SURVEY ON THE LIFE OF BUILDINGS IN JAPAN

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ABSTRACT

This paper discusses a method for the estimation of a building's life time, and presents the results of a field survey, implemented by the authors, on the actual state of a building's life time. The life time estimation method discussed here is based on the reliability theory and life table method. The field survey included wooden residential houses, reinforced concrete apartment houses and office buildings, and estimated their average lives.

１． INTRODUCTION

There may be many ways of estimating a building's life time, such as averaging the lives of demolished buildings, calculating it from material deteriorating speed and so on. As for the average life of a human, the life table is widely used, and the method of making it will be also effective to buildings. This is why we decided to study making a life table for buildings. We think that the ledgers prepared for fixed property taxes are very good as a data source for buildings life tables, because they include details of each building, such as built year, area, structure and so on. The ledgers of demolished ones are also available.

２． BASICS OF RELIABILITY THEORIES

If you want to examine the life of some item, you will get a group of specimens, put them into a testing environment and record the times when failures occur. In an ordinary life test, specimens are put into a test simultaneously, and many of the theories for analyzing life data are effective under this condition. Life data of buildings mentioned here, or generally life table style data, do not satisfy this condition, and we will discuss this later. Now let us take a general view of the basics about the theories of life data analysis. Some definitions of functions are as follows:

$R(t)$: *reliability function*, which represents the remaining rate of an item at time t.
$F(t)$: *unreliability function* or accumulated failure rate.
$f(t)$: *failure probability density function*.

$\lambda(t)$: *failure rate* at time t.

In this paper, $R(t)$ represents the remaining rate of a cohort of buildings, which is an imaginary group of buildings or houses built simultaneousry at a supposed time. These functions have the following relations.

$$F(t) = 1 - R(t)$$
$$f(t) = \frac{dF(t)}{dt} = -\frac{dR(t)}{dt}$$
$$\lambda(t) = \frac{f(t)}{R(t)} = -\frac{1}{R(t)}\frac{dR(t)}{dt}$$
$$\therefore R(t) = \exp\left\{-\int \lambda(t)dt\right\}$$

Sometimes $f(t)$ or $R(t)$ follows some stochastic distribution functions like "Normal distribution", "Log-normal distribution" or "Weibull's distribution". These are as follows:
Normal distribution:

$$f(t) = \frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{(t-\mu)^2}{\sigma^2}\right\}$$
$$R(t) = 1 - \frac{1}{\sqrt{2\pi}}\int_0^t \exp\left\{-\frac{(x-\mu)^2}{\sigma^2}\right\}dx$$

Log-normal distribution:

$$f(t) = \frac{1}{\sqrt{2\pi t}}\exp\left\{-\frac{(\ln t-\mu)^2}{\sigma^2}\right\}$$
$$R(t) = 1 - \frac{1}{\sqrt{2\pi t}}\int_0^t \frac{1}{x}\exp\left\{-\frac{(\ln x-\mu)^2}{\sigma^2}\right\}dx$$

Weibull's distribution (3 parameters)

$$f(t) = \frac{m}{\eta}\left(\frac{t-\delta}{\eta}\right)^{m-1}\exp\left\{-\left(\frac{t-\delta}{\eta}\right)^m\right\}$$
$$R(t) = \exp\left\{-\left(\frac{t-\delta}{\eta}\right)^m\right\}$$

３． LIFE TABLE DATA

Table 1 shows an example of building life data (or life table data) from the ledgers of fixed property tax. This data on wooden residential houses in Japan, contains the number of remaining units at Jan. 1st of 1987 and the number of demolished ones during the year of 1987. The ages shown in the left columns are counted from the newly built time (year) of buildings. The figures shown here are the maximum age of the group, i.e. age "1" means from age "0" up to under age "1", and "2" means "1" to "2". In other words, the group of houses built during the year of 1986 is regarded as having age "1" at the beginning of 1987, that of 1985 having age "2" and so on. These data were collected from 48 major cities from Hokkaido to Kyushu, including Sapporo, Aomori, Yokohama, Niigata, Osaka, Hiroshima, Takamatsu and Fukuoka. But the data from Tokyo and Nagoya relates to only one ward for each city.

Table 1. Wooden Residential Houses Data for Life Table (1987)

Age	Remaining Number	Demolished Number	Age	Remaining Number	Demolished Number	Age	Remaining Number	Demolished Number	Age	Remaining Number	Demolished Number
1	91390	36	26	68346	1983	51	12581	320	76	3522	143
2	95184	52	27	64181	1956	52	63813	1647	77	2475	91
3	100808	191	28	57278	1745	53	12439	405	78	1834	44
4	112899	142	29	52755	1716	54	12363	328	79	2538	91
5	117156	232	30	46765	1538	55	16456	499	80	5637	164
6	124779	245	31	40238	1341	56	14976	453	81	1393	42
7	153306	357	32	122133	3275	57	12998	351	82	1531	36
8	165980	387	33	33646	1170	58	10462	290	83	1599	46
9	171198	519	34	31380	1088	59	11848	362	84	1370	41
10	181484	713	35	28201	951	60	18091	504	85	8720	241
11	180475	894	36	28319	944	61	52251	1254	86	1945	68
12	165224	1016	37	27334	1016	62	9123	270	87	852	24
13	182270	1316	38	24068	955	63	10839	321	88	706	14
14	187542	1467	39	24566	908	64	8487	257	89	1353	48
15	177227	1653	40	22118	883	65	15464	413	90	2294	64
16	175975	1992	41	22987	938	66	11462	304	91	622	19
17	177279	2288	42	57275	2034	67	4588	113	92	681	14
18	170243	2462	43	8239	292	68	3800	120	93	837	31
19	155233	2581	44	7336	273	69	4971	161	94	673	26
20	139896	2637	45	9299	354	70	6304	180	95	4479	127
21	130374	2616	46	10546	365	71	5066	141	96	924	26
22	123468	2639	47	9541	337	72	2946	66	97	1938	34
23	112715	2592	48	9045	256	73	3751	120	98	615	34
24	99258	2472	49	9784	325	74	3797	135	99	1177	19
25	77057	2147	50	16673	496	75	11821	416	100	1469	44

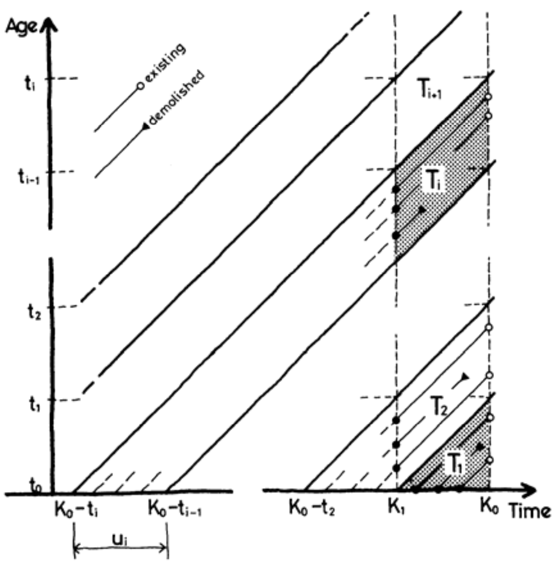

figure 1. Model of Building Life Data

Strictly speaking, units in a same year old group do not have the same age. For example, an unit built on Jan. 1st of 1980 and another built on Dec. 31 of 1980 are grouped in the same 7 year old group here, but their ages differ by almost one year from each other. Figure 1. shows a model of life table data. A sloping line from bottom left to upper right represents a unit built at a point of time which the horizontal axis represents. A vertical line drawn at the point K_1 is the time of observation. The sloped lines (or units) crossing this vertical line are remaining or "living" units at time K_1. K_0 is the end of the period for counting demolished or "dead" units. The *i* year old group's number of demolished units is represented by cut lines included in the dotted area T_i. The vertical axis represents age while the horizontal does time. The projection of a sloped line onto the vertical axis means the aging of a unit. Then the length of a cut sloped line is proportional to the age of a demolished unit. It will be clear from this model that each unit demolished in T_i has a different beginning and end of life, and then has a different age. The scale in the statistical method for life estimation should be "age", then this type of data should be arranged according to the age axis, not to tthe time axis.

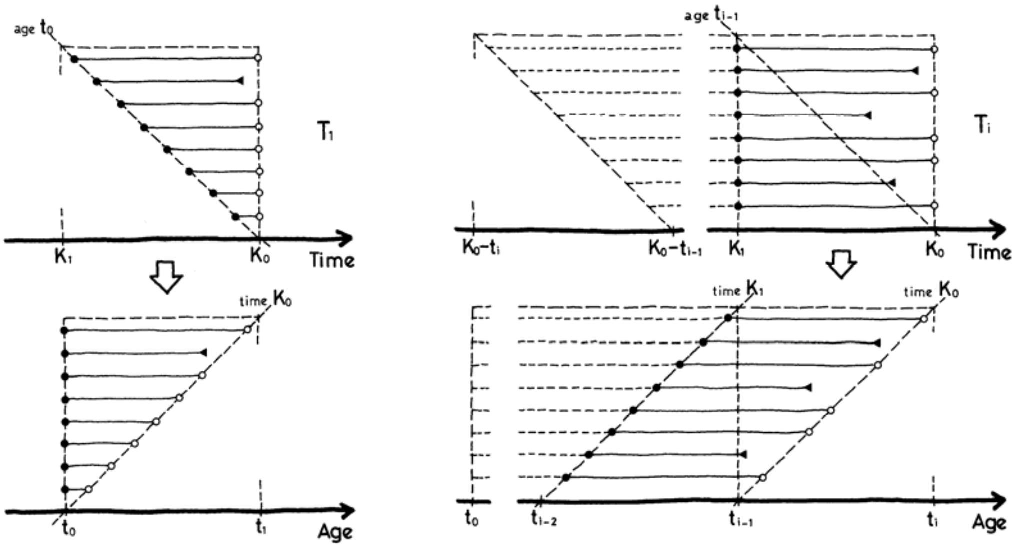

figure 2. Transformation from the time axis plot to the age axis plot

Figure 2. shows the transformation of building life data expression from the time axis to the age axis. Here, a unit is shown as a horizontal line, while it was a sloped line in figure 1. In the time axis, the end point of each line is aligned vertically, i.e. is simultaneous, but in the age axis the beginning point is vertical.

4． LIFE TIME ESTIMATION METHOD OF BUILDINGS

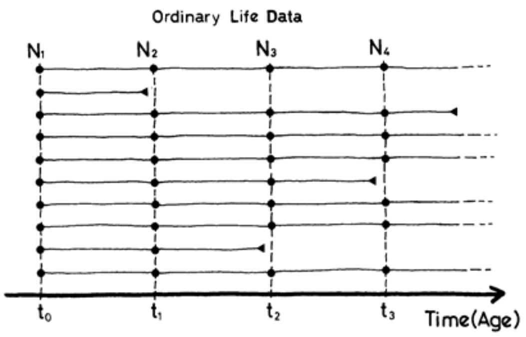

figure 3a. Model of Ordinary Life Test

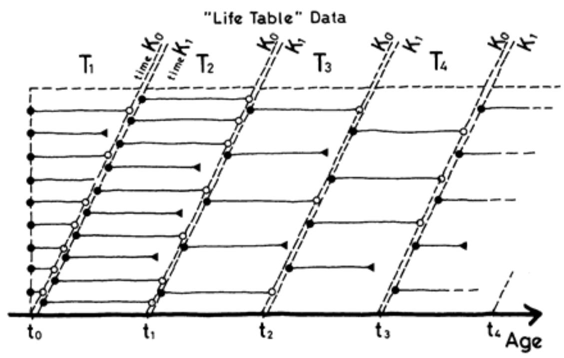

figure 3b. Model of Building Life Data

Figure 3a shows a model for ordinary data of life test, and figure 3b the life table data. In figure 3a, the remaining number at time t_1 of the specimens is N_2 , while N_1 is the remaining number at time t_0 and this is the total of specimens. The remaining probability at time t_1, exactly of the period from time t_0 up to t_1 and let it be $\Pr\{t_1|t_0\}$, is

$$\Pr\{t_1|t_0\} = \frac{N_2}{N_1} = \frac{N_1 - d_1}{N_1}$$

Generally, the remaining rate from time t_{i-1} up to t_i , $\Pr\{t_i|t_{i-1}\}$, is

$$\Pr\{t_i|t_{i-1}\} = \frac{N_{i+1}}{N_i} = \frac{N_i - d_i}{N_i}$$

Then the remaining probability from the beginning of test t_0 up to t_i ,$\Pr\{t_i|t_0\}$ is

$$\begin{aligned}\Pr\{t_i|t_0\} &= \Pr\{t_1|t_0\}\Pr\{t_2|t_1\}\cdots\Pr\{t_{i-2}|t_{i-1}\}\Pr\{t_i|t_{i-1}\}\\ &= \frac{N_1 - d_1}{N_1}\cdot\frac{N_2 - d_2}{N_2}\cdots\frac{N_{i-1} - d_{i-1}}{N_{i-1}}\cdot\frac{N_i - d_i}{N_i}\\ &= \frac{N_2}{N_1}\cdot\frac{N_3}{N_2}\cdots\frac{N_i}{N_{i-1}}\cdot\frac{N_{i+1}}{N_i}\\ &= \frac{N_{i+1}}{N_1}\end{aligned}$$

In the life table model shown in figure 1, let the remaining number of any newly built year group at time K_1 be N_i, and the number of units newly built between time K_1 and K_0 be N_1, and let the demolished number of any group between K_1 and K_0 be d_i. N_i and d_i are obtained as field data, as for this paper, from the ledgers. As shown in figure 3b, the vertical line at K_1 in figure 1 is expressed as a sloped line along the age axis. The remaining probability through the area T_1 in figure 3b is that from age (not time) t_0 up to time (not age) K_0, and can be expressed as $\Pr\{K_0|t_0\}$. As for T_i, the remaining probability through it is that of from time K_1 up to time K_0, and expressed as $\Pr\{K_0|K_1\}$. Using remaining numbers and demolished numbers, each can be expressed as follows;

$$\Pr\{K_0|t_0\} \cong \frac{N_1 - d_1}{N_1}$$

$$\Pr\{K_0|K_1\} \cong \frac{N_i - d_i}{N_i}$$

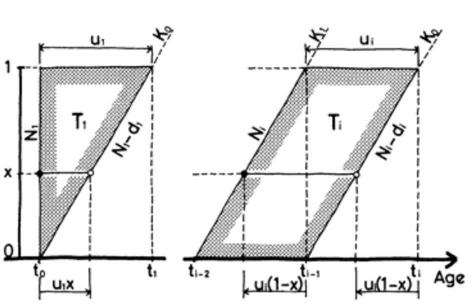

figure 4. Estimation of remaining probability

Assume that a remaining function $R(t)$ exists. Figure 4 contains two areas T_1 and T_i of figure 3b. In T_1, the remaining probability of a unit (shown as a line of height x) from age t_0 (the vertical line on the left side) to time K_0 (the sloped line on the right side) is $R(u_1 x)$. T_1 is a triangle and suppose that the number of units in it is enough, the remaining probability through T_1 can be expressed as

$$\Pr\{K_0|t_0\} = \int_0^1 R(u_1 x)dx$$

and the right side of the equation is approximated using the trapezoidal rule as shown

$$\int_0^1 R(u_1 x)dx \cong \frac{1}{2}\left\{R(0)+R(u_1\cdot 1)\right\}$$
$$=\frac{1}{2}\left\{1+R(t_1)\right\} \quad (\because R(0)=1)$$

Then

$$\frac{1}{2}\left\{1+R(t_1)\right\} \cong \frac{N_1-d_1}{N_1}$$
$$R(t_1) \cong \frac{N_1-2d_1}{N_1}$$

In T_i $(i \geq 2)$, the remaining probability of a unit (at height *x*) from time K_0 (the sloped line on the left side) to time K_0 (the sloped line on the right side) is

$$\frac{R(t_i-u_i(1-x))}{R(t_{i-1}-u_i(1-x))}$$

because this unit has lived until time $t_{i-1}-u_i(1-x)$ and lives up to $t_i-u_i(1-x)$.

Then

$$\Pr\left\{K_0|K_1\right\} \cong \int_0^1 \frac{R(t_i-u_i(1-x))}{R(t_{i-1}-u_i(1-x))}dx$$

Suppose that the period is small enough and the failure rate $\lambda(t)$ can be regarded as constant $(\lambda(t)\cong\lambda)$, then *R*(*t*) is approximated by an exponential distribution, i.e. $R(t)\cong \exp(-\lambda t)$. Then

$$\frac{R(t_i-u_i(1-x))}{R(t_{i-1}-u_i(1-x))} \cong \frac{\exp\left[-\lambda\left\{t_i-u_i(1-x)\right\}\right]}{\exp\left[-\lambda\left\{t_{i-1}-u_i(1-x)\right\}\right]}$$
$$=\frac{\exp(-\lambda t_i)}{\exp(-\lambda t_{i-1})}$$
$$\cong \frac{R(t_i)}{R(t_{i-1})}$$

So this equation does not contain *x*,

$$\Pr\left\{K_0|K_1\right\} \cong \frac{R(t_i)}{R(t_{i-1})}$$

Then

$$\frac{R(t_i)}{R(t_{i-1})} \cong \frac{N_i - d_i}{N_i}$$

$$R(t_i) \cong R(t_{i-1}) \cdot \frac{N_i - d_i}{N_i} \qquad (i \geq 2)$$

Let R_i be the estimation of $R(t_i)$,

$$R_1 = \frac{N_1 - 2d_1}{N_1}$$

$$R_i = R_1 \cdot \prod_{x=2}^{i} \frac{N_x - d_x}{N_x} \qquad (i \geq 2)$$

５． LIFE TIME ESTIMATION OF JAPANESE HOUSES AND BUILDINGS

The following figures show the results of the remaining rate analyses of Japanese houses and office buildings using above-mentioned equations. Figure 5 is for wooden residential houses, and figure 6 is for reinforced concrete (r.c.) apartment houses, this curve is extended by fitting parametric distribution (Weibull's distribution) by the least square method. Figure 7 is for office buildings of r.c. structure.

６． CONCLUSION

We proposed a life time estimation method using data like the life table. If average life can be defined as the expected years for a half of the cohort demolish, that of Japanese wooden residential houses is estimated at 38.2 years in 1987 and r.c. offices at 34.8. As for r.c. apartment houses, though the data on old buildings is not sufficient, the extrapolated curve shows that their average life would be 50.6 years.

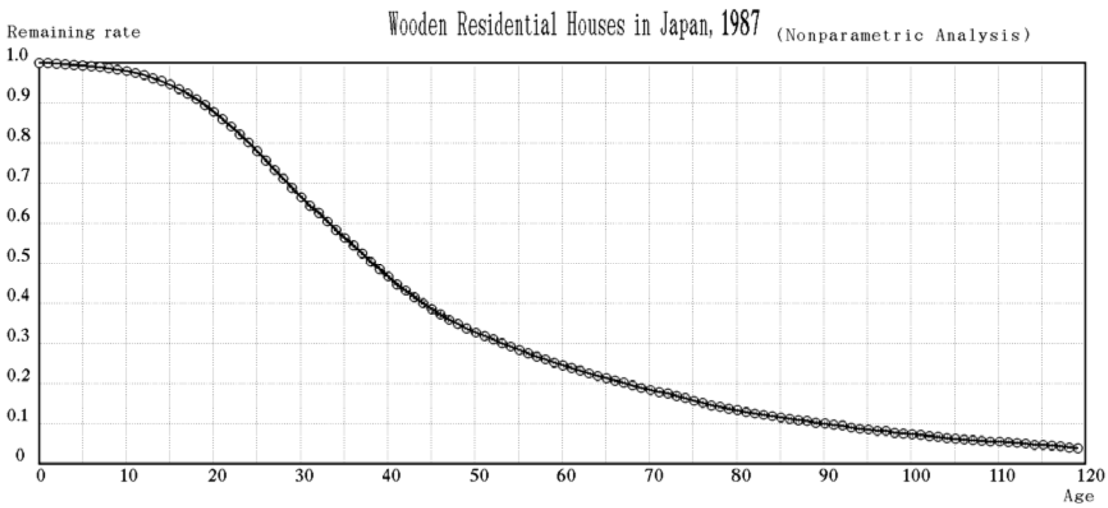

figure 5. Remaining Rate of Wooden Residential House

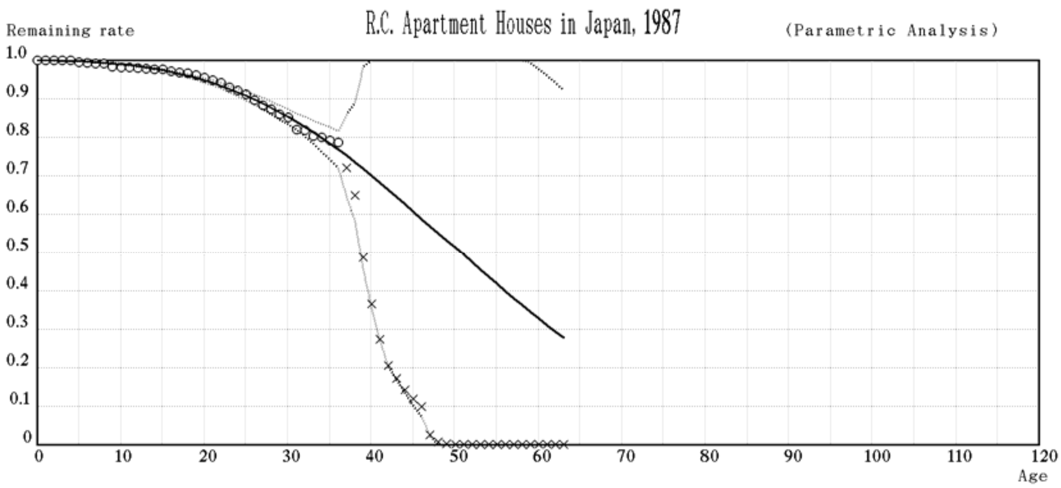

figure 6. Remainig Rate of R.C. Apartment Houses

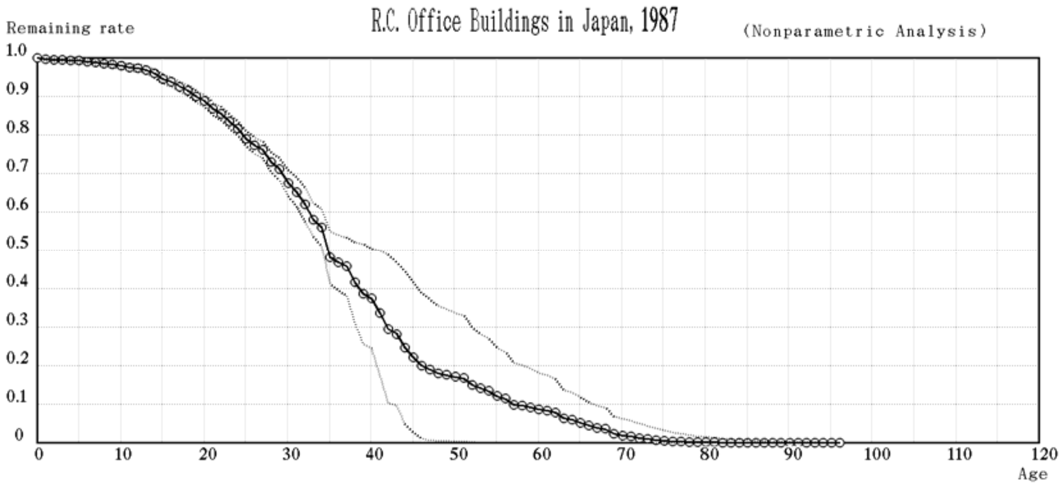

figure 7. Remaining Rate of R.C. Office Buildings

７． REFERENCES

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