Under this specification, the distribution of trade status over the unit interval of varieties is determined by the relative rental rate and the trade costs in the two countries. By embedding this mechanism in the two-country Acemoglu-Ventura framework, we can see how the growth rates of capital in the two countries interact with the distribution of trade status over time.[5]

After establishing the existence, uniqueness, and global stability of a balanced growth path, we show that, compared with the old balanced growth path, a permanent fall in the trade cost in any one country: (i)

[1]The problem is conceptually different from a direct comparison between autarky and free trade allocations (e.g., Rivera-Batiz and Romer, 1991). While the latter involves a large-scale discrete change in trade policy, the former is concerned with its marginal effects.

[2]Acemoglu and Ventura (2002) estimate using two-stage least squares that a 1 percentage point rise in a country's growth rate of GDP lowers its growth rate of terms of trade by around 0.6 percentage points.

[3]They first assume that the varieties of intermediate goods are exogenously differentiated by origin as in Armington (1969), and later allow for endogenous differentiation by monopolistically competitive firms to show that the dynamic properties are unaffected.

[4]The two-country, perfect-competition model of Dornbusch et al. (1977) is extended to a many-country world with probabilistic technologies by Eaton and Kortum (2002), which in turn is extended to deal with Bertrand competition allowing for positive markups by Bernard et al. (2003) We stick to the simplest Dornbusch et al. (1977) setup because we aim to obtain qualitative effects of unilateral trade liberalization, rather than to make calibrations and counterfactual experiments as Eaton and Kortum (2002) and Bernard et al. (2003) do.

[5]Eaton and Kortum (2001) extend the multi-country, continuum-good Ricardian model of Eaton and Kortum (2002) to build an R&D-based model of trade and growth. Because of its semi-endogenous-growth character, the long-run growth rate of technology in a country depends on its population growth rate, but not on trade costs.

raises the growth rates of capital in all countries for all periods; (ii) increases both the range of the imported varieties and that of the exported varieties in all countries for all periods; and (iii) raises welfare in all countries. The main driving force behind these results is the fall in the relative rental rate in the liberalizing country caused by its increased growth potential. This raises the growth rate of capital in the partner country by improving its terms of trade. On the other hand, unilateral trade liberalization in a country not only increases the range of its imported varieties through increased import competition, but it also increases the range of its exported varieties through decreased production costs. Since the existence of the nontraded varieties due to the trade costs is the only distortion in our model, unilateral trade liberalization raises welfare in both countries by narrowing the range of the nontraded varieties from both sides. Our findings that even unilateral trade liberalization is both growth- and welfare-enhancing in all countries is in stark contrast to the aforementioned literature on R&D-based endogenous growth models, where the growth and welfare effects of multilateral trade liberalization are mostly ambiguous. We will finally argue that our theoretical predictions are qualitatively consistent with the empirical evidence regarding trade liberalization and economic growth, trade costs and import shares, and transitional dynamics and exported varieties.

The rest of this paper is organized as follows. Section 2 sets up the model. Section 3 characterizes the equilibrium path. Section 4 examines the effects of unilateral trade liberalization. Section 5 discusses some extensions and supportive evidence. Section 6 concludes.

2 The model

Our endogenous growth model is based on Acemoglu and Ventura (2002). Consider a world with two countries. In each country $j(=1,2)$, there is one nontradable final good, which is consumed or invested. The final good is produced from a continuum of tradable intermediate goods indexed by $i(\in [0,1])$ under constant returns to scale and perfect competition. Each variety i is produced using nontradable capital under constant returns to scale and perfect competition, where its unit capital requirement $a_j(i)$ can be different across countries and varieties. Capital is the only primary factor.

2.1 Households

The representative household in country j maximizes its utility $U_j = \int_0^\infty \ln C_{jt} \exp(-\rho_j t)dt$, subject to the budget constraint:

$$p_{jt}^Y (C_{jt} + \dot{K}_{jt}) = r_{jt} K_{jt}, \tag{1}$$

with $\{p_{jt}^Y, r_{jt}\}_{t=0}^\infty$ and K_{j0} given, where C_j is consumption; ρ_j is the subjective discount rate; p_j^Y is the price of the final good; K_j is the capital stock; r_j is the rental rate of capital; and a dot over a variable represents differentiation with respect to time t (e.g., $\dot{K}_{jt} \equiv dK_{jt}/dt$). We omit the time subscripts whenever there is no confusion. Since capital depreciation does not affect qualitative results, we assume it away for simplicity. As a result of dynamic optimization, we obtain the Euler equation $\dot{C}_{jt}/C_{jt} = r_{jt}/p_{jt}^Y - \rho_j$ and the transversality condition $\lim_{s\to\infty} \exp(-\int_0^s (r_{jv}/p_{jv}^Y)dv)K_{js} = 0$. Moreover, the logarithmic form of the instantaneous utility function implies a simple form of the consumption function $C_{jt} = \rho_j K_{jt}$.[6] Therefore,

[6]It is obtained by integrating the budget constraint (1) from $s = t$ to infinity, and using the Euler equation and the transversality condition.

capital always grows at the same (but not necessarily constant) rate as consumption given by the Euler equation:

$$\dot{K}_{jt}/K_{jt} = \dot{C}_{jt}/C_{jt} = r_{jt}/p_{jt}^Y - \rho_j \forall t \in [0, \infty). \tag{2}$$

2.2 Final good firms

The representative final good firm in country j maximizes its profit $\Pi_j^Y = p_j^Y Y_j - \int_0^1 p_j(i)x_j(i)di$, subject to the production function $Y_j = \left(\int_0^1 x_j(i)^{(\sigma_j-1)/\sigma_j} di\right)^{\sigma_j/(\sigma_j-1)}$; $\sigma_j > 1$, with p_j^Y and $\{p_j(i)\}_{i=0}^1$ given, where Y_j is the supply of the final good; $p_j(i)$ is the demand price of variety i; $x_j(i)$ is the demand for variety i; and σ_j is the elasticity of substitution between any two varieties. Cost minimization yields:

$$\int_0^1 p_j(i)x_j(i)di = P_j Y_j; P_j(\{p_j(i)\}_{i=0}^1) \equiv \left(\int_0^1 p_j(i)^{1-\sigma_j} di\right)^{1/(1-\sigma_j)}, \tag{3}$$

where P_j is the minimized cost to obtain one unit of Y_j. Using the intermediate good price index function (3), the first-order condition for profit maximization is simply:

$$p_j^Y = P_j. \tag{4}$$

This also means that the maximized profit is zero.

2.3 Intermediate good firms

The representative intermediate good firm producing variety i in country j maximizes its profit $\Pi^x(i_j) = p(i_j)x(i_j) - r_j K^x(i_j)$, subject to the production function $x(i_j) = K^x(i_j)/a_j(i_j)$, with $p(i_j)$ and r_j given, where $p(i_j)$ is the supply price of variety i; $x(i_j)$ is the supply of variety i; $K^x(i_j)$ is the demand for capital by that firm. Using the production function to eliminate $K^x(i_j)$, the first-order condition for profit maximization allowing for the possibility of shutdown is given by:

$$p(i_j) - r_j a_j(i_j) \leq 0, x(i_j) \geq 0, x(i_j)(p(i_j) - r_j a_j(i_j)) = 0.$$

The pattern of specialization in the intermediate good sector can be analyzed in the same way as Dornbusch et al. (1977).[7] We index varieties so that the relative productivity of capital in country 1 to country 2 is diminishing with i:

$$A(i) \equiv a_2(i)/a_1(i); A'(i) < 0, i \in [0, 1].$$

We also introduce iceberg trade costs for imports: residents in country j have to buy $\tau_j(> 1)$ units of each variety from country $j'(\neq j)$ to get one unit of that variety at home because $\tau_j - 1$ units melt away in transit.[8] Then variety i_1 is produced in country 1 if and only if it is no more expensive for the representative final good firm in country 1 to buy that variety domestically than to import it from abroad: $r_1 a_1(i_1) \leq \tau_1 r_2 a_2(i_1)$, or

[7] Dornbusch et al. (1977) formulate a static Ricardian model where two countries trade a continuum of final goods produced from labor.

[8] It is very common for recent trade models such as Eaton and Kortum (2002) and Melitz (2003) to interpret a decrease in iceberg trade costs as trade liberalization, although the former extends their model to include revenue-generating import tariffs. All R&D-based endogenous growth models with heterogeneous firms mentioned in the Introduction follow this interpretation. In section 5.1, we discuss what will happen if we use import tariffs instead of iceberg trade costs.

$r_1/r_2 \leq \tau_1 A(i_1)$. Under the assumed productivity distribution, any variety $i_1 \in [0, I_1]$ is produced in country 1 with its supply price given by:

$$p(i_1) = r_1 a_1(i_1), i_1 \in [0, I_1], \tag{5}$$

where the cutoff variety I_1 is implicitly determined by:

$$r_1/r_2 = \tau_1 A(I_1). \tag{6}$$

Similarly, variety i_2 is produced in country 2 if and only if $\tau_2 r_1 a_1(i_2) \geq r_2 a_2(i_2)$, or $r_1/r_2 \geq A(i_2)/\tau_2$. Then we have:

$$p(i_2) = r_2 a_2(i_2), i_2 \in [I_2, 1]; \tag{7}$$

$$r_1/r_2 = A(I_2)/\tau_2. \tag{8}$$

For both I_1 and I_2 to be in the interior of $[0, 1]$, we impose the range of r_1/r_2 as follows:[9]

Assumption 1

$$r_1/r_2 \in (\tau_1 A(1), A(0)/\tau_2).$$

Fig. 1, which is essentially the same as Dornbusch et al. (1977, Fig. 3), summarizes the distribution of trade status. There are two downward-sloping curves, curve $\tau_1 A(i)$ and curve $A(i)/\tau_2$. Since $\tau_j > 1 \forall j$, curve $\tau_1 A(i)$ is higher than curve $A(i)/\tau_2$ for all i. The cutoff varieties I_1 and I_2 are found at the intersections of the horizontal line representing the relative rental rate in country 1 to country 2 r_1/r_2 and the corresponding downward-sloping curves, respectively. It is diagrammatically clear that $I_1 > I_2$. For country 1, relatively the least productive varieties in $[I_1, 1]$ are not produced but imported from country 2. Of produced varieties in $[0, I_1]$, relatively the most productive varieties in $[0, I_2]$ are exported to country 2, whereas the remaining varieties in $[I_2, I_1]$ are supplied to meet only domestic demand. Similarly, country 2 imports relatively the least productive varieties in $[0, I_2]$, exports relatively the most productive varieties in $[I_1, 1]$, and supplies the remaining varieties in $[I_2, I_1]$ only domestically.

We can use Fig. 1 to do some comparative statics for the distribution of trade status. First, a fall in τ_1 shifts down curve $\tau_1 A(i)$, which in turn decreases I_1. Similarly, a fall in τ_2 increases I_2. In other words, trade liberalization in a country increases the range of its imported varieties. Second, a fall in r_1/r_2 increases both I_1 and I_2. That is, a fall in the relative rental rate in a country, while indicating a deterioration in its terms of trade $p(i_j)/p(i_{j'}) = (r_j/r_{j'})a_j(i_j)/a_{j'}(i_{j'})$, increases both the range of its domestically produced varieties and that of its exported varieties. One important lesson from these exercises is that, ceteris paribus, unilateral trade liberalization in itself only increases the range of its imported varieties, but it does not increase the range of its exported varieties. It is not until we completely characterize the equilibrium path of the relative rental rate that can we understand how unilateral trade liberalization affects the distribution of trade status over time.

[9] As the unit capital requirement of the most productive variety approaches zero for all countries, that is, $a_2(1) \to 0$ and $a_1(0) \to 0$, the lower and upper bounds of r_1/r_2 approach zero and infinity, respectively.

2.4 Markets

The demand prices of the domestically produced and imported varieties are expressed as, respectively:

$$p_j(i_j) = p(i_j), \tag{9}$$

$$p_j(i_{j'}) = \tau_j p(i_{j'}), j, j' = 1, 2, j' \neq j. \tag{10}$$

The market-clearing conditions for the final goods, capital, and the intermediate goods are given by, respectively:

$$Y_j = C_j + \dot{K}_j, j = 1, 2, \tag{11}$$

$$K_1 = \int_0^{I_1} K^x(i_1) di_1, \tag{12}$$

$$K_2 = \int_{I_2}^1 K^x(i_2) di_2,$$

$$x(i_1) = x_1(i_1) + \tau_2 x_2(i_1), i_1 \in [0, I_2] \subset [0, I_1], \tag{13}$$

$$x(i_1) = x_1(i_1), i_1 \in [I_2, I_1] \subset [0, I_1], \tag{14}$$

$$x(i_2) = x_2(i_2) + \tau_1 x_1(i_2), i_2 \in [I_1, 1] \subset [I_2, 1],$$

$$x(i_2) = x_2(i_2), i_2 \in [I_2, I_1] \subset [I_2, 1].$$

For country 1, relatively the most productive varieties in $[0, I_2]$ are demanded by both countries (cf. Eq. (13)), whereas the mediocre varieties in $[I_2, I_1]$ have only domestic demand (cf. Eq. (14)). The same applies to country 2.

Finally, Eqs. (1), (3), (4), (5), (7), (9), and (10) imply Walras' law: the sum of the values of excess demands for all markets is identically zero. Thus we can choose any one of the eight types of goods and factors as the numeraire, and remove its market-clearing condition from equilibrium characterization.

3 Equilibrium

3.1 Dynamic system

We reduce the equations in the previous section to derive the dynamic system. From Eqs. (5), (7), (9), and (10), the demand prices of the domestically produced and imported varieties for the two countries are rewritten as:

$$p_1(i_1) = p(i_1) = r_1 a_1(i_1), i_1 \in [0, I_1],$$

$$p_1(i_2) = \tau_1 p(i_2) = \tau_1 r_2 a_2(i_2), i_2 \in [I_1, 1] \subset [I_2, 1],$$

$$p_2(i_2) = p(i_2) = r_2 a_2(i_2), i_2 \in [I_2, 1],$$

$$p_2(i_1) = \tau_2 p(i_1) = \tau_2 r_1 a_1(i_1), i_1 \in [0, I_2] \subset [0, I_1].$$

Using these expressions, Eq. (3) is simplified to:

$$P_1 = [(\tau_1 r_2)^{1-\sigma_1} \int_{I_1}^{1} a_2(i_2)^{1-\sigma_1} di_2 + r_1^{1-\sigma_1} \int_0^{I_1} a_1(i_1)^{1-\sigma_1} di_1]^{1/(1-\sigma_1)} \equiv Q_1(\tau_1 r_2, r_1, I_1), \tag{15}$$

$$P_2 = [(\tau_2 r_1)^{1-\sigma_2} \int_0^{I_2} a_1(i_1)^{1-\sigma_2} di_1 + r_2^{1-\sigma_2} \int_{I_2}^{1} a_2(i_2)^{1-\sigma_2} di_2]^{1/(1-\sigma_2)} \equiv Q_2(\tau_2 r_1, r_2, I_2). \tag{16}$$

The simplified intermediate good price index function $Q_j(\tau_j r_{j'}, r_j, I_j)$ is increasing and homogeneous of degree one in the foreign and domestic rental rates, with the former multiplied by the trade cost. The cutoff variety I_j appears as limits of integrals for the unit capital requirements embodied in the demand prices. Linear homogeneity of $Q_j(\cdot)$ in Eqs. (15) and (16) implies two key relationships. First, the rate of return to capital r_j/p_j^Y in the Euler equation (2) is rewritten using Eq. (4) as:

$$r_j/p_j^Y = r_j/Q_j(\tau_j r_{j'}, r_j, I_j) = 1/Q_j(\tau_j r_{j'}/r_j, 1, I_j). \tag{17}$$

Eq. (17) means that, with I_j given, the rate of return to capital in country j is decreasing in $\tau_j r_{j'}/r_j$. In other words, a fall in the trade cost in country j or a rise in the relative rental rate in country j, with the latter indicating an improvement in its terms of trade, raises the growth rate of capital in country j. Second, noting from Eq. (3) that the derived demand for an imported variety is expressed as $x_j(i_{j'}) = (\partial P_j/\partial p_j(i_{j'}))Y_j = P_j^{\sigma_j} p_j(i_{j'})^{-\sigma_j} Y_j$, the shares of the imported varieties in the total cost of the final good for the two countries are calculated as:

$$\begin{aligned} &\int_{I_1}^{1} p_1(i_2) x_1(i_2) di_2/(P_1 Y_1) \\ &\quad = Q_1(1, r_1/(\tau_1 r_2), I_1)^{\sigma_1 - 1} \int_{I_1}^{1} a_2(i_2)^{1-\sigma_1} di_2 \equiv \beta_1(\tau_1 r_2/r_1, I_1) \in (0,1), \end{aligned} \tag{18}$$

$$\begin{aligned} &\int_0^{I_2} p_2(i_1) x_2(i_1) di_1/(P_2 Y_2) \\ &\quad = Q_2(1, r_2/(\tau_2 r_1), I_2)^{\sigma_2 - 1} \int_0^{I_2} a_1(i_1)^{1-\sigma_2} di_1 \equiv \beta_2(\tau_2 r_1/r_2, I_2) \in (0,1). \end{aligned} \tag{19}$$

We can interpret $\beta_j(\cdot)$ as a natural indicator of openness in country j. Since $\sigma_j > 1 \forall j$, with I_j given, the openness in country j is decreasing in $\tau_j r_{j'}/r_j$. As the imported varieties get relatively more expensive, the representative final good firm substitutes the domestically produced varieties for the imported ones at a larger rate than the relative price change, thereby decreasing the cost share of the imported varieties.

We choose capital in country 2 as the numeraire: $r_2 \equiv 1$. Then our model is dramatically reduced to a two-dimensional autonomous dynamic system:[10]

[10]Eq. (20) is obtained immediately from Eqs. (2), (17), and $I_j = I_j(\cdot)$, where $I_1(\cdot)$ and $I_2(\cdot)$ are defined by inverting Eqs. (6) and (8), respectively. Eq. (21) is equivalent to $\beta_1(\cdot) r_1 K_1 = \beta_2(\cdot) r_2 K_2$, which is obtained by rewriting Eq. (12) using Eqs. (1), (4), (11), (13), (14), (18), and (19). We can reach the same expression by rewriting the market-clearing condition for capital in country 2 or the balanced trade condition for either country instead of Eq. (12).

$$\dot{\kappa} = \kappa(\gamma_1(\tau_1/r_1) - \gamma_2(\tau_2 r_1)); \kappa \equiv K_1/K_2, \tag{20}$$

$$\gamma_1(\tau_1/r_1) \equiv \dot{C}_1/C_1 = 1/Q_1(\tau_1/r_1, 1, I_1(\tau_1/r_1)) - \rho_1; I_1(\tau_1/r_1) \equiv A^{-1}(r_1/\tau_1),$$

$$\gamma_2(\tau_2 r_1) \equiv \dot{C}_2/C_2 = 1/Q_2(\tau_2 r_1, 1, I_2(\tau_2 r_1)) - \rho_2; I_2(\tau_2 r_1) \equiv A^{-1}(\tau_2 r_1),$$

$$\kappa = (\beta_2(\tau_2 r_1, I_2(\tau_2 r_1))/\beta_1(\tau_1/r_1, I_1(\tau_1/r_1)))/r_1. \tag{21}$$

Eq. (20) shows that the relative supply of capital in country 1 to country 2 κ evolves according to the difference between the common growth rates of consumption and capital $\gamma_j(\cdot)$ in the two countries. Eq. (21) states that the relative rental rate of capital in country 1 to country 2 r_1 is determined to equalize its relative supply and relative demand. All $Q_j(\cdot)$ and $\beta_j(\cdot)$ included in this system depend on $\tau_j r_{j'}/r_j$ directly with I_j given, and indirectly through the change in I_j as examined in section 2.3. For all $t \in [0, \infty)$, with τ_1 and τ_2 exogenous and with κ_t predetermined, r_{1t} is determined from Eq. (21), and then $\dot{\kappa}_t$ is determined from Eq. (20).

3.2 Balanced growth path and transitional dynamics

We define a balanced growth path (BGP) as a path along which all variables grow at constant rates. From Eq. (20), r_1 should be constant. This and Eq. (21) imply that κ should be constant. Setting $\dot{\kappa} = 0$, a BGP is implicitly determined by:

$$\gamma_1(\tau_1/r_1^*) = \gamma_2(\tau_2 r_1^*),$$

$$\kappa^* = (\beta_2(\tau_2 r_1^*, I_2(\tau_2 r_1^*))/\beta_1(\tau_1/r_1^*, I_1(\tau_1/r_1^*)))/r_1^*,$$

where an asterisk over a variable represents a BGP. The former equation is solved for r_1^*, and then the latter equation gives κ^*.

Before characterizing a BGP, we reveal the properties of the $Q_j(\cdot)$ and $\beta_j(\cdot)$ functions governing our dynamic system:

Lemma 1 *$Q_1(\tau_1/r_1, 1, I_1)$ and $Q_2(\tau_2 r_1, 1, I_2)$ are independent of I_1 and I_2, respectively.*

Proof. See Appendix A. ■

To interpret this result, take Eq. (15) for example. An increase in I_1 causes the demand price of a domestically produced variety $p_1(i_1) = r_1 a_1(i_1)$ to replace that of an imported variety $p_1(i_2) = \tau_1 a_2(i_2)$ at $i_1 = i_2 = I_1$. However, by definition of I_1 in Eq. (6), these demand prices must be equal right there. Therefore, any change in I_1 caused by any change in τ_1/r_1 does not affect Q_1. The same is true for country 2. An implication of this lemma is that, just like the envelope theorem, the total effect of a change in $\tau_j r_{j'}/r_j$ on r_j/p_j^Y equals its direct effect discussed around Eq. (17).

Lemma 2 *$\beta_1(\tau_1/r_1, I_1(\tau_1/r_1))$ and $\beta_2(\tau_2 r_1, I_2(\tau_2 r_1))$ are decreasing in τ_1/r_1 and $\tau_2 r_1$, respectively.*

Proof. See Appendix B. ■

Suppose that r_1 falls, so that the varieties produced in country 1 become relatively cheaper. This makes country 1 more closed (i.e., decreases β_1) but makes country 2 more open (i.e., increases β_2) through the direct substitution effects explained around Eqs. (18) and (19). On the other hand, it increases both the range of the domestically produced varieties and that of the exported varieties in country 1 (i.e., increases both I_1 and I_2), which further decreases the cost share of the imported varieties in country 1 but increases that in country 2. Consequently, both the decrease in β_1 and the increase in β_2 work to increase the relative demand for capital in country 1.

With the help of these lemmas, the following proposition shows the existence, uniqueness, and stability of a BGP:

Proposition 1 *Suppose that* $\lim_{r_1 \to \tau_1 A(1)}(\gamma_1(\tau_1/r_1) - \gamma_2(\tau_2 r_1)) < 0$ *and* $\lim_{r_1 \to A(0)/\tau_2}(\gamma_1(\tau_1/r_1) - \gamma_2(\tau_2 r_1)) > 0$. *Then there exists a unique and globally stable interior balanced growth path.*

Proof. From Lemma 1, $\gamma_1(\tau_1/r_1)$ and $\gamma_2(\tau_2 r_1)$ are decreasing in τ_1/r_1 and $\tau_2 r_1$, respectively, which in turn implies that $\gamma_1(\tau_1/r_1) - \gamma_2(\tau_2 r_1)$ is increasing in r_1. Hence there exists a unique $r_1^* \in (\tau_1 A(1), A(0)/\tau_2)$ such that $\gamma_1(\tau_1/r_1^*) - \gamma_2(\tau_2 r_1^*) = 0$. On the other hand, from Lemma 2, $(\beta_2/\beta_1)/r_1$ is decreasing in r_1. This immediately guarantees the uniqueness of $\kappa^* = (\beta_2^*/\beta_1^*)/r_1^*$. Moreover, for all $\kappa >$ (or $<$)κ^*, we have $r_1 <$ (or $>$)r_1^* from Eq. (21), and $\dot{\kappa} = \kappa(\gamma_1(\tau_1/r_1) - \gamma_2(\tau_2 r_1)) <$ (or $>$)0 from Eq. (20). This ensures the global stability of our dynamic system. ■

Example Suppose that the unit capital requirement functions are given by:

$$a_1(i) = \varepsilon_1 + i,$$
$$a_2(i) = 1 + \varepsilon_2 - i,$$

where ε_1 and ε_2 are positive constants. Then $A(i) = a_2(i)/a_1(i)$ is decreasing and convex in i. Suppose also that ε_1 and ε_2 approach zero. Then we have $A(0) = \infty$ and $A(1) = 0$. This implies that the lower and upper bounds of r_1 become zero and infinity, respectively. As r_1 approaches its lower bound of zero, I_1 and I_2 approach unity from Eqs. (6) and (8), and $Q_1(\tau_1/r_1, 1, I_1)$ and $Q_2(\tau_2 r_1, 1, I_2)$ approach $[\int_0^1(\varepsilon_1 + i_1)^{1-\sigma_1} di_1]^{1/(1-\sigma_1)}$ and zero from Eqs. (15) and (16), respectively. On the other hand, as r_1 approaches its upper bound of infinity, I_1 and I_2 approach zero, and $Q_1(\tau_1/r_1, 1, I_1)$ and $Q_2(\tau_2 r_1, 1, I_2)$ approach zero and $[\int_0^1(1 + \varepsilon_2 - i_2)^{1-\sigma_2} di_2]^{1/(1-\sigma_2)}$, respectively. Therefore, from Eq. (20), we have $\lim_{r_1 \to 0}(\gamma_1(\tau_1/r_1) - \gamma_2(\tau_2 r_1)) = -\infty < 0$ and $\lim_{r_1 \to \infty}(\gamma_1(\tau_1/r_1) - \gamma_2(\tau_2 r_1)) = \infty > 0$, thereby ensuring the existence of a BGP. By continuity, the two boundary conditions for the existence of a BGP are satisfied as long as ε_1 and ε_2 are sufficiently small.

The existence, uniqueness, and stability of a BGP are illustrated in Fig. 2, where the relative rental rate in country 1 and the common growth rates of consumption and capital in the two countries are measured on the horizontal and vertical axes, respectively. Curve $\gamma_1(\tau_1/r_1)$ (dashed) is upward-sloping because an increase in r_1 improves the terms of trade in country 1. By the symmetric reasoning, curve $\gamma_2(\tau_2 r_1)$ is downward-sloping. A BGP is found at point A: (r_1^*, γ^*), the intersection of these curves. As long as curve $\gamma_1(\tau_1/r_1)$ is lower (or higher) than curve $\gamma_2(\tau_2 r_1)$ at the lower (or upper) bound of r_1 for interior I_1 and I_2, that intersection exists uniquely. Once r_1^* is determined at point A, κ^* is determined accordingly from Eq. (21). As for transitional dynamics, suppose that capital in country 1 is relatively scarce in the initial period: $\kappa_0 < \kappa^*$. Then the equilibrium relative rental rate in country 1 must be high enough to reflect this scarcity: $r_{10} > r_1^*$. At that r_{10}, to the right of point A, country 1 with relatively high terms of trade starts to grow faster than country 2. Since the higher growth in country 1 causes κ_t to increase, r_{1t} and hence the terms of trade in country 1 start to fall. This pulls down the growth rate of capital in country 1 but pushes up that in country 2 along curve $\gamma_1(\tau_1/r_1)$ and curve $\gamma_2(\tau_2 r_1)$, respectively. This process continues until these growth rates are equalized at point A, the BGP. Exactly the opposite occurs when $\kappa_0 > \kappa^*$. Since our dynamic system reaches the unique BGP for any initial condition κ_0 (so long as the corresponding r_{10} satisfies Assumption 1), the BGP is globally stable.

Combining Fig. 2 with Fig. 1, we can see how the distribution of trade status evolves over time. When $\kappa_0 < \kappa^*$, due to relatively high production costs caused by high r_{10}, country 1 initially imports a wide range of varieties (i.e., I_{10} is low) but exports a narrow range of varieties (i.e., I_{20} is low). As that country develops faster than the rest of the world during the transition, more and more imported varieties are substituted by domestic production (i.e., I_{1t} increases to I_1^*), whereas more and more varieties which were supplied only domestically begin to be exported (i.e., I_{2t} increases to I_2^*).

4 Effects of unilateral trade liberalization

4.1 Growth rates of capital

Having characterized the equilibrium path in the previous section, we now examine how it is affected by unilateral trade liberalization. In Fig. 2, suppose that the world economy is originally at point A, the old BGP, and that the trade cost in country 1 falls permanently from τ_1 to τ_1'. This shifts curve $\gamma_1(\tau_1/r_1)$ up to curve $\gamma_1(\tau_1'/r_1)$. In the long run, the new BGP comes to point D: $(r_1^{*\prime}, \gamma^{*\prime})$, the intersection of curve $\gamma_1(\tau_1'/r_1)$ and curve $\gamma_2(\tau_2 r_1)$. Since point D is located to the northwest of point A along curve $\gamma_2(\tau_2 r_1)$, unilateral trade liberalization raises the balanced growth rate of capital.

Things become more complicated in the short run. One might think that, in the initial period, transition to the new BGP starts from $r_{10} = r_1^*$. In fact, since the fall in τ_1 increases the openness in country 1 and hence decreases the relative demand for capital in country 1, its equilibrium relative rental rate must fall. Depending on how far r_{10} falls, we consider two cases in turn:[11]

Case 1: $r_1^{*\prime} < r_{10}' < r_1^*$

This case is described in Fig. 2. Country 1 and country 2 initially jump from point A to point B and point C, respectively. Point B is higher than point C, which in turn is higher than point A. Since the higher growth in country 1 itself starts to deteriorate its terms of trade, the growth rate of capital in country 1 goes down but that in country 2 goes up along curve $\gamma_1(\tau_1'/r_1)$ and curve $\gamma_2(\tau_2 r_1)$, respectively, until they converge at point D, the new BGP. While the growth rate of capital in country 2 rises monotonically to $\gamma^{*\prime}(> \gamma^*)$, the growth rate of capital in country 1 exhibits an overshooting behavior.

Case 2: $r_{10}' < r_1^{*\prime}$

In this case, point B and point C indicating initial jumps on curve $\gamma_1(\tau_1'/r_1)$ and curve $\gamma_2(\tau_2 r_1)$, respectively, are located to the left of point D. Although point B is now lower than point C, it is still higher than point A. This is because r_{10} must fall at a smaller rate than τ_1 in order to satisfy Eq. (21) for $t = 0$.[12] Since country 1 grows more slowly than country 2, the terms of trade in country 1 starts to improve. This

[11] Appendix C shows that r_{10} may or may not fall at a larger rate than r_1^*.

[12] In Eq. (21) for $t = 0$, a fall in r_{10} increases $\beta_2(\tau_2 r_{10}, I_2(\tau_2 r_{10}))/r_{10}$ from Lemma 2. For that equation to be satisfied, $\beta_1(\tau_1/r_{10}, I_1(\tau_1/r_{10}))$ should also increase. Then again from Lemma 2, τ_1/r_{10} should fall. This logic is confirmed by Eq. (C.2) in Appendix C.

continues to narrow the growth gap until it is eliminated at point D. Now country 2 grows in an overshooting manner.

Our examination of the growth effects of unilateral trade liberalization is summarized as follows:

Proposition 2 *Compared with the old balanced growth path, a permanent fall in the trade cost in any one country raises the growth rates of capital in all countries for all periods.*

To highlight this result, consider the case of symmetric trade liberalization, where τ_1 and τ_2 are reduced by the same rate. Since this shifts up both curve $\gamma_1(\tau_1/r_1)$ and curve $\gamma_2(\tau_2 r_1)$ in Fig. 2, r_1^* does not change so much from its old value.[13] Faster growth in each country comes mainly from the reduction in its own trade cost. In contrast, Proposition 2 reveals that unilateral trade liberalization speeds up growth of not only the liberalizing country itself but also the non-liberalizing country through the terms of trade improvement for the latter.

4.2 Distribution of trade status

Considering from Eqs. (6) and (8) that $I_1(\tau_1/r_1)$ is increasing in τ_1/r_1 whereas $I_2(\tau_2 r_1)$ is decreasing in $\tau_2 r_1$, we can say that $I_1(\tau_1/r_1)$ moves in the opposite direction of $\gamma_1(\tau_1/r_1)$ whereas $I_2(\tau_2 r_1)$ moves in the same direction as $\gamma_2(\tau_2 r_1)$. Combining this fact with the result in the previous subsection, we can trace the effect on the distribution of trade status over time. In the initial period, we have $I'_{10} < I_1^*$ and $I'_{20} > I_2^*$, implying the wider ranges of both the imported and exported varieties in all countries. In case 1, where country 1 grows faster than country 2 during the transition, both I'_{1t} and I'_{2t} get larger and larger, pushing the former back toward I_1^*. In the long run, however, we still have $I_1^{*\prime} < I_1^*$ and $I_2^{*\prime} > I_2^*$ just like the initial period. Although the transitional movements of I'_{1t} and I'_{2t} in case 2 are just the opposite, the long-run consequence is the same:

Proposition 3 *Compared with the old balanced growth path, a permanent fall in the trade cost in any one country increases both the range of the imported varieties and that of the exported varieties in all countries for all periods.*

Trade liberalization in a country not only increases the range of its imported varieties, but it also increases the range of its exported varieties through the fall in its relative rental rate resulting from its increased growth potential.

4.3 Welfare

Considering that welfare in country j is the discounted sum of its instantaneous utility $\ln C_{jt} = \ln C_{j0} + \int_0^t \gamma_{js} ds = \ln \rho_j + \ln K_{j0} + \int_0^t \gamma_{js} ds$, we immediately find from Proposition 2 that the instantaneous utility is higher than the old BGP in all countries for all periods, and so is welfare:

Proposition 4 *Compared with the old balanced growth path, a permanent fall in the trade cost in any one country raises welfare in all countries.*

In our model, the trade costs distort the representative final good firms' choices between domestic purchases and imports in favor of the former. This creates the range of the nontraded varieties $[I_2, I_1]$, which

[13] As Appendix C shows, even the reductions in τ_1 and τ_2 by the same rate can change r_1^* in general, but the latter is unchanged when the two countries are completely symmetric.

could not exist without the trade costs. Unilateral trade liberalization in country 1, for example, alleviates the import distortion in country 1 (i.e., decreases I_1) by exposing its domestic firms to cheaper imports. In addition, it also corrects the import distortion in country 2 (i.e., increases I_2) by making firms in country 1 relatively more competitive in their export market. Since there are no other distortions such as pricing above marginal cost and knowledge spillovers, which are prevalent in the literature on R&D-based endogenous growth models, even unilateral trade liberalization is Pareto-improving in our model.

5 Discussions

5.1 Import tariffs

Since the trade costs are modeled as pure wastes so far, reducing them is likely to be beneficial by simply decreasing the wasted resources. How are our strong results affected when the trade costs consist mainly of import tariffs? To see this, suppose that $\tau_j - 1$ here represents an ad valorem tariff rate in country j, and that the tariff revenue is transferred to the representative household in a lump-sum manner. Then the government budget constraints in country 1 and country 2 are given by, respectively:

$$T_1 = \int_{I_1}^{1} (\tau_1 - 1)p(i_2)x_1(i_2)di_2,$$
$$T_2 = \int_{0}^{I_2} (\tau_2 - 1)p(i_1)x_2(i_1)di_1,$$

where T_j is the lump-sum transfer in country j. The lump-sum transfer adds to the right-hand side of Eq. (1), but the representative household takes it as given. Finally, unlike Eq. (13), the market-clearing conditions for the traded intermediate goods now do not involve τ_j.

Replacing the iceberg trade costs with the import tariffs causes two major changes in our analysis. First, unlike Eq. (2), capital does not always grow at the same rate as consumption during the transition due to the presence of the tariff revenue. This means that we have to add the consumption/capital ratios in the two countries to our dynamic system. Second, a fall in the tariff of a country may decrease its tariff revenue and hence consumption in the short run from Eq. (1). This will partly lower welfare in the liberalizing country. However, since the growth rates of consumption and capital in both countries must be equalized along the BGP, the balanced growth rate is still determined solely by $\gamma_1(\tau_1/r_1^*) = \gamma_2(\tau_2 r_1^*)$ as in section 3.2. This implies that even unilateral tariff reduction raises the balanced growth rate in the long run. Moreover, as long as the subjective discount rates are sufficiently low, the long-run welfare gains thanks to faster consumption growth outweigh the possible short-run welfare losses due to decreased tariff revenue.[14] In this case, unilateral tariff reduction is still Pareto-improving.

5.2 More than two countries

One possible way to extend our model to more than two countries will be to employ the probabilistic formulation of technologies developed by Eaton and Kortum (2002). Suppose that there are $N(\geq 2)$ countries. The demand price of variety i produced in country j and delivered to country n is given by $p_{nj}(i) =$

[14] If we use a more general instantaneous utility function with a constant elasticity of intertemporal substitution, we can predict that the long-run welfare gains are more dominant, the higher is the elasticity of intertemporal substitution.

$\tau_{nj}r_j a_j(i), i \in [0,1], j, n = 1, ..., N$, where τ_{nj} is the iceberg trade cost of delivering one unit of each variety from country j to country n. It is assumed that $\tau_{jj} = 1 \forall j, \tau_{nj} > 1 \forall n \neq j$, and $\tau_{nj} \leq \tau_{nk}\tau_{kj} \forall j, k, n$. Since the representative final good firm in country n buys each variety from the country offering the lowest demand price, its actual demand price of variety i is expressed as $p_n(i) = \min\{\{p_{nj}(i)\}_{j=1}^N\}$.

Let A_j represent an independent and identically distributed random variable for $a_j(i)$. And, let $P_{nj} = \tau_{nj}r_j A_j$ and $P_n = \min\{\{P_{nj}\}_{j=1}^N\}$ denote i.i.d. random variables for $p_{nj}(i)$ and $p_n(i)$, respectively. Following Eaton and Kortum (2002), we assume that the productivity of capital $1/A_j$ has a Fréchet distribution $F_j(1/a) \equiv \Pr(1/A_j \leq 1/a) \equiv \exp(-b_j(1/a)^{-\theta})$, where $b_j > 0$ and $\theta > 1$. The parameter b_j captures country j's overall state of technology (i.e., absolute advantage), whereas θ governs the variability of productivity (i.e., comparative advantage). Then the distribution functions for P_{nj} and P_n are derived as $G_{nj}(p) \equiv \Pr(P_{nj} \leq p) = 1 - \exp(-p^\theta b_j(\tau_{nj}r_j)^{-\theta})$ and $G_n(p) \equiv \Pr(P_n \leq p) = 1 - \exp(-p^\theta \Phi_n)$, where $\Phi_n \equiv \sum_{j=1}^N b_j(\tau_{nj}r_j)^{-\theta}$, respectively.

In the same way as Eaton and Kortum (2002), we obtain three important properties. First, the probability that country n buys a variety from country j is expressed as:

$$\pi_{nj} = b_j(\tau_{nj}r_j)^{-\theta}/\Phi_n.$$

Remembering that there are a continuum of intermediate goods in $[0,1]$, this is equal to country n's cost share of varieties produced in country j (corresponding to β_n and $1-\beta_n$ in the two-country case). Second, the conditional distribution of P_{nj}, given that country n buys a variety from country j, is the same as $G_n(p)$. Third, the intermediate good price index $Q_n = \left(\int_0^1 E(P_n^{1-\sigma_n})di\right)^{1/(1-\sigma_n)}$ is calculated as:

$$Q_n(\{\tau_{nj}r_j\}_{j=1}^N) = c_n\Phi_n^{-1/\theta} = c_n\left[\sum_{j=1}^N b_j(\tau_{nj}r_j)^{-\theta}\right]^{-1/\theta}; c_n \equiv \Gamma(1+(1-\sigma_n)/\theta)^{1/(1-\sigma_n)},$$

where $\Gamma(1+(1-\sigma_n)/\theta)$ is the Gamma function, with the restriction that $1+(1-\sigma_n)/\theta > 0$.

Noting that $Q_n(\{\tau_{nj}r_j\}_{j=1}^N)$ is homogeneous of degree one in $\{\tau_{nj}r_j\}_{j=1}^N$, the rate of return to capital in country n, corresponding to Eq. (17) in the two-country case, is rewritten as:

$$r_n/p_n^Y = r_n/Q_n(\{\tau_{nj}r_j\}_{j=1}^N) = 1/Q_n(\{\tau_{nj}r_j/r_n\}_{j=1}^N).$$

This implies that, for all $j \neq n$, a fall in τ_{nj} or a rise in r_n/r_j raises the growth rate of capital in country n, just like the two-country case.

Finally, to close the model, we should choose capital in the last country N as the numeraire, and derive the capital market-clearing conditions for the first $N-1$ countries relative to country N. Although characterization of the equilibrium path in general is beyond the scope of this paper, we can say more for the case where the direct negative effect of $\kappa_j \equiv K_j/K_N$ on r_j is dominant. First, an increase in κ_n lowers r_n and hence the growth rate of capital in country n. Second, for all $j \neq n$, an increase in κ_j lowers r_j, which in turn raises the growth rate of capital in country n. Since a fall in the rental rate in a faster-growing country pulls down its own growth rate of capital but pushes up those in all other countries, the BGP is still stable in the extended model.

5.3 Comparing predictions to data

In this subsection, we discuss how well our theory matches the empirical evidence in three points: trade liberalization and economic growth, trade costs and import shares, and transitional dynamics and exported varieties.

Since Rodriguez and Rodrik (2000) questioned the robustness of the preceding empirical literature reporting the positive relationship between trade liberalization and economic growth, some researchers have tried to overcome their skepticism by using more recent data and more sophisticated empirical strategies. Wacziarg and Welch (2008) calculate an openness dummy defined by Sachs and Warner (1995) up until 1998, and apply fixed-effects regressions to the panel of 136 countries during 1950-1998. Estevadeordal and Taylor (2008) collect data for two periods, 1975-1989 and 1990-2005, and construct a liberalizer dummy taking the value of one if a country lowered its average tariffs after 1985 in tandem with the GATT's Uruguay round, to run difference-in-difference regressions. Romalis (2007) estimates the effect of openness (i.e., the trade/GDP ratio) of developing countries on their growth rates, using the tariffs of the USA as instruments for the openness. The three more recent papers provide more solid evidence than before that trade liberalization does promote growth, supporting our theory.

Although we do not find direct econometric evidence that import shares are decreasing in trade costs, we can test it indirectly by seeing if the price elasticity of demand for traded goods, or the elasticity of substitution among traded goods, is larger than unity. Limão and Venables (2001) estimate that (the absolute value of) the elasticity of trade volume with respect to the transport cost factor is 2.24 from the full model, and 3.11 from the fixed-effects model. Anderson and van Wincoop (2004) survey that the elasticities of substitution estimated by several researchers range from five to ten. Since the estimated elasticities in question are well above unity, we expect that falling trade costs contribute to increased import shares.

In the last paragraph of section 3, we see that the range of a country's exported varieties, also called its "extensive margin of exports", increases as it grows faster than the rest of the world during the transition. Is there empirical evidence for this theoretical prediction? Hummels and Klenow (2005) define the extensive margin as a weighted fraction of varieties a country exports to another country. Using a cross section of 126 exporting countries, 59 importing countries, and 5,017 six-digit HS (Harmonized System) codes in 1995, they find that a 1 percentage point increase in the GDP of an exporting country relative to the rest of the world increases its extensive margin of exports by 0.61 percentage points. Kehoe and Ruhl (2009) sort the four-digit SITC (Standard International Trade Classification) codes by the value of exports in ascending order, and make ten sets each representing one-tenth of total exports, for each pair of exporting and importing countries. They then use a change in the share of the "least traded goods", that is, the set of varieties with the smallest export values accounting for the first 10% of total exports in the initial period, as an indicator of a change in the extensive margin of exports. As a result, they report that the shares of the least traded goods exported from Chile (1975-1985), China (1995-2005), and Korea (1975-1985) to the United States increased to 24.4%, 24.7%, and 60.4% during their decades of rapid economic growth. Our model is qualitatively consistent with these empirical results.

6 Concluding remarks

Formulating a simple two-country endogenous growth model with endogenous trade status, we show that reallocations across intermediate good firms induced by unilateral trade liberalization always raises both aggregate growth and welfare in all countries. This is in contrast to the existing literature on R&D-based endogenous growth models, where multilateral trade liberalization has at best ambiguous growth and welfare effects. The difference in results comes from the difference in assumptions: in our Ricardian framework, the intermediate good firms are perfectly competitive with no fixed R&D costs. This suggests that the effectiveness of trade liberalization depends on the market structure of tradable industries.

As discussed in section 5, our two-country model with iceberg trade costs can be extended to more than two countries and import tariffs. This allows us to apply our framework to a variety of problems concerning trade and growth. For example, we can examine the qualitative impacts of trade liberalization in one country on other countries having different characteristics. Another example will be to see the growth effects of regional trade agreements on member and non-member countries. In the theoretical literature on trade and growth, this paper may serve as an alternative to the R&D-based endogenous growth models.

Acknowledgements

I am grateful to the co-editor Timothy J. Kehoe, two anonymous referees, and conference and seminar participants at IEFS Japan Meeting 2010, Tohoku University, Doshisha University, Waseda University, Osaka University, and Kansai Macroeconomics Workshop for their helpful comments and suggestions. I also appreciate IR/PS, UCSD for great research environment, and JSPS Postdoctoral Fellowship for Research Abroad and KAKENHI #22730190 for financial support. All remaining errors are mine.

Appendix A. Proof of Lemma 1

We logarithmically differentiate Eq. (15) to obtain:

$$\begin{aligned} dQ_1/Q_1 = {} & [1/(1-\sigma_1)](1/Q_1^{1-\sigma_1}) \\ & \times \{(1-\sigma_1)\tau_1^{-\sigma_1}\int_{I_1}^{1} a_2(i_2)^{1-\sigma_1}di_2 d\tau_1 + (1-\sigma_1)r_1^{-\sigma_1}\int_0^{I_1} a_1(i_1)^{1-\sigma_1}di_1 dr_1 \\ & + [\tau_1^{1-\sigma_1}(-a_2(I_1)^{1-\sigma_1}) + r_1^{1-\sigma_1}a_1(I_1)^{1-\sigma_1}]dI_1\}. \end{aligned}$$

Noting that Eq. (6) is rewritten as $r_1 a_1(I_1) = \tau_1 a_2(I_1)$, the two terms in the brackets before dI_1 cancel each other out, thereby neutralizing the effect of any change in I_1 on Q_1. Using the definition of β_1 in Eq. (18), the above expression is simplified to:

$$dQ_1/Q_1 = \beta_1 d\tau_1/\tau_1 + (1-\beta_1)dr_1/r_1. \tag{A.1}$$

We can similarly show from Eqs. (8), (16), and (19) that any change in I_2 does not affect Q_2, yielding:

$$dQ_2/Q_2 = \beta_2(d\tau_2/\tau_2 + dr_1/r_1). \tag{A.2}$$

Appendix B. Proof of Lemma 2

We logarithmically differentiate Eq. (18) to have:

$$d\beta_1/\beta_1 = (\sigma_1-1)(dQ_1/Q_1 - d\tau_1/\tau_1) + (1/\int_{I_1}^{1} a_2(i_2)^{1-\sigma_1}di_2)(-a_2(I_1)^{1-\sigma_1})dI_1,$$

where $Q_1(1, r_1/\tau_1, I_1) = Q_1(\tau_1, r_1, I_1)/\tau_1$ is used. Using Eq. (A.1) and the logarithmically differentiated form of Eq. (6), this is rewritten as:

$$d\beta_1/\beta_1 = -B_1(d\tau_1/\tau_1 - dr_1/r_1); \tag{B.1}$$
$$B_1 \equiv (\sigma_1 - 1)(1-\beta_1) - (I_1 a_2(I_1)^{1-\sigma_1}/\int_{I_1}^{1} a_2(i_2)^{1-\sigma_1} di_2)A/(A'I_1) > 0.$$

Therefore, $\beta_1(\tau_1/r_1, I_1(\tau_1/r_1))$ is decreasing in τ_1/r_1.

Similarly, from Eqs. (8), (19), and (A.2), we obtain:

$$d\beta_2/\beta_2 = -B_2(d\tau_2/\tau_2 + dr_1/r_1); \tag{B.2}$$
$$B_2 \equiv (\sigma_2 - 1)(1-\beta_2) - (I_2 a_1(I_2)^{1-\sigma_2}/\int_{0}^{I_2} a_1(i_1)^{1-\sigma_2} di_1)A/(A'I_2) > 0.$$

This implies that $\beta_2(\tau_2 r_1, I_2(\tau_2 r_1))$ is decreasing in $\tau_2 r_1$.

Appendix C. Comparing changes in r_1^* and r_{10}

The rate of change in r_1^* caused by a change in τ_1 is derived from $\gamma_1(\tau_1/r_1^*) = \gamma_2(\tau_2 r_1^*)$. Using Eq. (2), (4), (A.1), and (A.2), changes in the growth rates of capital in the two countries are generally obtained as $d\gamma_1 = (\gamma_1 + \rho_1)\beta_1(dr_1/r_1 - d\tau_1/\tau_1)$ and $d\gamma_2 = -(\gamma_2 + \rho_2)\beta_2(d\tau_2/\tau_2 + dr_1/r_1)$. Substituting them into $d\gamma_1^* = d\gamma_2^*$, and solving it for dr_1^*/r_1^*, we obtain:

$$dr_1^*/r_1^* = \{1/[(\gamma_1^* + \rho_1)\beta_1^* + (\gamma_2^* + \rho_2)\beta_2^*]\}[(\gamma_1^* + \rho_1)\beta_1^* d\tau_1/\tau_1 - (\gamma_2^* + \rho_2)\beta_2^* d\tau_2/\tau_2].$$

This expression informs us of two features. First, symmetric changes in τ_1 and τ_2 (i.e., $d\tau_1/\tau_1 = d\tau_2/\tau_2$) generally affects r_1^*, but the latter is unaffected if the two countries are completely symmetric so that $(\gamma_1^* + \rho_1)\beta_1^* = (\gamma_2^* + \rho_2)\beta_2^*$. Second, the elasticity of r_1^* with respect to τ_1 is less than unity:

$$(dr_1^*/r_1^*)/(d\tau_1/\tau_1) = (\gamma_1^* + \rho_1)\beta_1^*/[(\gamma_1^* + \rho_1)\beta_1^* + (\gamma_2^* + \rho_2)\beta_2^*] \in (0,1). \tag{C.1}$$

On the other hand, the rate of change in r_{10} is calculated from Eq. (21) for $t = 0$. Logarithmically differentiating that equation, using Eqs. (B.1) and (B.2), and considering that $d\kappa_0/\kappa_0 = 0$, we obtain:

$$dr_{10}/r_{10} = [1/(B_{10} + B_{20} + 1)](B_{10} d\tau_1/\tau_1 - B_{20} d\tau_2/\tau_2).$$

With $d\tau_2/\tau_2 = 0$, we have:

$$(dr_{10}/r_{10})/(d\tau_1/\tau_1) = B_{10}/(B_{10} + B_{20} + 1) \in (0,1). \tag{C.2}$$

A fall in τ_1 lowers r_{10} at a smaller rate than r_1^* as in Fig. 2 if and only if Eq. (C.2) is smaller than Eq. (C.1). This is not always guaranteed. For example, when $r_1^* = r_{10}$ is very low, $\beta_1^* = \beta_{10}$ and hence Eq. (C.1) become very small from Eq. (18), whereas B_{10} and thus Eq. (C.2) become very large from Eqs. (6) and (B.1). In that case, r_{10} is likely to fall at a larger rate than r_1^*.

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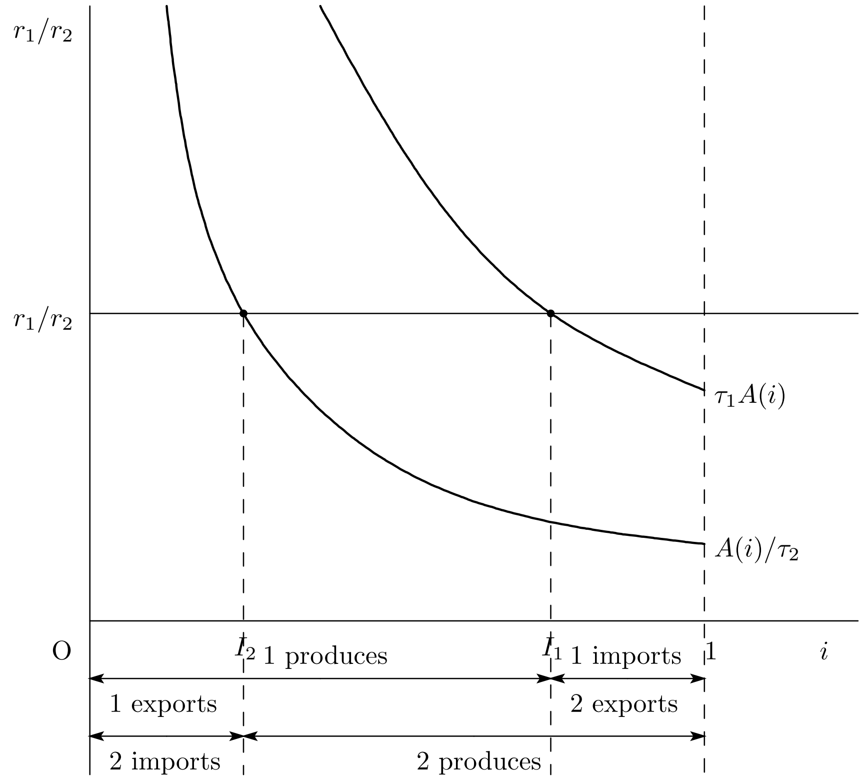

Fig. 1. Distribution of trade status.

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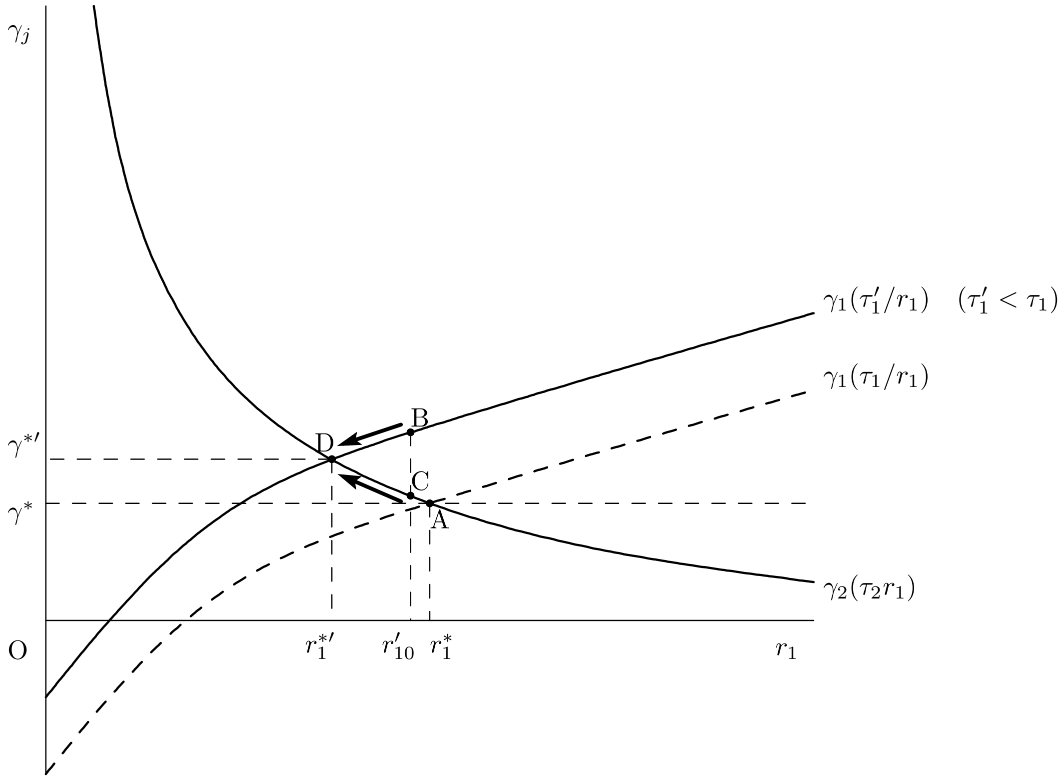

Fig. 2. Growth effects of a fall in the transport cost in country 1: case 1.